We responded to the comments of Reviewer #1 on January 25. We thought deeply about the comments and the related issues these days. Now, we consider that the perspective of parameter extensions within the Budyko framework will provide a clearer answer to the referee comments, and highlight the contributions of our study. We appreciate the reviewer's comments, which prompted these further considerations and clarifications.

The original non-parametric Budyko equation expresses the evapotranspiration ratio as a function of the dryness index at the mean annual catchment scale (Budyko, 1974, 1958) (Eq. (1)). Subsequent studies introduced one or more model parameters into the non-parametric Budyko equation, leading to the development of a series of parametric equations, such as the Budyko-MCY equation (Mezentsev, 1955; Choudhury, 1999; Yang et al., 2008) (Eq. (2)).

$$\frac{E}{P} = \sqrt{\frac{E_P}{P}\tanh\left(\frac{E_P}{P}\right)^{-1}\left[1-\exp\left(-\frac{E_P}{P}\right)\right]} \qquad (1)$$

$$\frac{E}{P} = \left[1+\left(\frac{E_P}{P}\right)^{-n}\right]^{-1/n} \qquad (2)$$

To date, there has been no unified specification regarding the estimation of potential evapotranspiration (E_P) in applications of the Budyko framework (Zhang and Brutsaert, 2021; Pimentel et al., 2023). A variety of estimation methods have therefore been adopted, among which the Penman method is one commonly used approach.

$$E_P = E_{P-Pen} \qquad (3)$$

This study proposes the concept of Budyko E_P (E_{P-B}), which is considered a form of E_P that is specially suitable for the Budyko framework. Corresponding to Eqs. (1) and (2), two types of Budyko E_P are defined, namely the reference Budyko E_P (E_{P-Bref}) and the adjustable Budyko E_P (E_{P-Badj}). The Budyko E_P still embodies the characteristic of maximum evapotranspiration and is therefore related to and comparable with meteorological E_P. Because of differences in estimation methods and regional conditions, their values are not equal, but a correlation function can be established between them. Based on the current analysis results, their correlation may be positive (MOPEX catchments) or negative (CLP catchments). Considering general applicability and simplicity, the relationship can be expressed in a linear form, which we refer to as the conversion function in this study. If the Penman E_P is used as an input variable in the Budyko model, it should first be adjusted by the corresponding conversion function.

$$E_P = aE_{P-Pen} + b \qquad (4)$$

By combining Eq. (4) with Eq. (1), the coefficient a and the intercept b can be calibrated using catchment-scale P-Q observations together with meteorological data. In this case, Eq. (4) is equivalent to the conversion function between E_{P-Bref} and E_{P-Pen}. Similarly, by combining Eq. (4) with Eq. (2), and calibrating n, a, and b using catchment-scale P-Q observations and meteorological data, Eq. (4) then represents the conversion function between E_{P-Badj} and E_{P-Pen}.

We next compare the performance of Eq. (3) and Eq. (4) in E estimation, respectively. From the perspective of parameter extension, using E_P from Eq. (3) results in a zero-parameter Budyko model corresponding to Eq. (1) and a one-parameter model (with parameter n) corresponding to Eq. (2). When using E_P from Eq. (4), the Budyko models become two-parameter (a, b) and three-parameter (n, a, b) formulations, respectively. This leads to four distinct parameter schemes under the Budyko framework.

For the MOPEX catchments used in this study, a comparison of the performance of E estimation under the four parameter schemes is shown in Fig. R1. The results indicate that, the mean absolute error (MAE) of E estimation decreases from 163.32 mm yr^{-1} under the zero-parameter scheme to 90.30 mm yr^{-1} under the one-parameter scheme. In contrast, the two-parameter and three-parameter schemes exhibit a high degree of consistency, with MAE values of 60.41 and 59.05 mm yr^{-1}, respectively. Figure R2 further presents the distribution of catchment water balance data points in the Budyko space under the four schemes. The deviation of the data points from the Budyko curve is quantified by the mean absolute difference between the observed evapotranspiration ratio and the corresponding value on the Budyko curve at the same dryness index (Ibrahim et al., 2025; Fang et al., 2016). The results show that data points are relatively scattered under the zero-parameter and one-parameter schemes, whereas more closely clustered around the Budyko curve under the two-parameter and three-parameter schemes, with noticeably fewer outliers.

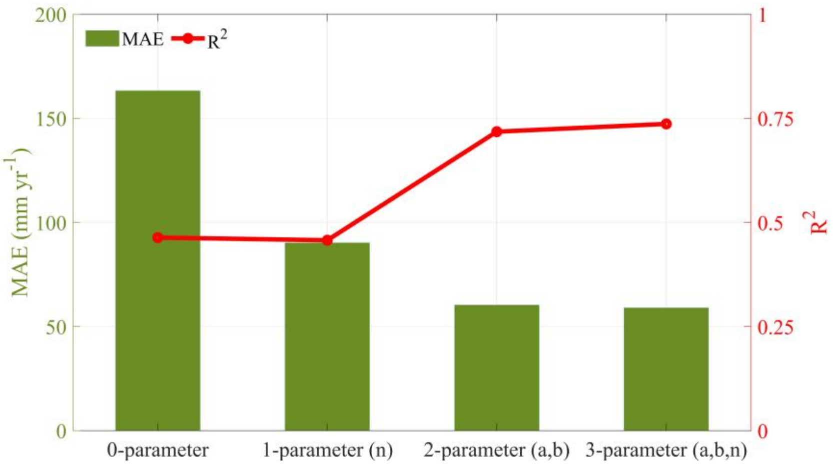

Figure R1. Comparison of mean absolute error (MAE) and coefficient of determination (R^2) of E estimation under four parameter schemes for the Budyko model: (a) 0-parameter: Eq. (1) + Eq. (3); (b) 1-parameter (n): Eq. (2) + Eq. (3); (c) 2-parameter (a, b): Eq. (1) + Eq. (4); (d) 3-parameter (a, b, n): Eq. (2) + Eq. (4).

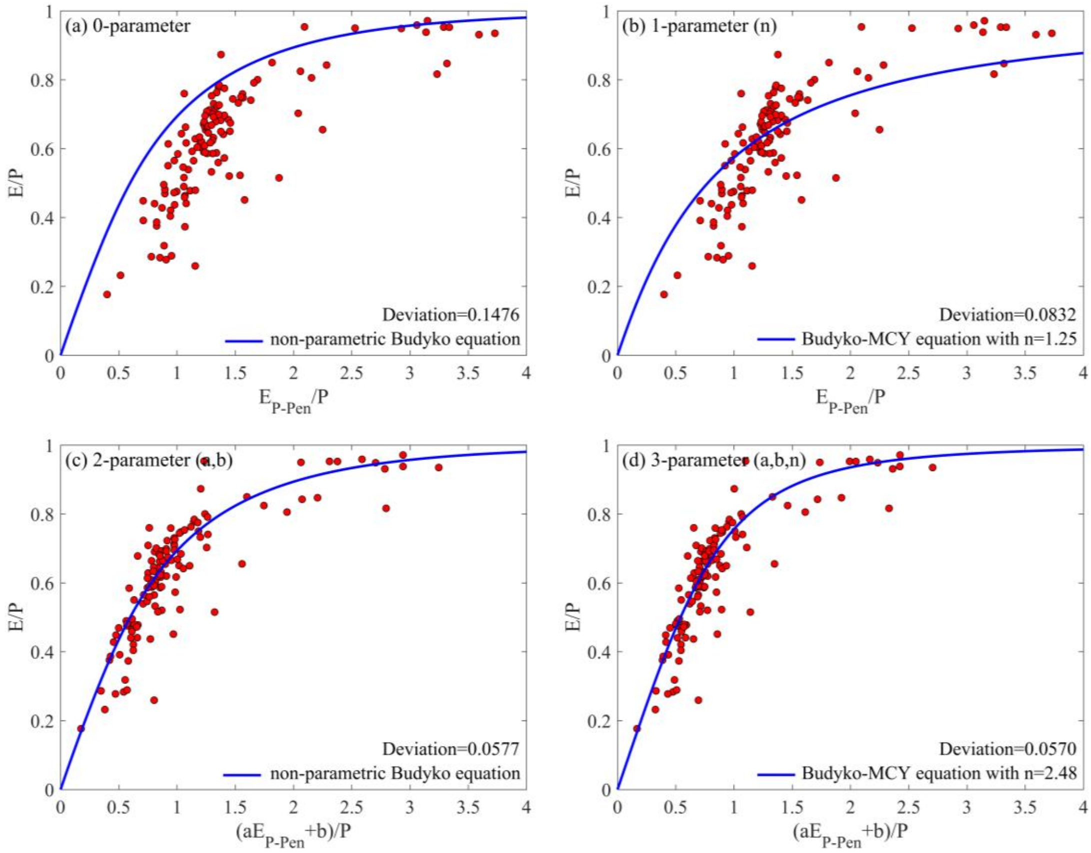

Figure R2. Comparison of the distribution of water balance data points in the Budyko space under four parameter schemes for the Budyko model: (a) 0-parameter: Eq. (1) + Eq. (3); (b) 1-parameter (n): Eq. (2) + Eq. (3); (c) 2-parameter (a, b): Eq. (1) + Eq. (4); (d) 3-parameter (a, b, n): Eq. (2) + Eq. (4).

Relative to the zero-parameter scheme, the reduction in MAE is more pronounced under the two-parameter scheme than under the one-parameter scheme. This indicates that, for the study catchments, introducing parameters a and b yields greater improvement in E estimation than introducing parameter n. Furthermore, although the three-parameter scheme yields a lower MAE than the two-parameter scheme, the difference is relatively small. Therefore, with a reasonable E_P, model performance becomes less sensitive to the structural form of the Budyko equation. To some extent, this suggests that E_P conversion is of greater importance at both theoretical and practical levels.

In the Budyko-MCY equation, the parameter n regulates the overall relationship between the evapotranspiration ratio and the dryness index under a given set of data points. It adjusts the vertical position of the curve in the Budyko space, allowing it to better fit the distribution of the data points. As n increases, the Budyko-MCY curve moves upward. In particular, when n=1.9, the curve is broadly consistent with that of the original non-parametric Budyko equation (Roderick et al., 2014; Yang et al., 2019). By contrast, the conversion function modifies E_P and thereby adjusts the horizontal position of the data points in the Budyko space. Notably, when a=1 and b=0, Eq. (4) becomes identical to Eq. (3), implying that the zero-parameter and one-parameter schemes are special cases of the two-parameter and three-parameter schemes, respectively, which is unlikely to occur under real-world conditions.

In general, without introducing additional information, appropriately increasing the number of parameters enhances model flexibility and can, to some extent, improve model performance (Schoups et al., 2008; Wood et al., 2017). This characteristic is intuitively reflected in the extension from the non-parametric Budyko equation to the parametric one. Therefore, the advantage of the conversion function approach arises not only from the fact that the converted E_P is numerically and conceptually better aligned with the Budyko framework, but also from the increased degrees of freedom, which strengthen the model's ability to constrain errors.

We will further elaborate on the understanding of parameter extension within the Budyko framework in the revised manuscript, and change the title of our manuscript to "Estimation of potential evapotranspiration from hydrological observation and the conversion function in the Budyko framework".

Thank you again for your time and careful review.

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