Content

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- **Movie S1:** Evolution of centerline ice-sheet and bed profiles demonstrating the occurrence of R-tipping.
- **Movie S2:** Evolution of centerline ice-sheet and bed profiles demonstrating the occurrence of self-sustaining oscillations between collapsed and advanced ice-sheet states.

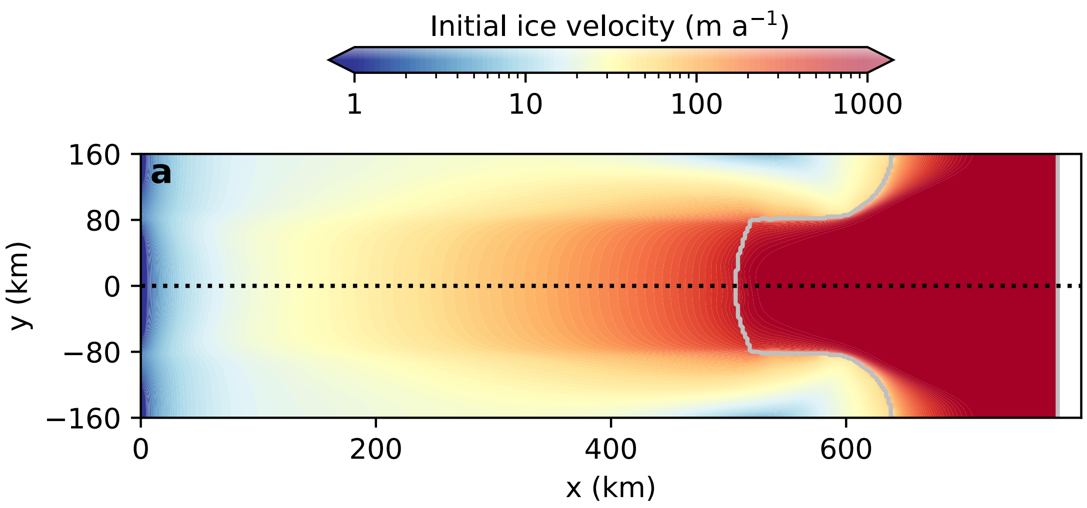

Figure S1. Initial field of ice-flow speed for the case of a deep bed depression ($D_{\mathrm{BD}} = 900$ m; see Fig. 1 of main manuscript).

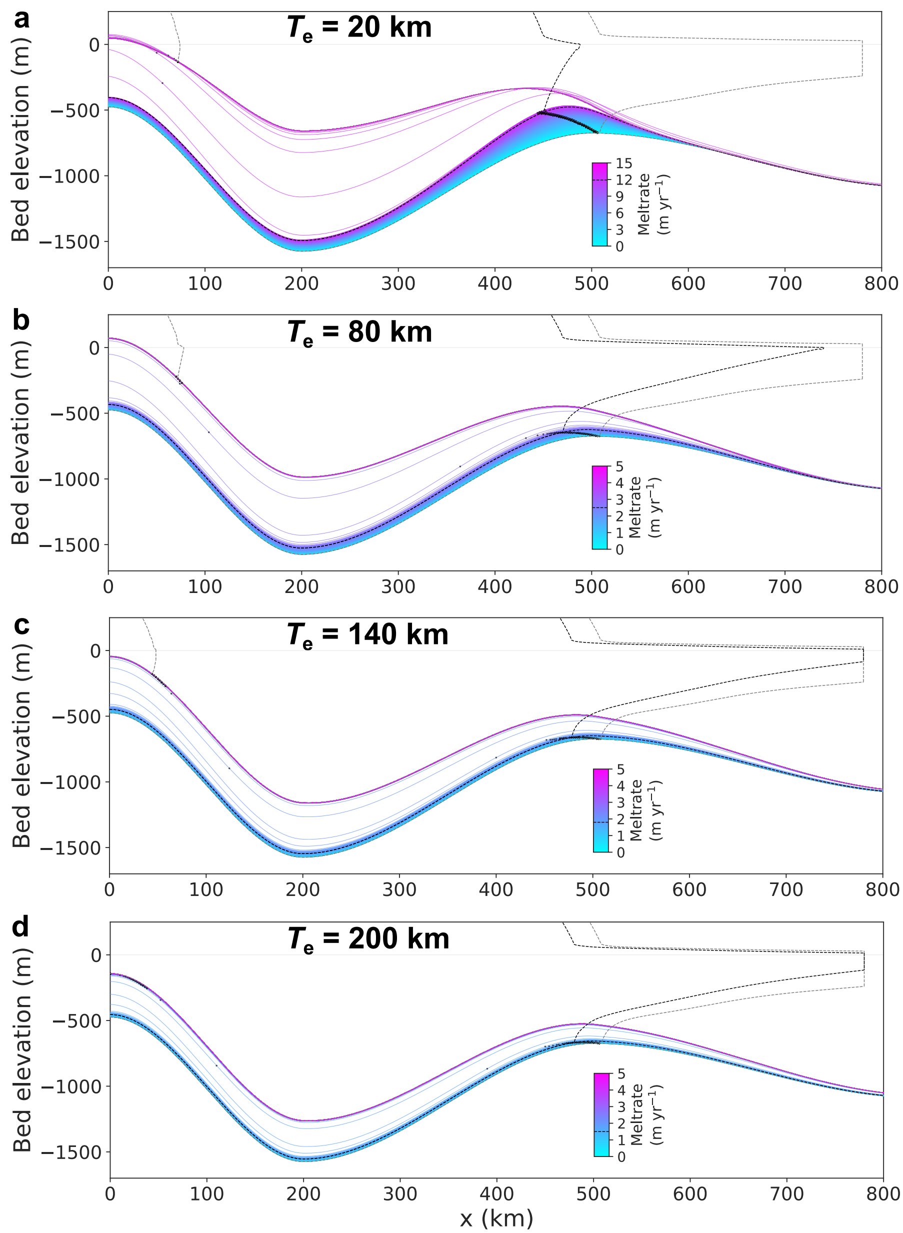

Figure S2. Comparison of the evolution of the centerline bed topography during slow forcing ramp-up for the four applied lithosphere thicknesses, T_e (fixed upper-mantle viscosity $\eta = 5 \cdot 10^{19}$ Pa s). Profile snapshots have an interval of 1,000 yr with GL positions marked by small circles. Dashed contours represent ice-sheet and bed profiles for initial and final equilibrium state (grey) and the last stable state before collapse (black). The B-tipping threshold is marked by a dashed line in the colorbars. Note the increase in the locality of the be d uplift with decreasing lithosphere thickness, i.e., from bottom to top.

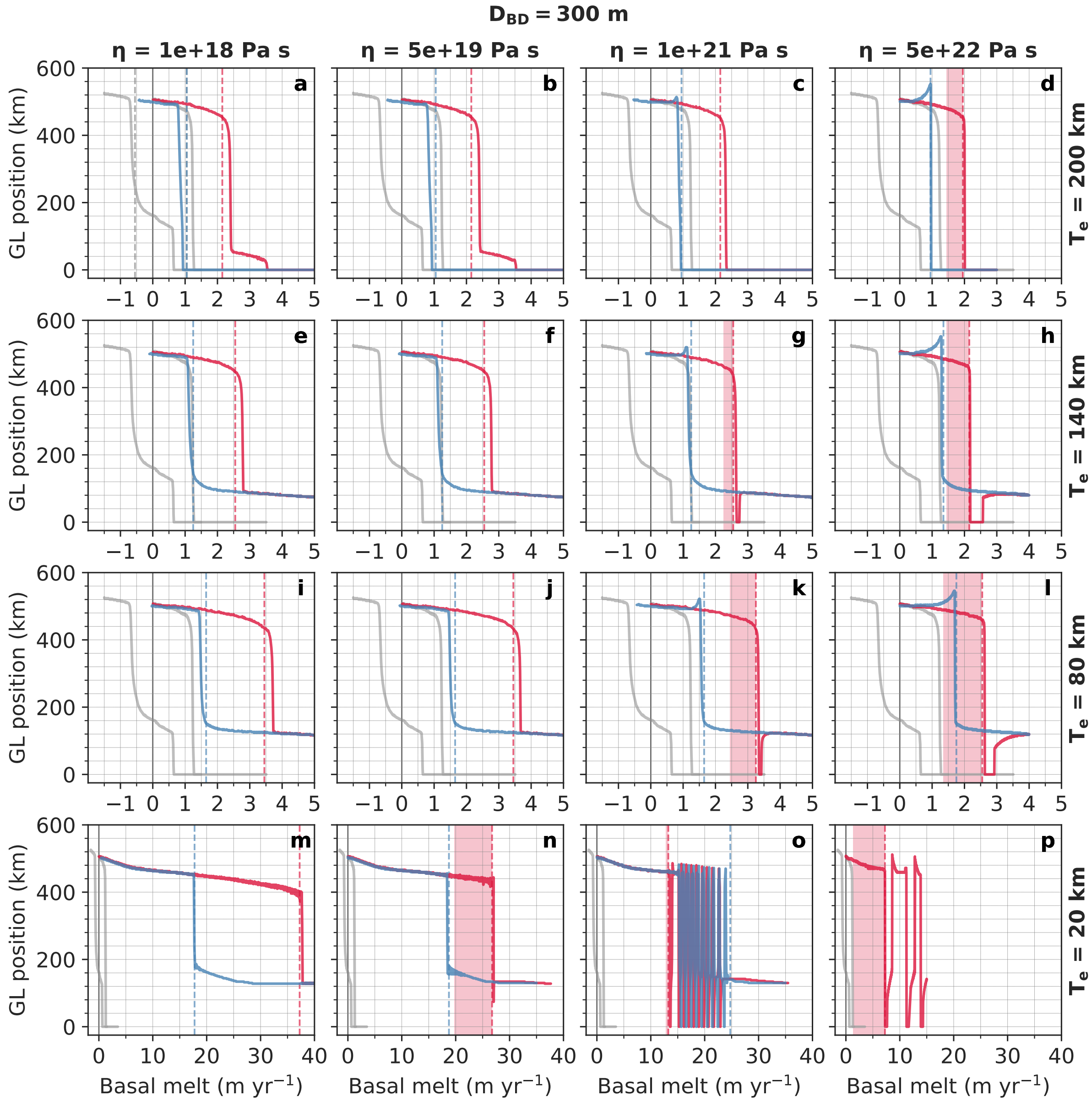

Figure S3. Hysteresis curves of the centerline GL position for the case of a shallow initial bed depression ($D_{\mathrm{BD}} = 300$ m), analogous to Fig. 5. Note that for the model configuration exhibiting self-sustaining oscillations for $\eta = 5 \cdot 10^{22}$ Pa s (panel p) we cannot provide the full forward and backward branches, due to the very slow forcing ramp-up/down (spannning several $10,000000$ model years) and the associated very long duration of the simulations.

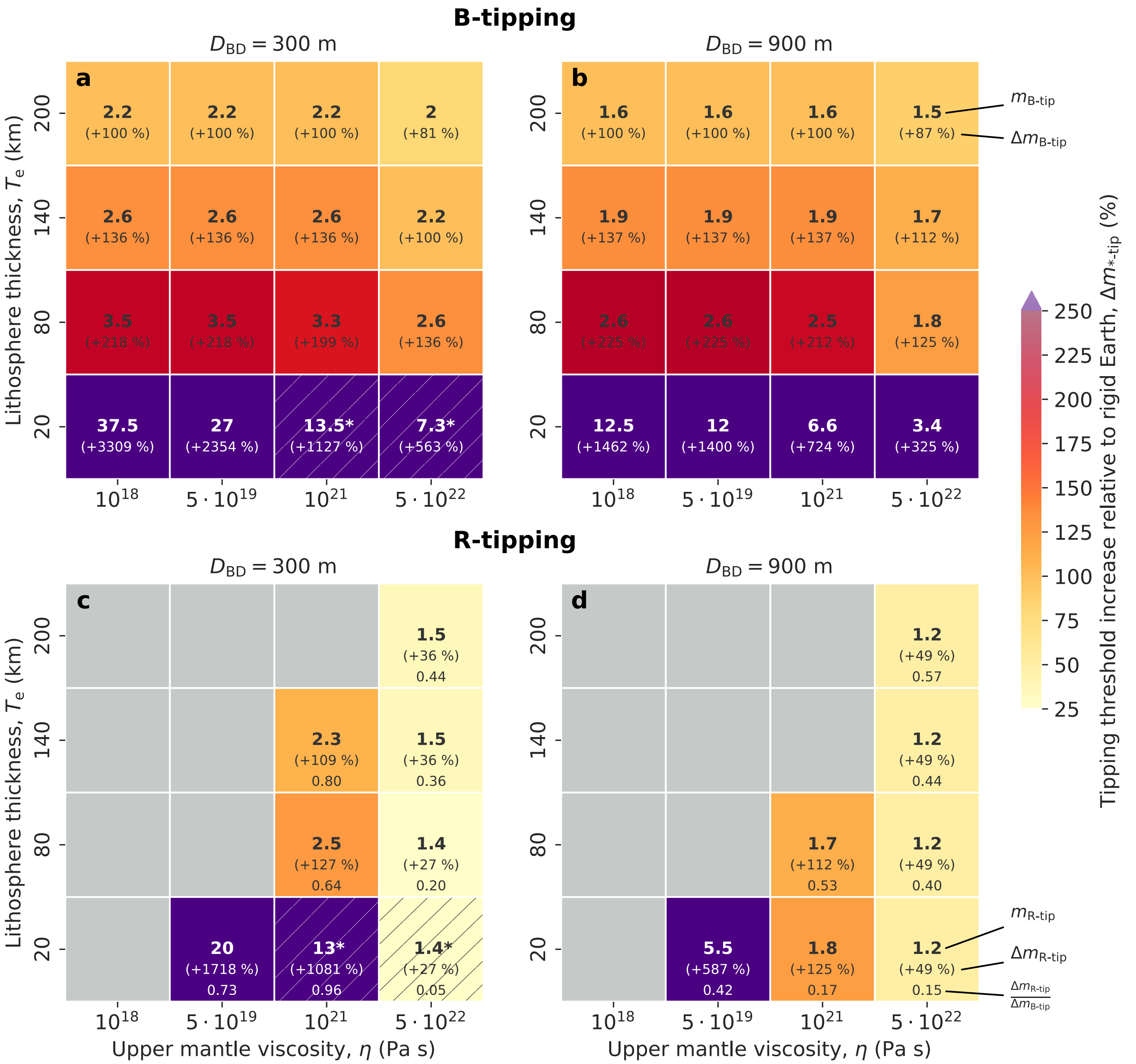

Figure S4. Comparison of heat maps for **(a)**, **(c)** a shallow and **(b)**, **(d)** a deep bed depression, i.e., a shallow and a steep retrograde bed slope, respectively. Colors indicate the shift in the critical meltrate threshold relative to the fixed-bed case for B-tipping (panels a and b) and potential R-tipping (panels c and d). The absolute values of the respective tipping thresholds, $m_{\mathrm{B-tip}}$ and $m_{\mathrm{R-tip}}$, are given by bold numbers and the values of tipping threshold increase with respect to the fixed-bed case, $\Delta m_{\mathrm{B-tip}}$ and $\Delta m_{\mathrm{R-tip}}$, are given in brackets (see annotations in panels (b) and (d) and Eqs. 1 and 2). The third row in each tile of panels c and d states the ratio between $\Delta m_{\mathrm{R-tip}}$ and $\Delta m_{\mathrm{B-tip}}$. Hatched squares denote tipping into self-sustaining oscillations (Sect. 3.6, Fig. S3o,p and Movie S2; instead of direct collapse). The reference tipping thresholds of the fixed-bed case are $m_{\mathrm{B-tip,fixed}} = 1.1\ \mathrm{m\,yr^{-1}}$ and $0.8\ \mathrm{m\,yr^{-1}}$ for the shallow and the steep slope, respectively. Note the extended range of the x axis (basal meltrate) for $T_e = 20$ km due to the occurrence of comparatively large B-tipping thresholds.

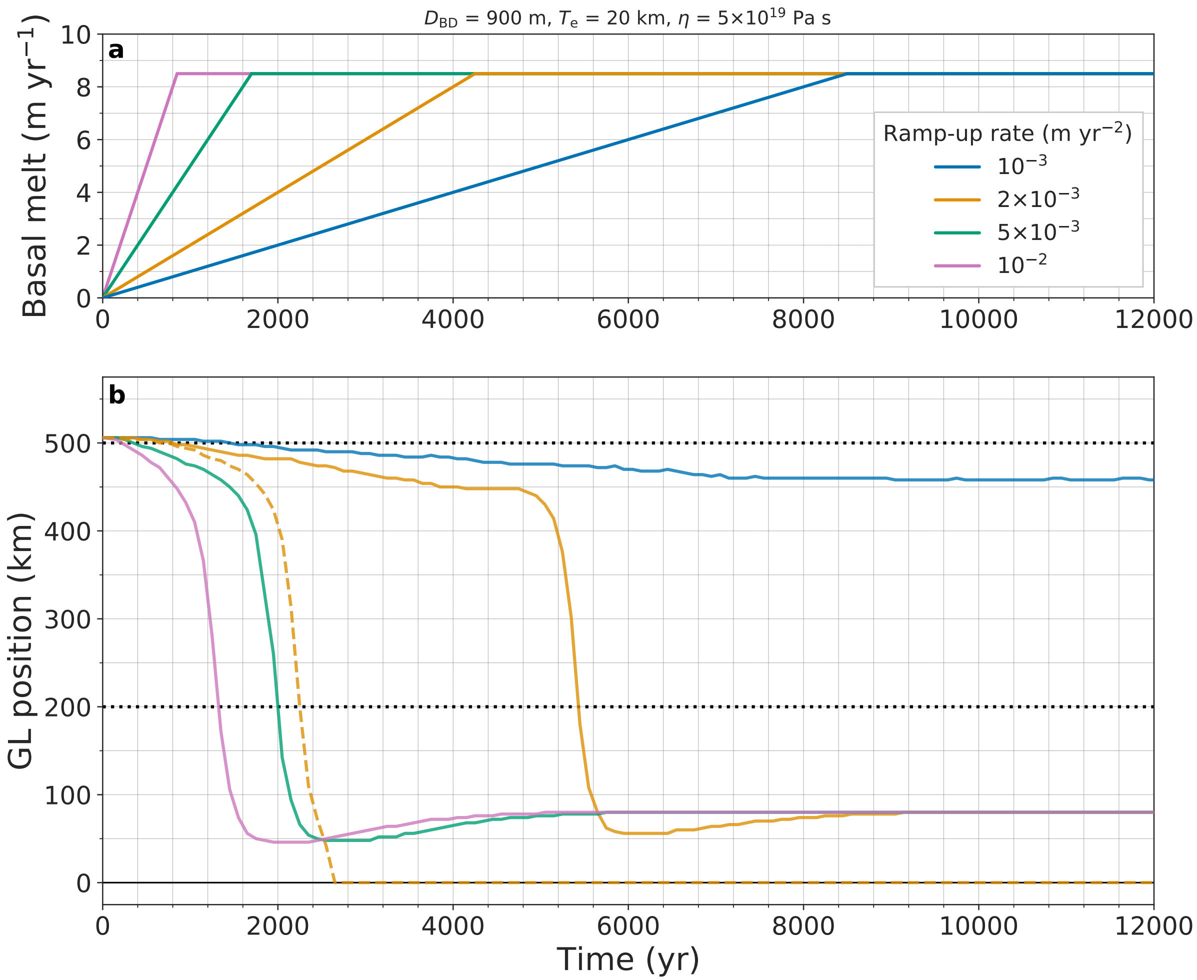

Figure S5. Timeseries of **(a)** different ramp-up rates of applied sub-ice-shelf melt forcing and **(b)** associated response of the centerline GL position, highlighting the rate-dependency of a possible ice-sheet destabilization. Dashed line represents the case of a fixed bed. Dotted horizontal lines mark the range of the retrograde slope section. Figure analogous to Fig. 8.

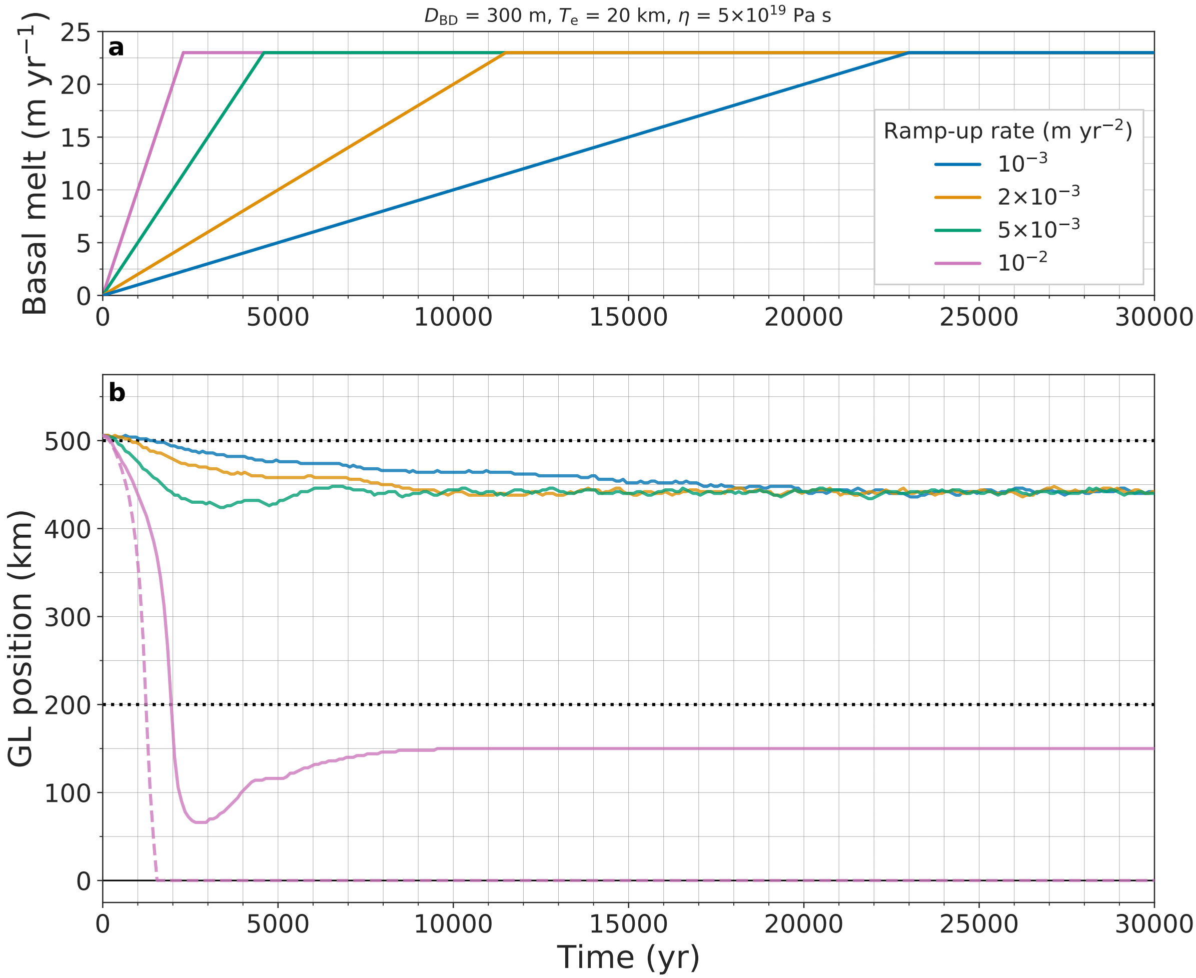

Figure S6. Timeseries analogous to Figs. 8, S5 and S7.

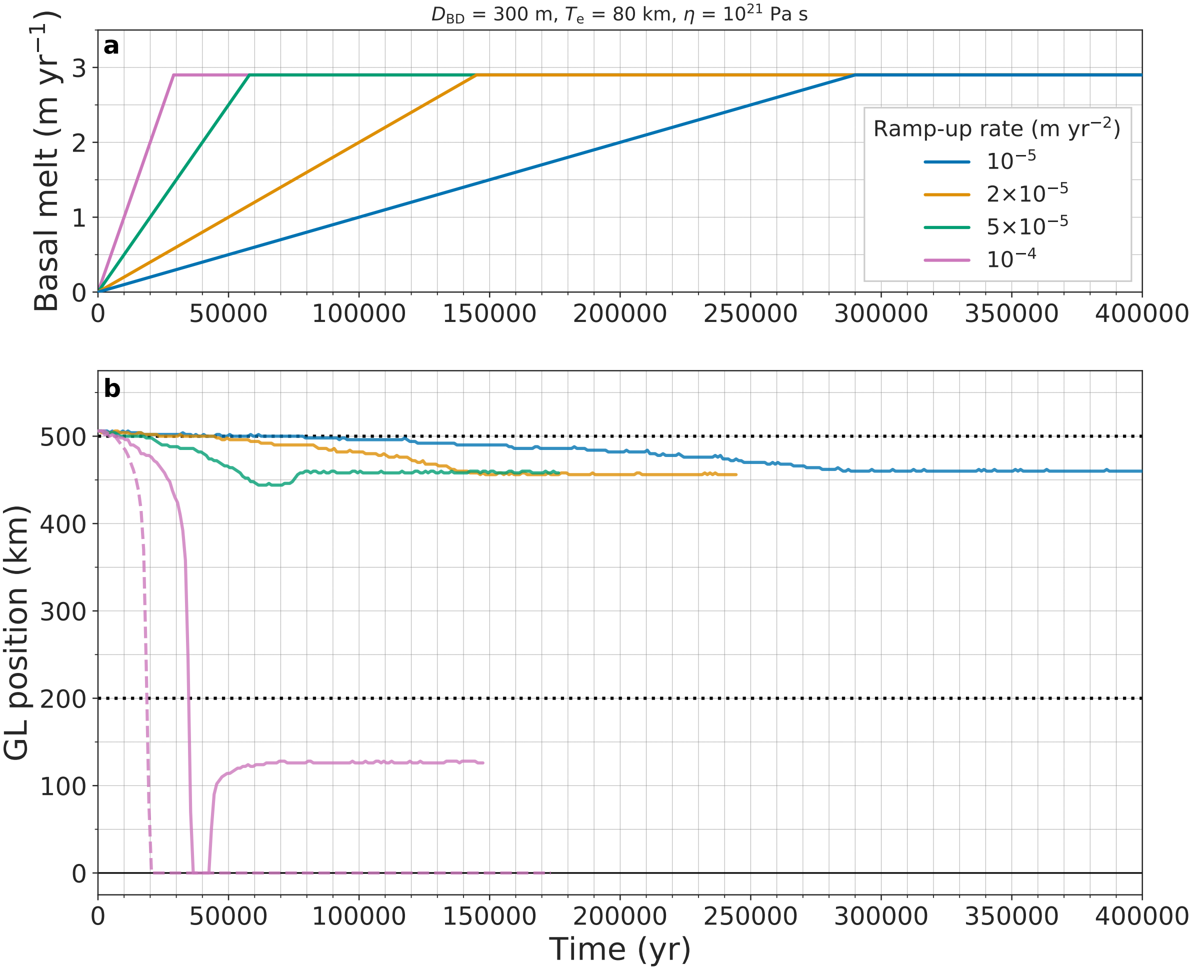

Figure S7. Timeseries analogous to Figs. 8, S5 and S6.

Movie S1. Evolution of **(a)** applied forcing ramp-up of basal ice-shelf meltrate and **(b)** response of ice-sheet and bed profiles along the centerline of the setup, demonstrating the occurrence of R-tipping. All simulations start from the same initial equilibrium state at a meltrate of zero, which is ramped up to a value of 2.9 $\mathrm{m\,yr^{-1}}$ and thus slightly below the B-tipping threshold of 3.2 $\mathrm{m\,yr^{-1}}$ (marked black). The ramp-up rate is either $10^{-4}\ \mathrm{m\,yr^{-2}}$ (orange) or $5 \cdot 10^{-5}\ \mathrm{m\,yr^{-2}}$ (halved rate, green). The associated response of ice sheet and bed is shown in panel b in the respective colors. Dashed profiles indicate the case of a fixed bed (no bed deformation, only shown for the fast forcing rate), grey profiles show the initial state. While the ice sheet stabilizes in the case of the slower (halved) forcing rate it collapses in the case of the faster forcing rate, indicating R-tipping. Relevant parameters of this model configuration: $T_e = 80\ \mathrm{km}$, $\eta = 10^{21}\ \mathrm{Pa\,s}$, $D_{\mathrm{BD}} = 300\ \mathrm{m}$.

Movie S2. (a) Prescribed constant basal ice-shelf meltrate and **(b)** response of ice-sheet and bed profiles along the centerline of the setup, demonstrating the occurrence self-sustaining oscillations. The simulation starts from a branched-off state from the slow (quasi-equilibrium) forcing ramp-up at a meltrate of $13.5\ \mathrm{m\,yr^{-1}}$ which is held constant over the entire duration of the experiment (see red dashed line with triangle in Fig. 3 for a sketch of the branch-off). The resulting self-sustaining cyclic evolution of ice-sheet and bed profiles is shown in black, the initial state is highlighted in grey. Relevant parameters of this model configuration: $T_e = 20\ \mathrm{km}$, $\eta = 10^{21}\ \mathrm{Pa\,s}$, $D_{\mathrm{BD}} = 300\ \mathrm{m}$.