Response to Referee#2

We would like to thank Referee#2 for the careful reading of our manuscript, for the helpful comments and suggestions as well as for the constructive criticism. We are delighted by the Referee's overall positive assessment of our study and will address the points made by the Referee in a revised version of the manuscript. Please find below the *Referee's comments in italics* and our response in blue.

Sincerely,
Johannes Feldmann et al.

Reviewer report egusphere-2025-5859

Rate-induced tipping of ice sheets due to visco-elastic Earth response under idealized conditions

January 31, 2026

The present work studies how the interaction between ice-sheet dynamics and the visco-elastic response of the solid Earth affects the tipping dynamics of marine-terminating ice sheets in idealised geometries. A bifurcation point is found via quasi-equilibrium forcing in ocean melt rates. If the forcing rates are increased, a glacier collapse occurs for melt rates below the bifurcation points for certain parameter combinations, thus illustrating R-tipping behaviour. The last message of the study exhibits self-sustained oscillations driven solely by the non-linear interaction between ice flow and solid Earth.

Overall, I find that the message of the paper is successfully conveyed. Idealised geometries are fundamental to isolate mechanism and understand the underlying physical processes. Results presented are clear and well structured, providing valuable conclusions for the scientific community.

Nevertheless, there are a number of major remarks that I would like the authors to address and further elaborate on the manuscript. Particularly, the work falls short in certain aspects that I have listed in the Major remarks section. Moreover, I have included a list with other minor remarks at the end of the document.

1 Major remarks

- ***Motivation.***
  Line 69 in the introduction reads: "While the concept of R-tipping has been examined for other tipping elements [...], to date it remains largely unstudied in the MISI context". However, there are previous studies that have already explored the idea that timescale separation between solid-Earth and ice sheets permits R-tipping. For instance, Swierczek-Jereczek et al. (2025): "Here we propose that sectors subject to MISI can undergo R-tipping: when forcing rates are high and the bedrock uplift is slow, the grounding-zone retreat is not stabilised as efficiently as in equilibrium simulations, triggering MISI at warming levels below the bifurcation point". Currently, the manuscript under review is motivated by the apparent absence of previous studies where MISI is investigated within the context of R-tipping. Instead, I would suggest framing the motivation in terms of a lack of experiments in idealised geometries where physical mechanisms can be isolated.

  We thank the Referee for this valuable suggestion and will revise the Introduction accordingly.

- ***Spin-up.***

According to Section 2.2.2, bed deformation is switched off during spin-up and is later justified by: "[...] enabling bed deformation after the system has equilibrated does not lead to any changes in the ice and bed geometries". Why is this the case? I guess the GIA model is set up to reflect load changes referring to the ice thickness equilibrated with a fixed bed. I understand that this allows simulations to start from the same grounding line position, but it is only one choice and should be detailed in the text.
I propose an additional approach to potentially enrich the discussion: performing the spin-up with bed deformation. Equilibrium grounding lines will therefore be different for each combination of Te and η. It is quite informative to observe the equilibrated ice sheet profiles with their corresponding relaxed bed geometry (analogous to Fig. 4b). This will give an idea of how far the grounding line position with a fixed geometry lies from the GIA counterpart. The spin-up could start from a 2 km block of ice (as currently performed) or perhaps from a thin layer of ice that grows until equilibrium (Pattyn et al, 2012). In total, this would only introduce 32 additional simulations, feasible considering this idealised domain.

We followed the Referee's suggestion and carried out 32 spinups that account for bed deformation, leaving the rest of the model configuration identical to the original fixed-bed spinups. In the case of a deformable bed, the spun-up ice sheets have a greater horizontal extent (GL more advanced in main flow direction) which also leads to an overall thicker ice body: while the initial ice block evolves, the deformable bed allows for subsidence where the ice sheet is grounded and uplift on the oceanward side of the coastal sill, where there the ice is floating. This pattern is more pronounced for thinner lithospheres due to the more localized response, resulting in more advanced, thicker ice sheets. We will add these results to the manuscript, providing figures that compare the centerline ice-sheet profiles and discussing the differences.

- ***Lithospheric thickness Te.***
  I find striking the steady-state bed displacement sensitivity to lithospheric thickness. To my understanding, this parameter mainly dictates the horizontal extent and the transient response of the vertical displacement, thus only driving minor changes in the steady-state displacement (Coulon et al., 2021; Swierczek-Jereczek et al., 2025). Figure S2 shows the contrary: the equilibrium displacement is apparently dictated by Te . Why is this? I encourage the authors to elaborate on such behaviour, seemingly contradicting previous studies. The consequences of this behaviour are notable, since the reported R-tipping is therefore highly sensitive to Te . I would suggest moving Fig. S2 to the main text since the ultimate physical mechanism is therein illustrated.

  We thank the Referee pointing to the significant role of the lithosphere thickness, T_e, in our simulations. Currently, we are running additional simulations to further explore the mentioned strong sensitivity of our results to T_e. As suggested by the Referee, we will move Fig. S2 to the main manuscript and give a deeper discussion of the underlying mechanism, also based on the outcome of the additional simulations.

- ***Shift in the critical melt rate threshold.***
  The study defines the shift relative to the fixed-bedrock geometry mB−tip,fixed (Eq. 2 and Fig. 6). Nevertheless, one must look at simulations with identical parameter choice to illustrate the contribution of forcing rates (i.e., Te and η). Hence, Eq. 2 must be modified to reflect that the threshold shift is compared between simulation with identical GIA parameter values and different melt rates:

  $$\Delta mR{-}tip(Te, \eta) = (mR{-}tip(Te, \eta) - mB{-}tip(Te, \eta)) / mB{-}tip(Te, \eta) \qquad (1)$$

  With this new definition, the threshold shift only reflects the contribution of the rate of forcing, rather than the influence of considering bedrock displacement.

  We see the advantage of the Referee's suggested quantification and will revise our manuscript accordingly.

- ***GIA domain and boundary conditions.***
  The horizontal scale of the bed displacement might reach beyond the edges of the domain. If boundary conditions impose a null vertical displacement at the edges, it would introduce a bias (particularly relevant at large Te). Ideally, the vertical displacement should vanish near the edges to avoid this bias. I suggest an easy test to quantify this potential error by simply evaluating the horizontal gradient of the vertical displacement. If these are significantly non-zero as the edge is approached, the GIA domain definition should be extended further, specially along the y-axis. If boundary conditions are different (e.g., Robin-type $\partial u/\partial n + u = 0$), the choice should be justified and detailed in the text.

  This is indeed an important point brought up by the Referee. In our simulations the extent of the GIA domain is 4 times the extent of the ice-sheet model domain in both horizontal dimensions. The GIA model applies periodic boundary conditions at the edges of the GIA domain. Following the Referee's suggestion, we found that the vertical displacement does not vanish at the GIA domain boundaries in the case of a very thick lithosphere (T_e=200km). We carried out simplified simulations (as suggested by Referee#1) with a larger GIA model domain (factor 10 instead of 4) for which the vertical displacement vanishes at the domain boundary also for T_e=200km. We find that the deviation in the response in bed uplift between the two GIA model domain sizes is 3 % at maximum (located in the setup center), suggesting that the size of our original GIA model configuration is sufficient, while reducing the computational cost of the simulations. We will revise our manuscript based on these deeper insights, adding more details on the choice of the GIA model domain configuration and boundary conditions and how it effects the results.

- ***Additional stabilizing feedbacks.***
  The are additional stabilizing feedbacks that could be discussed in the manuscript. For

example, heterogeneous precipitation (Fig. 2 in Sergienko, 2022) and the temperature dependency of ice viscosity (Fig. 6 in Moreno-Parada et al., 2025). Sergienko (2022) demonstrated that the feed-back between precipitation, atmospheric surface temperature and ice-sheet surface elevation provides additional stability. Moreno-Parada et al. (2025) showed that the bistability of the system is increased (i.e., a widen hysteresis loop) if the modelled ice temperature and viscosity are fully coupled, due to velocity corrections as the thermal structure adjusts to the changing ice thickness. Both mechanisms are absent in this study, since ice softness and surface accumulation are assumed to be constant.

We thank the Referee for pointing us to these important stabilizing feedbacks that we missed to discuss in our study. We will revise the manuscript accordingly.

- ***Grid resolution.***
  All simulations are run with $\Delta x = 2$ km following Feldmann and Levermann (2023). Have you performed a convergence analysis where bedrock displacement is included? Given that the model domain is idealised, higher resolutions are feasible. I would suggest taking two parameter choices of the pair T e and η (corresponding to the presence and absence of R-tipping behaviour) to perform a number of simulations for, at least, $\Delta x = 0.5$, 1, 4 km. This would be ideal to compare both the hysteresis loops and the forcing rates exhibiting R-tipping as a function of Δx, ultimately justifying (or not) the resolution choice if bedrock displacement is considered. The convergence test will further provide robustness to the results.

  As suggested by the Referee, we are currently running a convergence study choosing the two (T_e, η)-tuples (80 km, 1e+21 Pa s) and (80 km, 5e+19 Pa s) – the former exhibiting R-tipping, the latter not. Preliminary results show that the resolution of course has an influence but that results on the 2-km grid are quite close to the 1-km results, quantitatively as well as qualitatively, i.e., in terms of B-tipping thresholds, (non-)occurrence of R-tipping and the R-tipping range. We will present and discuss these results in the revised version of the manuscript. Note that conducting the whole hysteresis loop is relatively costly in terms of computational time already for the 2-km simulations and thus we won't be able to provide results for the suggested grid size of 0.5 km.

2 Minor comments

- *For the self-sustained oscillations, it would be informative to estimate if a parameter combination of η and Te characterizing the Laurentide Ice Sheet gives rise to such an oscillatory behaviour.*

  This is indeed an interesting comparison. The parameter ranges we found in the literature (Zhao, 2013, Argus et al., 2021) are T_e=100-180 km, η=0.2-0.5 10^{21} Pa s, thus not fitting the parameter combinations of our simulations that lead to self-sustained oscillations. We will add a statement on that in the Discussion.

- *The caption of Figure 8 should read: "The two dotted horizontal lines mark the range of retrograde bed slope for a fixed bed."*
  Will be done.

- *Figure 9: "reverse MISI ". I would simply use MISI. The instability does not imply a specific direction, but rather to the absence of stable equilibrium positions over retrograde slopes (Schoof, 2007).*
  Will be done.

References

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