Response to Reviewer 2

Manuscript: *A tempered fractional Hawkes framework for finite-memory drought dynamics*
Mauricio Herrera-Marín

Dear Reviewer,

Thank you for your careful and constructive evaluation. Your comments were particularly valuable in highlighting the need for consistency between theory and implementation, stronger validation, and a more cautious interpretation of physical meaning.

1. Interpretation of α

You noted that the interpretation of α was inconsistent.

Response. This has been fully corrected. The manuscript now explicitly states that:

$$\phi(t) \sim t^{\alpha-1} = t^{-(1-\alpha)},$$

so that smaller α implies faster decay. All results and discussion have been revised accordingly.

2. TFHP implementation

You pointed out that the tempered model was not actually estimated.

Response. This was a central issue and has been fully resolved. The revised manuscript now performs direct maximum-likelihood estimation of the full TFHP:

$$(\mu, \eta, \alpha, \theta),$$

with no reliance on post-hoc tempering.

3. Model validation

You requested stronger validation and comparison.

Response. The revised manuscript now includes:

- AIC-based comparison with Poisson,

- time-rescaling diagnostics,

- identifiability checks,

- spatial consistency analysis.

The claims regarding model performance have been made more conservative and evidence-based.

4. Physical interpretation

You raised concerns about overinterpretation.

Response. The revised manuscript now treats parameters as **effective descriptors** rather than directly measurable physical quantities. The discussion explicitly separates:

- statistical meaning,

- mathematical structure,

- and physical plausibility.

5. Scope and limitations

You highlighted missing discussion of applicability.

Response. The revised manuscript now explicitly discusses:

- limitations in sparse-event regimes,

- dependence on event definition,

- absence of spatial coupling,

- and extensions toward covariate-driven models.

Closing remark

Your comments significantly improved the clarity, rigor, and scope of the manuscript. The revised version now aligns theoretical formulation, statistical inference, and physical interpretation in a consistent framework.

Thank you again for your constructive feedback.

Sincerely,
Mauricio Herrera-Marín