

# Response to the Editor

Manuscript: *A tempered fractional Hawkes framework for finite-memory drought dynamics*  
Mauricio Herrera-Marín

Dear Editor,

Thank you for your careful assessment of the manuscript and for the clear guidance provided in your editorial recommendation.

I fully acknowledge that the original submission contained important conceptual and methodological inconsistencies, particularly regarding (i) the interpretation of the fractional exponent, (ii) the operator-theoretic formulation, and (iii) the lack of direct estimation of the tempered model. These were substantive issues, and I appreciate that they were identified clearly in the review process.

In response, the manuscript has undergone a **complete reconstruction rather than a superficial revision**. The revised version now addresses all central concerns raised by the reviewers:

- The mathematical framework has been rewritten from first principles using a consistent tempered fractional formulation, including a clear definition of the tempered Caputo operator and its relation to the Volterra representation.
- The empirical model is now the *actual tempered fractional Hawkes process*, with direct joint estimation of  $(\mu, \eta, \alpha, \theta)$  rather than post-hoc derivation of tempering.
- The likelihood is evaluated using the exact analytic compensator, improving both correctness and numerical stability.
- Model validation has been substantially strengthened through AIC comparison, time-rescaling diagnostics, and identifiability analysis.
- The interpretation of parameters has been revised to distinguish clearly between mathematical meaning, statistical estimation, and physical plausibility.

Importantly, these changes are not incremental. The revised manuscript is now internally consistent: the theory, inference, validation, and interpretation all refer to the same model and are aligned with one another.

For these reasons, I respectfully submit that the manuscript has evolved into a substantially stronger and methodologically sound contribution. I therefore hope that, in light of the extensive revisions undertaken, the manuscript may still be considered suitable for evaluation within the journal.

If, however, the editorial decision remains that a new submission is required, I will of course respect that judgment. In that case, I hope this response demonstrates that the concerns raised have been fully addressed and that the revised work now meets the expected scientific standards.

Thank you again for the constructive and rigorous evaluation.

Sincerely,

Mauricio Herrera-Marín