



A monograph in astrogeodetic orientation in the current era of satellite positioning technology

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Abstract. In recent years, classic surveying astronomy has begun to take interest in published literature. The original authoritative textbooks and manuals on astrogeodetic surveying are perceived to be outdated and obsolete since the advent of satellite positioning and are scarce or impossible to find. Before satellite positioning, surveyors relied on rigorous drawn-out methods of precise astronomical surveying. This recent surge of interest appears to be directed toward astronomical levelling in the determination of the deflection of the vertical in modelling the geoid. Additionally, there has been a spate of literature on astronomic, geodetic, and grid azimuths. An azimuth is a line measured clockwise from the north point. Astronomic azimuth is the line measured in a clockwise direction from the north astronomic pole to a celestial body, and the line pointing to the celestial body forms a great circle through the zenith of the observer, intersecting the observer's astronomic longitude. The astronomic azimuth is referenced to a local (astronomic) horizon coordinate system perpendicular to the local gravity vector. To illustrate the procedures for astronomical observations, an actual night observation of a star done by the author in May of 1998 was used. This work is intended to show the modern surveyor how field astrometric observations were done before the advent of satellite positioning, and to understand the fundamentals of astrogeodetic methods that has become a lost art and science.

20 Key words: earth-centered earth-fixed coordinate systems, astronomical coordinate systems, geodetic coordinate systems, geoid, deflection of the vertical, astronomical time scales, Laplace correction, convergence of the meridians, astronomic azimuth, grid azimuth

1 Introduction

25 “When I look up, I see heavenly bodies blanketing the night sky. These stellar wonders inspire a sense of warmth and comfort within me and a feeling of love and awe. It is this feeling that brings about one of the greatest joys in my life, and I want to share this with the universe. There is no greater feeling of love God has given to humankind.”

—David A. Rolbiecki

This monograph aims to help the modern surveyor understand astrogeodetic methods and the process of obtaining a grid azimuth in the absence of Global Navigation Satellite System (GNSS) technology. Authoritative textbooks on geodetic astronomy from the late to mid-twentieth century (e.g., Hayford, 1898, C&GS, 1947; Nassau, 1948; Roelofs, 1950; Davis et al., 1966; Thomson, 1981; Elgin et al., 2001) have been set aside, marginalizing this scientific discipline in favor of satellite

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geodesy. As satellite positioning revolutionized modern surveying, astrogeodetic positioning and orientation remains vital today, primarily if for some unseen event, GNSS service is non-existent. Astrogeodetic positioning is also useful in astronomic levelling in the determination of the deflection of the vertical, which represents the angular difference between the gravity vector through a point on the earth normal to the ellipsoid surface (Featherstone, 1999). Surveyors using GNSS technology are, by default, performing geodetic surveying even if they are unaware. These unwitting surveyors collect geodetic points with GNSS and assume these points are simply on a certain state or provincial plane coordinate system with a grid bearing and accept it on face value. It is important to understand that each state plane coordinate value comes from a geodetic measurement. This measurement first produces earth-centered, earth-fixed Cartesian coordinates based on a reference ellipsoid in a terrestrial reference system.

Without the availability of GNSS, surveyors can only establish state plane grid azimuths using an optical theodolite either by coming off found survey control with published geodetic and grid values, or by direct observation of celestial bodies and performing a series of computations to get from astronomic to geodetic, then to grid azimuth.

Much of today's surveying profession focuses on GNSS technology to increase productivity and profit. Popular surveying journals emphasize accuracy and precision, giving little to no credit as to how modern technology applies geodesy in land surveying. Field astronomy for surveying receives even less attention; perhaps considered to be antediluvian. In a popular American surveying magazine, a well-known and respected surveying professional wrote about why geodesy matters in surveying today (Paiva, 2011). The message was clear and is consistent in defining a trend that today's surveyors rely totally on GNSS technology without understanding how and why they arrived at a grid coordinate and bearing basis on their survey. Fortunately, there is a rising trend of research articles dedicated to reintroducing surveying astronomy for the modern surveyor to be exposed to classic astrogeodetic methods and understand the history of this lost art and science—as it was an essential discipline of survey field astronomy—in in order to expand their interest, which can only benefit their career (Barazzetti, 2025a & 2025b).

Prior to commercially available GNSS technology, surveyors were dependent upon the use of first, second and third order survey control stations in the orientation of their surveys. These legacy monuments possessed geodetic values, and if found, their stability and visibility was questionable, because over time, they may have been disturbed by land development or visibility between two connected stations was reduced by buildings or trees that sprung up since the stations were established. Often as was the case, the surveyor performed astronomic observations from a geodetic station with no reference azimuth whereby an astronomic azimuth needed to be established—if not for the purpose of geodetic work—then for the purpose of determining the bearing basis of a boundary survey (Davis et al., 1966; Nassau, 1948; NOAA, 1977). If available (or affordable), a gyro theodolite was used to determine astronomic azimuth.

It was generally accepted that surveyors of the era before commercially available and affordable GNSS systems were adept in the science of astrometry and most surveying textbooks addressed low-precision and high-precision methods for determining astronomical position and azimuth (see Nassau, 1948; Davis et al., 1966; Elgin et al., 2001). The popular yearly celestial observation handbook and ephemerides published by Leitz and Sokkia were common stock items in surveying supply stores



available to surveyors until it went out of print in 2008 (Cheves, 2010). In this current era, emphasis leans towards advanced remote sensing technology and GNSS. Even then, there is still value in using astronomic azimuth observations as a quality control check on GNSS-derived azimuths (Chang & Tsai, 2006) and accounting for local gravity anomalies that GNSS cannot do alone. There is no argument that precision and accuracy of these systems supersede first order specifications, and the time and cost in operations saved along with increased productivity is a critical business decision when compared to the classical approach to astrogeodetic orientation methods of merely 35 years ago. Regardless, the modern surveyor should understand how the physical shape of the earth is represented on a grid surface and possess the practical skills to accomplish this in the absence of GNSS technology.

The aim of this work is to expose the modern surveyor to the art and science of field astronomy. It seeks to familiarize the modern land surveyor with astrogeodetic methods, specifically the process of obtaining a grid azimuth in the absence of GNSS technology. Formerly, determining an astronomic azimuth was the principal method. This work also explains how to reduce an astronomical azimuth to a geodetic azimuth and, finally, to a grid azimuth for survey project mapping projections. With today's emphasis is focused on advanced remote sensing technology and GNSS, can the modern surveyor explain how coordinate values are derived with the push of a button?

2 Fundamental Coordinate Systems

2.1 Earth-centered, earth-fixed coordinate systems

Before astrometric methods are discussed, a basic understanding is required, of a fundamental coordinate system by which a GNSS survey obtains a position on the earth in an earth-centered, earth-fixed (ECEF) Cartesian coordinate system (Fig. 1). ECEF coordinate systems define three-dimensional positions (X , Y , Z) with respect to the center of mass of the reference ellipsoid (Fig. 1). The Z -axis points toward the north geodetic pole, which is aligned with the International Earth Rotation Service (IERS) reference pole; the X -axis is perpendicular to the north geodetic pole and points toward 0° longitude on the IERS reference meridian intersecting at right angles to the geodetic equatorial plane. The Y -axis completes a right-handed orthogonal system by a plane 90° east of the X -axis and its intersection with the geodetic equatorial plane. However, ECEF coordinates of a point are of little value to a surveyor or project engineer having to deal with ΔX , ΔY , ΔZ in their set of engineering designs. Instead, the desired output would take the form of a coordinate system output tangible to the user, such as latitude, longitude, and height (L, L, H); or northing, easting, and elevation (N, E, E) on a grid system. In the reduction from ECEF coordinates to NEE, transformation parameters are needed to rotate, translate, and scale ECEF to geodetic and grid coordinate systems. For an in-depth search of ECEF literature, the reader may refer to Thomson, 1981, Stem, 1989, Hofmann-Wellenhof et al., 1996, Iliffe, 2000, NIMA, 2000, Rolbiecki, 2024, as this topic goes beyond the scope of this paper.

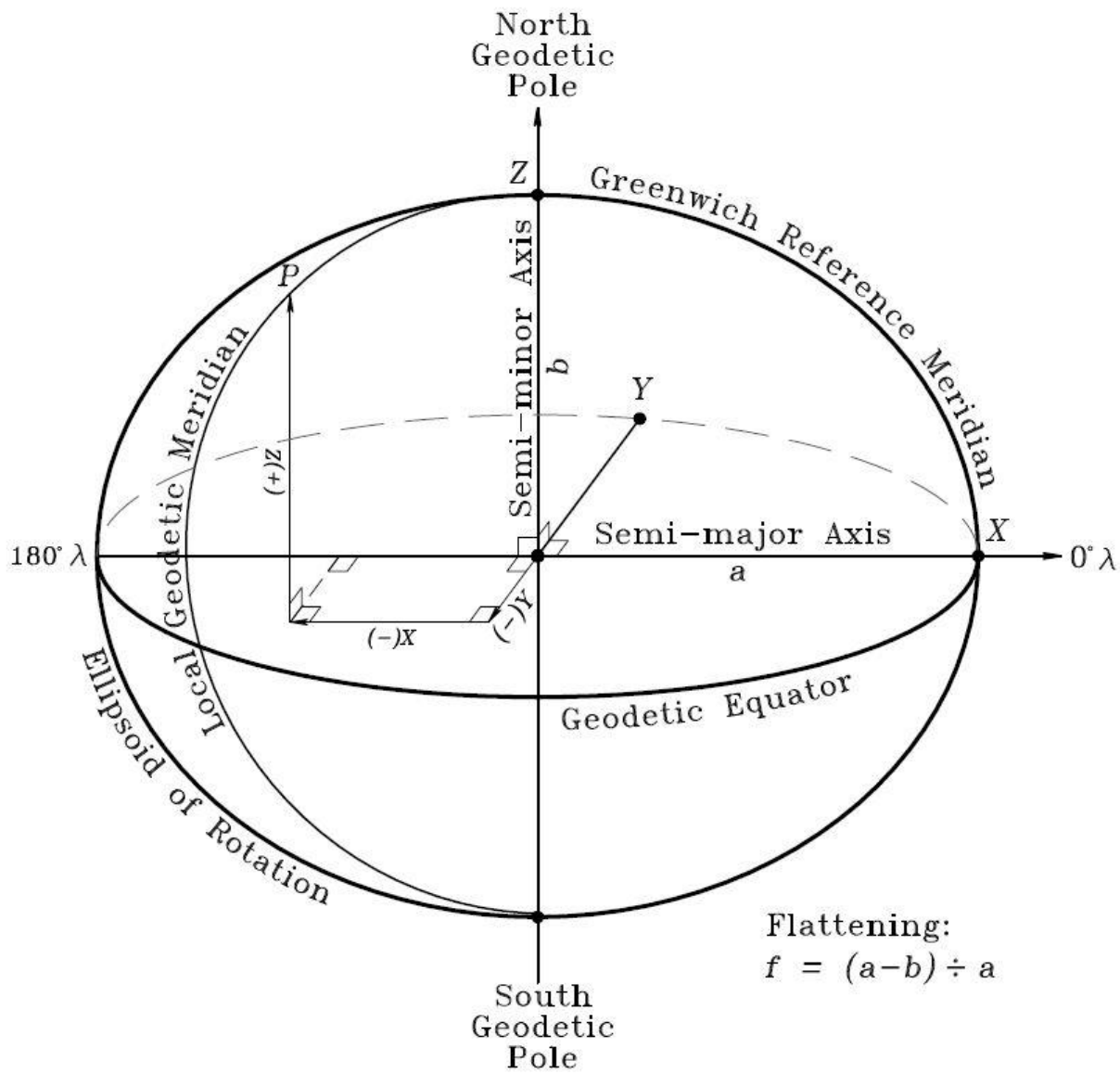




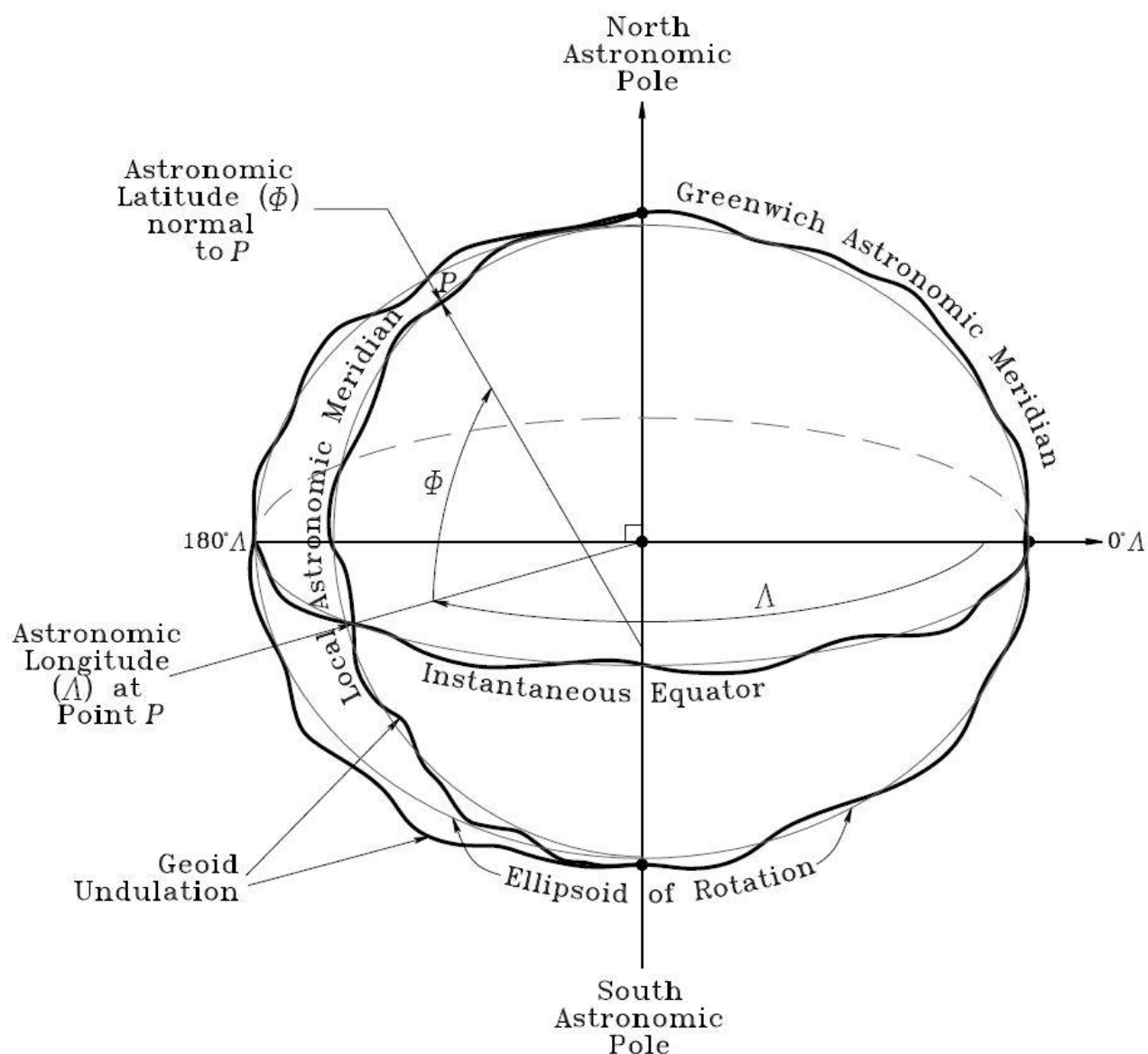
Figure 1: Earth-centered, earth-fixed coordinate system shown in northern and western hemispheres.

2.2 Astronomical Coordinate Systems

135 The Astronomic Coordinate System is a local curvilinear coordinate system determined by direct observations of celestial bodies using precise time, based on the instantaneous earth rotational axis about the equator (Thomson, 1981, Soler & Hothem, 1988; Dimal & Balicanta, 2009). It may be further broken down into two subsystems known as the Horizon System and Equatorial System (Nassau, 1948), also known as the Right Ascension System (Thomson, 1981). The Horizon System is a plane tangent to the topocentric surface of the earth aligned on the celestial sphere, which is the fundamental circle of this system whose radius is infinity with respect to stars (Nassau, 1948; Thomson, 1981). The poles are the *zenith* and *nadir* at the observer's station orthogonal to the topographic horizon. The nadir follows the direction of the plumb line of the observer and does not pass through the center of the earth, as will be seen below. Vertical circles passing through the zenith and nadir are great circles on the celestial spheroid and define the variables, azimuth and altitude of the observer.

The Equatorial System's fundamental circle is the equator, which passes through the center of the celestial sphere orthogonal to the north and south astronomic poles and referred to as a great circle. Secondary great circles orthogonal to the equator are known as hour circles, which defines the right ascension (RA) of a celestial body, the Greenwich astronomic meridian at 0° longitude, the Local Hour Angle (L.H.A.) between the celestial body and the observer's meridian, and the astronomic longitude of the observer. Small circles parallel to the equator defines the declination (δ) of a celestial body and the astronomic latitude of the observer (Nassau, 1948). In 1718 Edmund Halley discovered stars were not fixed, but moved across the sky relative to other stars, and changes in RA and δ will become displaced over time (≈ 10 arcsec per year). This displacement is called proper motion (μ), with the components μ_{RA} and μ_δ , respectively (Roelofs, 1950).

The Astronomic Coordinate System defines the latitude and longitude (their symbols are noted in upper case Φ and A , respectively) of a point on the topographic surface of the earth normal to the geoid and are influenced by local gravity anomalies. Astronomic latitude is the angular measurement north (positive, $0-90^\circ$) or south (negative, $0-90^\circ$) of, and at right angles to, the instantaneous equator along a great circle passing through the north and south astronomic poles. The zenith and nadir of the observer's pole forms the astronomic latitude Φ made by the gravitational vector of the plumb line to the earth's equator (Bowditch, 1802; Nassau, 1948; Thomson, 1981). A is the angular measurement along the instantaneous equator east (positive, $0-180^\circ$) or west (negative, $0-180^\circ$) of the Greenwich astronomic meridian (Fig. 2).



195 **Figure 2: Astronomic latitude and longitude normal to the geoid (N & W hemispheres).**

The vertical component in this so-called “natural” coordinate system (Soler & Hothem, 1988) is the orthometric height (H) measured above the geoid, whose surface approximates mean sea level (Rolbiecki & Lyle, 2008; Rolbiecki, 2024; Soler & Hothem, 1988; Stem, 1989). While conducting an astronomic observation, the zenith and plumb line of the observer is

perpendicular to the geoid—which at any point on the surface of the earth—represents the equipotential surface of the earth’s
 gravity field (Bowditch, 1802; Fig. 3, Fig. 4).

2.2.1 Deflection of the Vertical

The deflection of the vertical (DoV) on the earth’s surface is the angle at the topocentric point of observation made between
 the direction of the plumb line attracted by gravity and the direction normal to the reference ellipsoid (Featherstone, 1999;
 Hiroki, 2025). In relative terms, it is simply the angle formed between the astronomical vertical and the geodetic vertical (Fig.
 3). Astronomic and geodetic coordinate systems are not aligned with each other due to this vertical deflection (Roelofs, 1950),
 and a transformation process is needed. In order to transform from astronomic latitude and longitude (Φ, λ) to geodetic latitude
 and longitude (ϕ, λ), the deflection of the vertical is used with the following components: The north-south component ξ (ζ)
 and the east-west (prime-vertical) component η (η). These components at the topocentric surface are derived by the following
 expressions (Featherstone, 1999; Soler et al., 2014):

$$\xi = \Phi - \phi \quad (1)$$

$$\eta = (\lambda - \lambda') \cos \phi \quad (2)$$

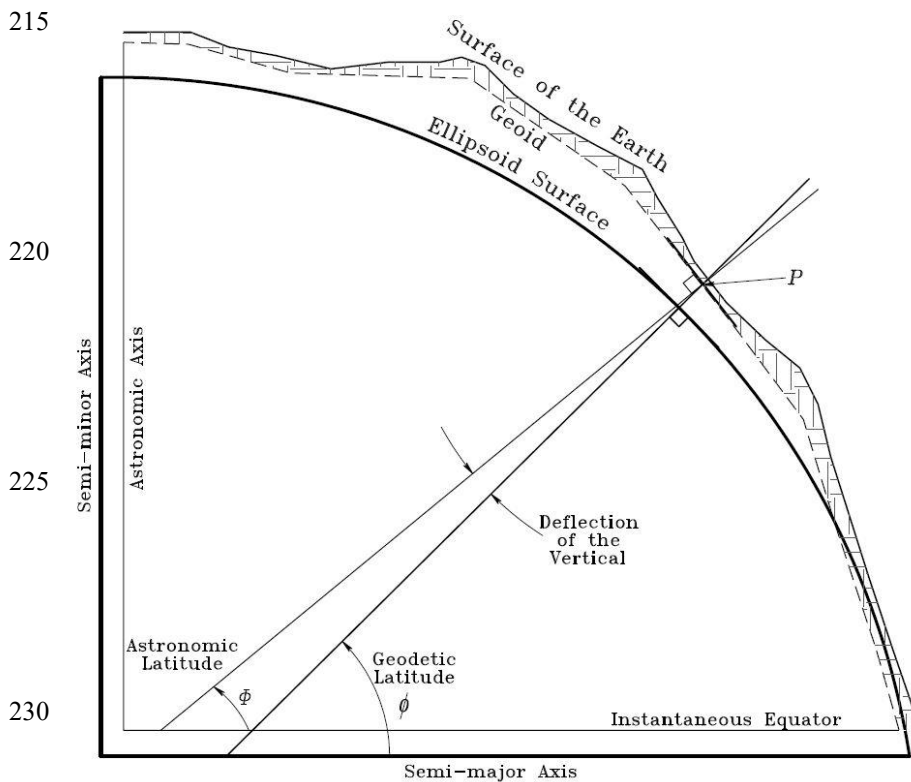
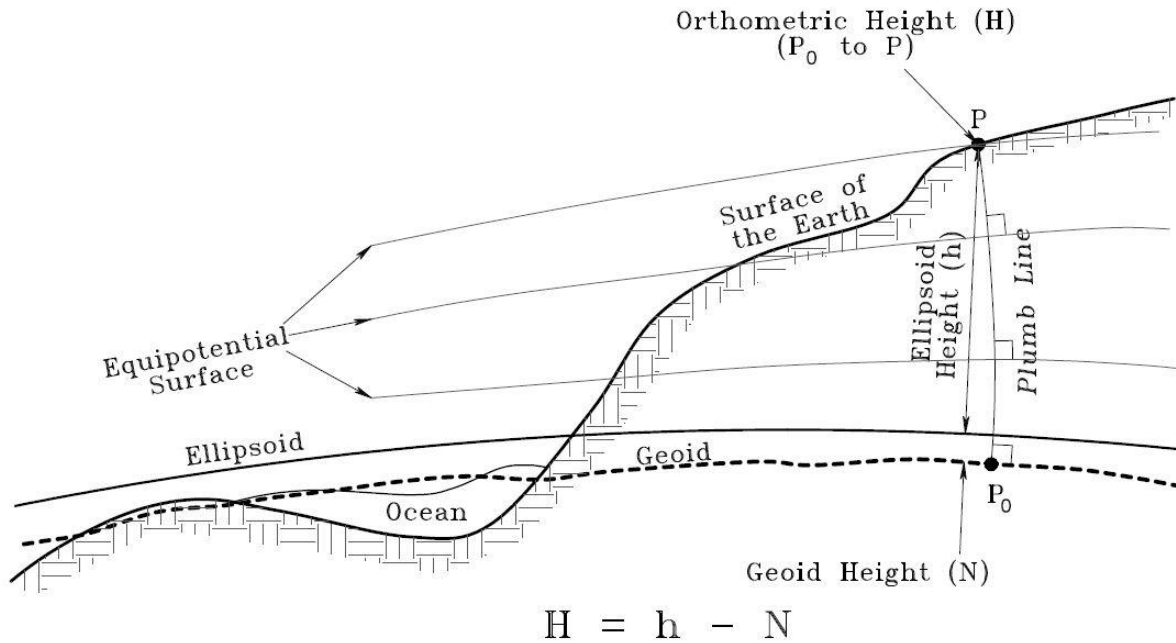


Figure 3: Deflection of the vertical at point P.



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Figure 4: Geoid-ellipsoid relationship and the direction of the plumbline perpendicular to the equipotential surface.

The components, ζ & η may be obtained for DoV by taking the difference between direct astronomic observations reduced to topocentric latitude and longitude and the geodetic latitude and longitude at the station. ζ & η may also be determined by
 240 commercially available GNSS post-processing software or specific software designed to compute the deflection of the vertical such as DEFLEC18 published by the National Geodetic Survey. A positive meridian component in ζ indicates Φ will fall north of φ , and a positive prime-vertical component of η indicates λ will fall east of λ .

You will see in Fig. 4 that ellipsoid and geoid surfaces are not coplanar, and a separation between the two is referred as the geoid height (N) and may be above or below the ellipsoid in different areas of the world (Rolbiecki & Lyle, 2008). Using the
 245 expression $h - N$ will obtain orthometric height H. For a review of ellipsoid-geoid relationships, refer to Stem (1989) and Thomson (1981).

There are places on the earth where there is either a dearth of gravity data to model a geoidal surface in that area, or there are places near mountains and other massive geological formations that may create inaccurate geoids at these locations due to large local attractions from the mass of the land feature. This can arguably be another reason for determining DoV at these
 250 locations by direct measurement of stars to define the absolute direction of gravity and the slope of the geoid (Hardy et al., 2020).



2.3 Geodetic Coordinate Systems

Geodetic coordinate systems are modelled after a reference ellipsoid (Fig. 5); e.g., the World Geodetic System of 1984 (WGS-84) and the Geodetic Reference System of 1980 (GRS-80); having the defined parameters, semi-major axis (a), semi-minor axis, (b), and flattening of the poles (f), whose value is derived by $f = (a - b) \div a$ (Thomson, 1981; Schwarz, 1989; Stem, 1989; Hoffman-Wellenhop et al., 1994; Iliffe, 2000; NIMA, 2000).

Isaac Newton knew by hypothesis that the earth took on a bulging of mass at the equator due to gravity and centrifugal force. His theory stated that the earth was oblate (flattening at the poles with the equatorial diameter greater than from pole-to-pole) as opposed to some European astronomers such as Giovanni Domenico Cassini, who believed the earth was prolate (the longest axis was from pole-to-pole).

In 1735-6, the Académie Royale De Sciences in Paris sent out two geodetic expeditions to measure the length of one degree of latitude along a meridian of longitude to compare the arc length of one degree of latitude—measured near the equator and near the North Pole—in order to ascertain the flattening aspect of the earth (NOAA, 1977; Danson, 2001; Alder, 2002). Louis Godin and Pierre Bouguer went to Mitad del Mundo, Peru to measure the meridian arc by astronomic observations. Bouguer measured the earth's density and gravitational variations by observing the deflection of the plumb bob from the vertical. In 1736, Pierre-Louis Moreau de Maupertuis led an expedition to the Tornio valley in Lapland to measure their arc. The Académie Royale De Sciences compared the results of both expeditions and concluded the earth was in fact oblate, having an equatorial radius of 3 282 350 toises (6 397 312 meters), and a flattening ratio of 1/216.8. This proved what Newton already knew, and in the essay titled “Discourse on Moderation,” the French philosopher Voltaire (an avid supporter of Newton) wrote this about Maupertuis and the Lapland expedition (Halsted, 1913; NOAA, 1977):

*“You have confirmed in regions drear
 What Newton discerned without going near.”*

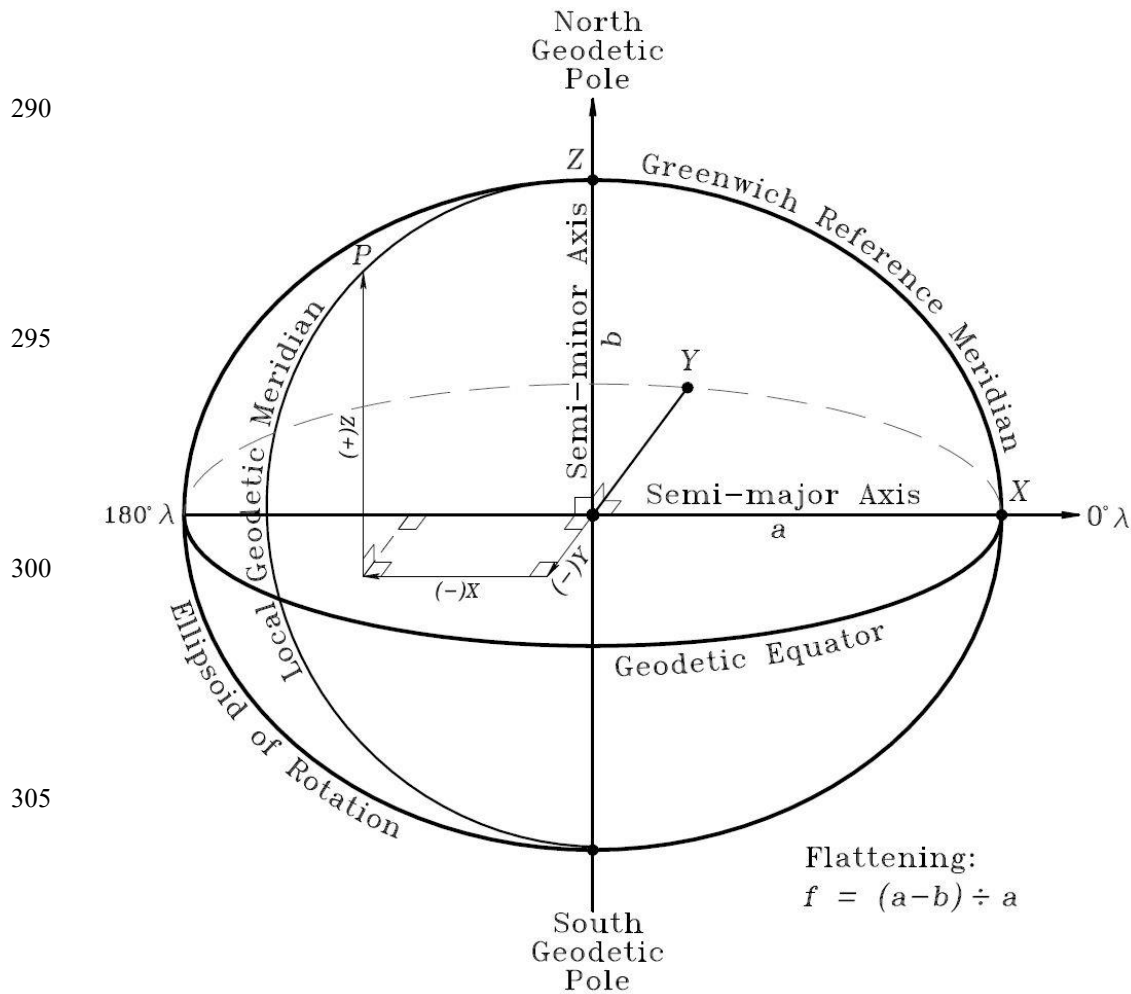


Figure 5: Geodetic latitude and longitude overlaid onto the earth-centered, earth-fixed frame. Points on the ellipsoid make up curvilinear coordinates (φ , λ) and X, Y, Z Cartesian coordinates.

The Geodetic Coordinate System refers to geodetic latitude and longitude (their symbols are noted in lower case φ and λ , respectively), and ellipsoid height (h) (Fig. 5). Geodetic latitude is the angular measurement north (positive, 0-90°), or south (negative, 0-90°) of the geodetic equator along the local ellipsoid meridian normal to point P orthogonal to the equator. On the reference ellipsoid, the earth has an infinite number of principal radii of curvature in the prime vertical plane cutting the polar axis of rotation at different locations, shown as P_I . Geodetic longitude is the angular measurement along the geodetic equator east (positive, 0-180°), or west (negative, 0-180°) of the Greenwich (Zero) Meridian. The vertical component (h) on the topographic surface is the height above the ellipsoid at point P as seen in Fig. 4.



3 Astronomical Time Scales

There are several time scales used in society that serves specific needs. Civil time is a standard based on the mean solar day used throughout the world for every-day purposes. For many astronomical applications, navigation and geodesy, astronomical time scales based on the earth's rotation and dynamical time scales based on the planetary motions in the solar system are used (Thomson, 1981; McCarthy, 1991; Hoffman-Wellenhof et al., 1994; Kaplan, 2005;). For our purposes in this paper, focus will be on sidereal time, Coordinated Universal Time, and longitude because these time scales are used exclusively with astronomical computations. Dynamical time scales are touched on only briefly; for a comprehensive review of the different time scales, see McCarthy (1991) and Kaplan, (2005).

3.1 Sidereal Time

The fundamental rotational time scale based on the apparent diurnal movement of the stars is *sidereal time*, which is defined as the L.H.A. of the *vernal equinox*—a fixed point among the stars—and the point of origin of the celestial reference system (Nassau, 1948; Roelofs, 1950; Thomson, 1981, McCarthy, 1991; USNO, 2005). The vernal equinox is also referred to as the *First Point of Aries* (Υ) and occurs when the sun enters the constellation Aries (the “ram”) and is the point of intersection between the ascending node of ecliptic plane (the orbital path of the earth around the sun) and the plane of the celestial equator, and marks the origin of the sidereal hour angle, S.H.A. (Bowditch, 1802). Out of the four seasons of the tropical year, Υ marks the first day of Spring.¹ As the earth moves along the ecliptic plane, Υ is advanced by $\approx 1^\circ$ from the mean sun in 24 sidereal hours, and the sidereal day gains approximately 00h 03m 56s on the mean solar day. A tropical year is 365.242 5 mean solar days and marks the interval of two successive transits of the sun through Υ . At the end of a tropical year, the sidereal clock will have gained 24 hours on the tropical year.

A sidereal day is the interval of time a given star crosses a longitude twice, and marks two successive upper transits of the vernal equinox at the same location. Therefore, *local sidereal time* (L.S.T.) is equal to the L.H.A. of Υ ; therefore, L.S.T. at any instant is equal to the *right ascension*, α of a celestial body plus its L.H.A. (Nassau, 1948; Roelofs, 1950; Thomson, 1981):

$$\text{L.S.T.} = \alpha + \text{L.H.A.} \quad (3)$$

Note: The symbol, α is also given for the brightest stars in constellations, so *RA* is used throughout this monograph. *RA*—expressed in hours-minutes-seconds (hh-mm-ss.sss)—is measured *eastward* along the celestial equator from Υ to the hour circle passing through a celestial body (USNO, 2005; Nassau, 1948).

3.2 Secular and Periodical Changes

¹ Six months after Υ is autumnal equinox, Ω (the first day of Fall), which is the descending node of ecliptic plane intersecting the plane of the celestial equator.



350 Secular and periodical changes in the rotation and orientation of the earth's polar axis are effects caused by the gravitational attractions of the sun, moon and planets and a variety of wind, tide and geodynamical actions, and causes changes in the motion of the astronomic pole and equator. This cause and effect are known as *precession of the equinoxes* and *nutation* (Fig. 6) of the astronomical pole about the ecliptic polar axis and changes the position of the vernal equinox with respect to the positions of the stars (Bowditch, 1802; Nassau, 1948; Roelofs, 1950; Thomson, 1981, USNO, 2005).

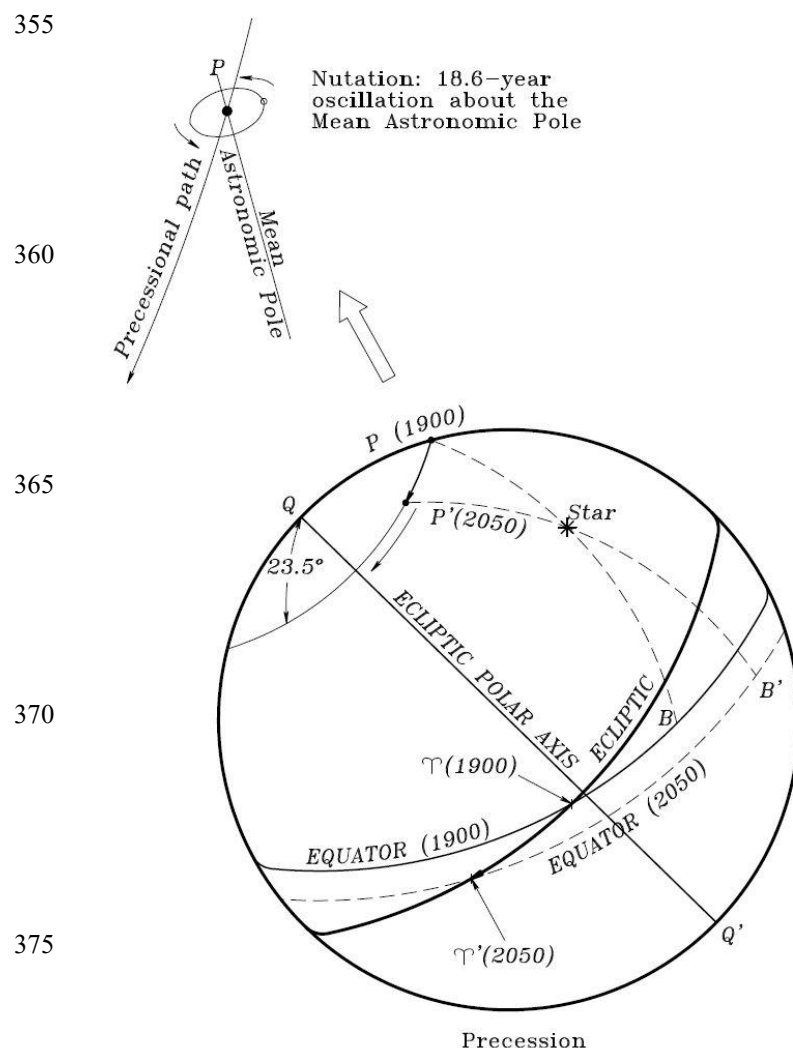


Figure 6: Author's rendering of Nassau's precession of the equinoxes from 1900 to year 2050 and periodic nutation about the mean astronomic pole P (Nassau, 1948).

380 Precession is a secular change with a uniform westerly motion of the vernal equinox of approximately 20 arcsec per year on the ecliptic caused by the “wobbling” of the astronomic pole. The wobble may be seen as a cone forming a circle traced by the mean astronomic pole around the ecliptic pole with a radius of approximately 23.5° . One revolution period of the circle takes approximately 25 800 years. In our current era, the star Polaris is nearly on the North Pole, but in another 12 000 years,



the star Vega will become our pole star. In Figure 6, let Q and Q' represent the ecliptic pole, Υ the vernal equinox, and P the mean astronomic pole in the year 1900. The hour circle passes through a star to the equator at B . By around the year 2050, the mean astronomic pole will be at P' and the hour circle through the star will be at B' with the vernal equinox at Υ' having moved west with the equator in a different position.

Nutation is a periodic change over a relatively short period of time and accounts for *apparent sidereal time* and *mean sidereal time* through the equation of the equinoxes (Thomson, 1981; USNO, 2005). Nutation is the “nodding” of the astronomic pole as it moves along the path around the ecliptic pole during precession (see upper left inset in Fig. 6). The nodding effect produces an epicyclical oscillation about the mean astronomic pole with an amplitude of 14-18 arcsec, completing a revolution about every 18.6 years (Kaplan, 2005). This coincides with the Metonic 19-year lunar tidal epoch based on the 18.6-year lunar nodal cycle (Cole, 1997; NOAA, 2003; Palmer et al., 2023).

3.3 Apparent and Mean Sidereal Time

Apparent sidereal time is the intersection of the true equator of date and the ecliptic of date and accounts for the equation of the equinoxes. Mean sidereal time is equal to apparent sidereal time minus the equation of the equinoxes (Davis et al., 1966; Thomson, 1981; Kaplan, 2005; USNO, 2005).

3.4 Longitude

Longitude (λ) of the observer is a time value in hh-mm-ss, which when multiplied by 15 (15° in each hour), is converted into angular format for computational purposes and is the measured difference between the L.S.T. at the base station of known longitude and L.S.T. at the field station of unknown longitude (C&GS, 1947). Longitude is reckoned east or west from the Greenwich astronomic meridian and may be determined directly by observing the movement of stars crossing the local meridian of the observer and recording the precise time of the crossing, usually by the stars upper transit (Thomson, 1981; C&GS, 1947; Roelofs, 1950). Constraints are put on this method in that the observer must know the time of upper transit and be set up and ready to observe at that instant. Times for upper and lower transits of stars may be obtained through an astronomical almanac or ephemeris by inputting the observer’s assumed latitude and longitude. Depending on the time of the year, transiting stars may occur in daylight, which will be difficult to see.

At the instant of upper meridian transit of the star crossing the observer’s local astronomic meridian, L.H.A. is zero, and L.S.T. is equal to the star’s right ascension (C&GS, 1947). When this occurs, the longitude of the observer may be determined by comparing the difference in sidereal time between the Greenwich astronomic meridian and the observer (Nassau, 1948). Considering the effect of nutation significant to astronomical observations, the equation of the equinoxes is taken into account (McCarthy, 1991; Kaplan, 2005) and the difference is expressed as:

$$\Delta \lambda = (G - \tau) \quad (4)$$

where $\Delta \lambda$ is the difference in longitude (hh.mm. ss) between the Greenwich astronomic meridian and the observer’s meridian, G is the Greenwich apparent sidereal time (G.A.S.T.), and τ is the Local apparent sidereal time (L.A.S.T.) on the observer’s meridian at the instant of upper transit.



3.5 Coordinated Universal Time

The civil time standard adopted by most countries is based on time at the Greenwich meridian, formerly known as Greenwich Mean Time (GMT). Universal Time (UT) has since replaced GMT. Because the rate of the rotation of the earth is not constant, civil use of time needed to coincide with the mean solar day from which Coordinated Universal Time (UTC) was adopted. UTC is an atomic time scale based on the International Atomic Time (TAI) SI second on the geoid at epoch 1958, January 1.00, when UTC – TAI was equal to zero (Thomson, 1981; McCarthy, 1991; Hoffman-Wellenhop et al., 1994; Kaplan, 2005; USNO, 2005). For astrometric applications, additional corrections to UTC need to be applied. In order to compensate for the non-uniform rotation rate of the earth and keep UTC aligned with sidereal time,² an adjustment from UTC to UT1 is necessary by incorporating the difference known as DUT1. DUT1 has an accuracy of 0.1 second, and positive or negative incremental corrections are applied to UTC throughout the year to keep DUT1 within ± 0.9 seconds of UT1. When DUT1 reaches ± 0.9 seconds, a leap second is applied—either on 31 December or 1 June. The last time a leap second occurred was when the IERS applied it on December 31, 2016, just before midnight, UTC (IERS, 2025).

UTC and DUT1 radio time signals can be heard on WWV in Fort Collins, Colorado or WWVH in Hawaii on shortwave frequencies, 2.5, 5, 10, 15 and 20 MHz. The signal comes through as a continuous tone accompanied by 1-second ticks. The 30-second mark is noted with the dropping of the tone at 29 seconds. The tone is dropped again after 45 seconds for both stations to announce the time of the upcoming UTC. At the 59th second, the 1-second tick is dropped in preparation for the top of the minute, which is announced as a loud “beep” at 60 seconds. After the top of each minute, a series of “double ticks” representing DUT1 is introduced between the 1st and 15th second, indicating how to correct UTC to UT1: between the 1st and 7th second, DUT1 is added; between the 9th and 15th second, DUT1 is subtracted.

By using the Greenwich astronomic meridian as the reference meridian east or west of the observer’s longitude, the Greenwich Hour Angle (G.H.A.) and Declination (δ) of the celestial body for any instant of UT1 is thus obtained (Thomson, 1981).

3.6 Dynamical Time

In some applications of astronomy, dynamical time scales are used to provide a more uniform time scale than UTC with regard to the earth’s orbit around the sun. Terrestrial Time (TT) is used for certain geocentric and topocentric applications based on the ephemeris meridian that defines a system of ephemeris longitudes independent of the irregular rotation of the earth and is currently ahead (east) of the Greenwich meridian (USNO, 2005). TT was formerly known as Terrestrial Dynamical Time (TDT), which replaced Ephemeris Time in 1979 when it became obsolete. The naming conventions have changed, but TT has remained consistent with Ephemeris Time (McCarthy, 1991). TT is uniform in rate with TAI on the geoid with an epoch offset of +32.184 seconds (McCarthy, 1991; Kaplan, 2005; USNO, 2005). The difference between UT1 and TT is known as ΔT and can be obtained from a current astronomical ephemeris such as the Multiyear Interactive Computer Almanac (MICA). For

² The fundamental rotational time scale based on the apparent diurnal movement of the stars is sidereal time, which is defined as the Local Hour Angle of the vernal equinox (Υ)—a fixed point among the stars—and the point of origin of the celestial reference system.

example, on June 21, 2011(Julian date 2 455 734.178 750), ΔT was 66.471 seconds. On August 25, 2025 (Julian date 2 460 913.131 227), ΔT was 73.497 seconds, (USNO, 2005).

450 Barycentric, Terrestrial Dynamical Time (TDB) is a time scale based on the *barycenter* (center of mass) of the solar system and is used in solar system ephemerides (Kaplan, 2005). TDB is another extension of Ephemeris Time defined by the International Astronomic Union (IAU) in 1976 and adopted for use in 1979 to account for relativistic terms in the transformation from TT to TDB (McCarthy, 1991; Kaplan, 2005).

4 Astronomic, Geodetic, and Grid Azimuths

455 Astronomic and geodetic azimuths and bearings are curvilinear; likewise, the meridians of longitude always converge towards the poles. Geodetic coordinate systems are modeled after a reference ellipsoid, e.g., WGS-84 and GRS-80. In surveys covering areas of great distance, the idea of square corners is therefore impossible, unless two great circles intersect one another.

4.1 Astronomic Azimuth

460 The astronomic azimuth of a celestial body is measured *clockwise* from the north or south astronomic pole orthogonal to the *instantaneous equator* (Fig. 2) of the observer on the geoid at point *P* (Fig. 4). The term instantaneous refers to a singular point on the earth orthogonal to the equipotential surface at point *P* in the direction of the local gravity vector at that instant of observation (refer to Section 2.2.1 and Fig. 4) due to DoV, and does not point directly the earth's center of mass.

4.2 Geodetic Azimuth

465 Geodetic azimuth is a line measured *clockwise* from the north geodetic pole, measured along a horizontal plane orthogonal to the geodetic longitude of the observer. Geodetic azimuth differs from astronomic azimuth for the reasons of the deflection of the vertical discussed above. Reduction of the astronomic azimuth between the previously mentioned survey control stations using a Laplace correction will produce a geodetic azimuth of that line. Geodetic azimuths will always have a Normal Section (NS) Forward and Back azimuth because they are measured along curvilinear lines between the projected geodetic meridian and the projected geodetic line Deakin, 2009a, Deakin, 2009b). The difference between the forward and back azimuths will 470 always exceed 180° and is referred to as *spherical excess*, whereas a reciprocal grid azimuth is exactly 180° opposite.

4.3 Grid Azimuth

Grid azimuth is a line measured from a point on the observer's grid meridian *clockwise* to another point on a mapping projection, such as a state plane or Universal Transverse Mercator (UTM) coordinate system. The observer's grid meridian is parallel to the central meridian of the mapping projection; however, the convergence (θ) increases east or west from the central 475 meridian the farther the survey project site becomes. θ is positive (*added*) for surveys east of the central meridian and negative (*subtracted*) for surveys west of the central meridian (further discussed in Section 9 and Fig. 13 & 14). It should be noted that the sign of these values works well with azimuths—and not bearings—for the simple fact that much confusion occurs when applying the correct sign in any one of the four cardinal bearing quadrants.

4.4 What is True North?

480 There are opinions as to what is the definition of true north, which can be argued “relative to what?” Contemporary wisdom would define it as being derived directly from measuring a celestial body such as *Polaris*; hence, true north = astronomic north.



However, this cannot be the case due to the position of the astronomical coordinate system not being aligned and in harmony with the ECEF coordinate system of a standardized geodetic reference system as discussed in Section 2, and from the effects of precession and nutation discussed in Section 3. Therefore, it can be inferred that *the definition of true north is the north geodetic pole*. This inference is supported by the fact that a stable, fixed geodetic pole (the rotational axis of the earth) is necessary for satellite positioning and navigation, and mapping.

5 Spherical Components of the Astronomical Triangle

The astronomical triangle is a system of spherical circles projected onto the celestial sphere with the earth at the center. The astronomic azimuth is determined by the solution of the astronomical triangle formed by the intersection of great circles whose vertices forms a Polar-Zenith-Star (*PZS*) triangle (Fig. 7). Each side of the triangle forms arcs on the celestial sphere complimentary (abbreviated “*CO*–”) to the angles that become the latitude (Φ) of the observer; the zenith distance (*ZD*)—the vertical angle measure from the observer’s zenith of the theodolite to the star and is the *CO*-altitude; and the declination (δ) of the star. The declination of a star is the angular measurement on an hour circle passing through the hour circle of the star to the equator and may be north (*positive*) or south (*negative*) of the equator depending on which star is observed. The Greenwich Hour Angle (G.H.A.) of a star = G.H.A. φ + S.H.A. of the star. Values of G.H.A. and S.H.A. can be found in the Nautical Almanac and the Astronomical Almanac published by the U.S. Naval Observatory.

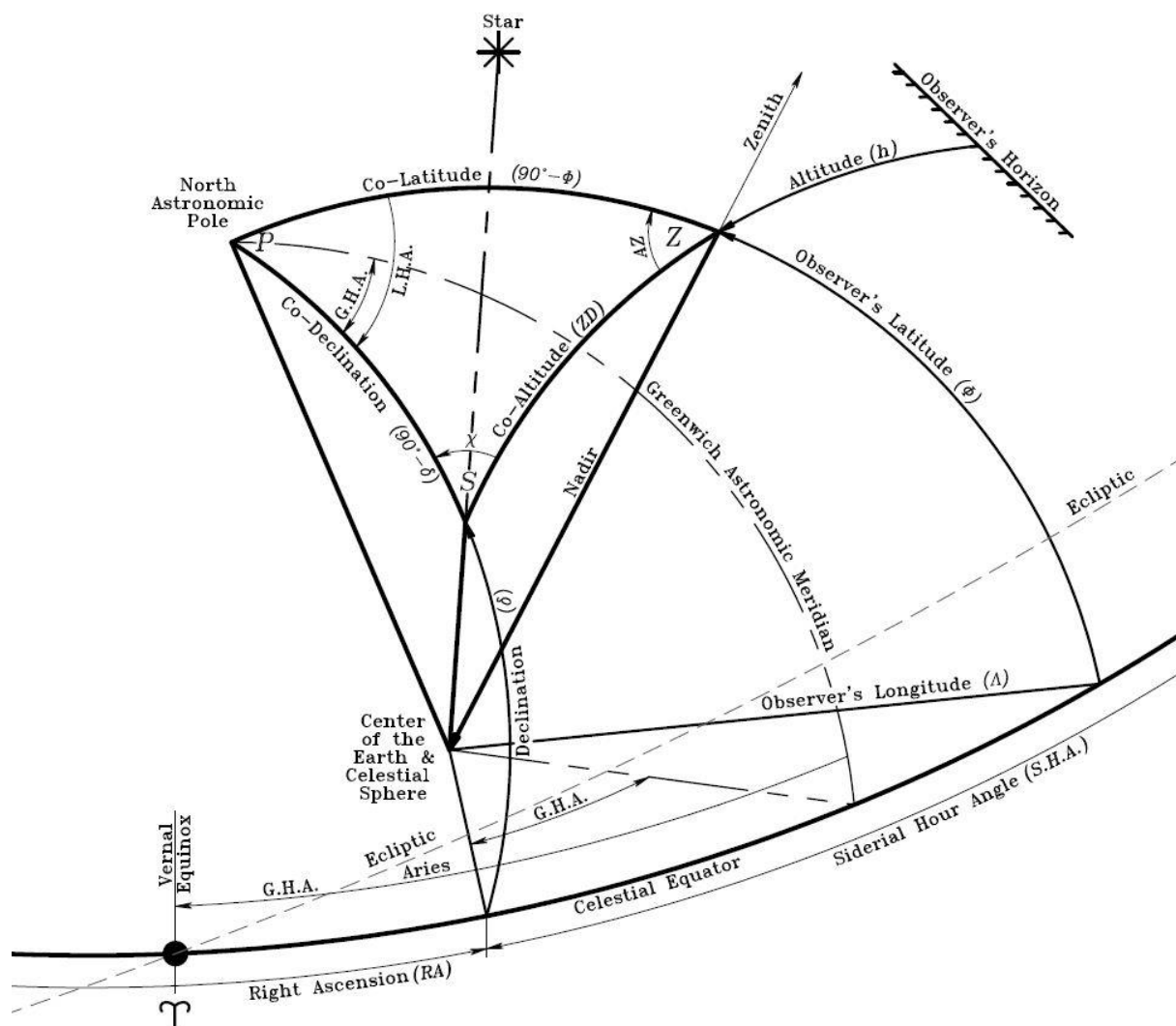
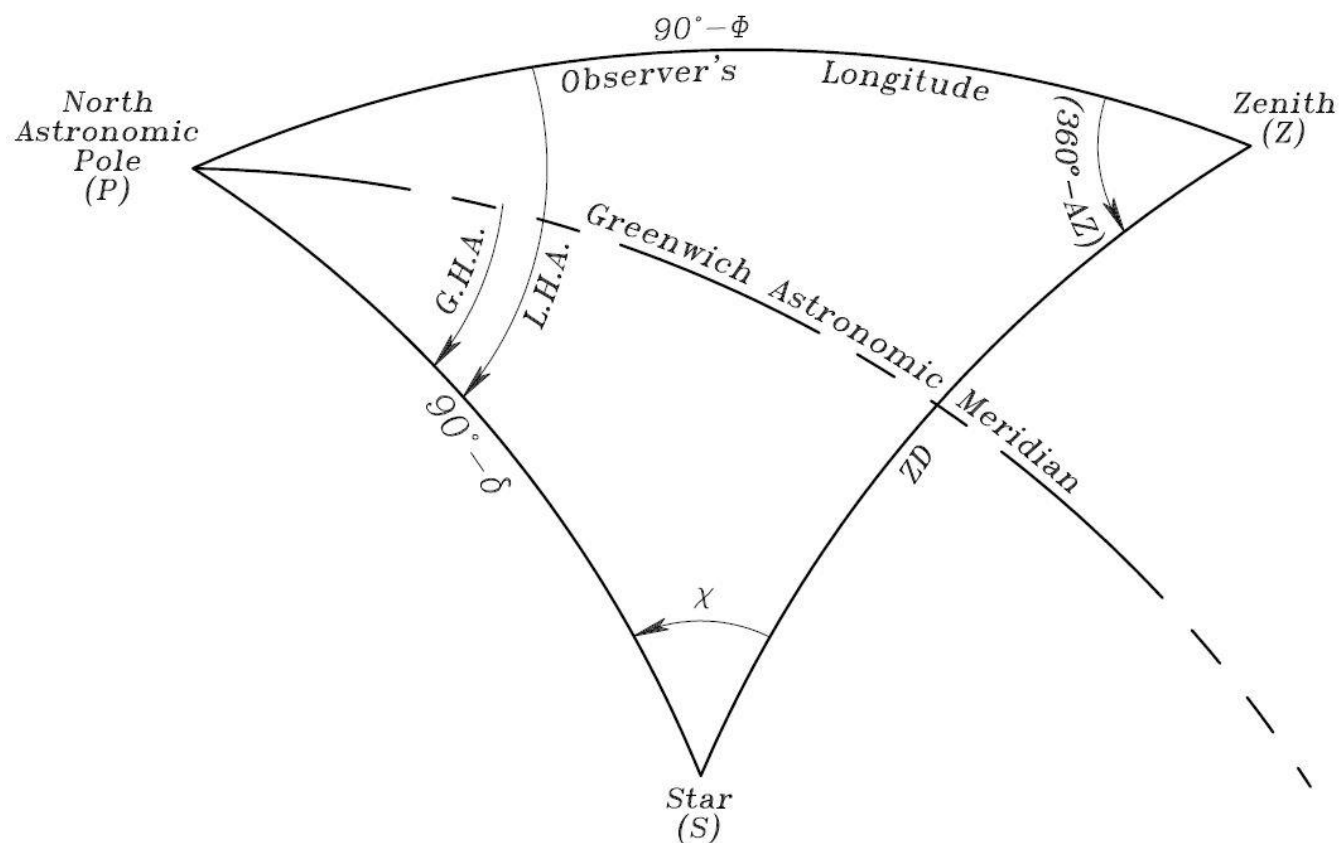


Figure 7: The PZS triangle. Observer is east of the Greenwich astronomic meridian.

In Fig. 7, the vertex P is the north astronomic pole and forms the L.H.A. between the sides $CO-\phi$ ($90^\circ - \phi$) and $CO-\delta$ ($90^\circ - \delta$) on the Equatorial System, and is always an interior angle at P . L.H.A. (also referred to as the t -angle), increases westward from 0° to 360° on the equator from the observer's longitude to the hour circle passing through the star. L.H.A. is determined by (Bowditch, 1802; USNO, 2025; see Fig. 8 & 9):

$$\begin{aligned} &+ \text{east} \\ \text{L.H.A.} &= \text{G.H.A. longitude} \\ &- \text{west} \end{aligned}$$

(5)



550 Figure 8: *PZS* triangle showing observer east of the Greenwich astronomic meridian. $L.H.A. = G.H.A. + E. Longitude$.

555

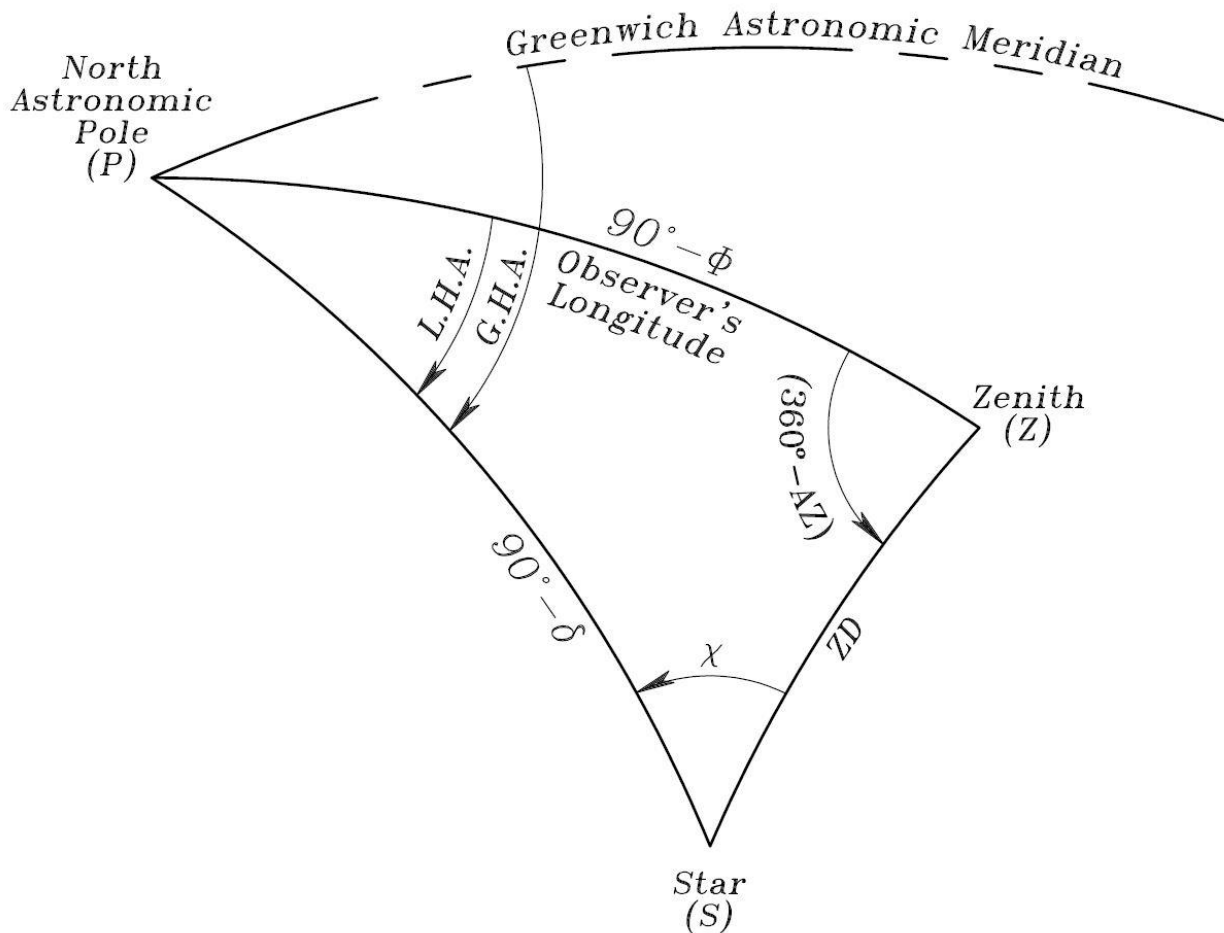


Figure 9: *PZS* triangle showing observer west of the Greenwich astronomic meridian. $L.H.A. = G.H.A. - W. Longitude$.

Referring to Figs. 8 & 9, the range of G.H.A. is from 0° to 360° *westward* beginning at 0° on the Greenwich astronomic meridian back to the Greenwich astronomic meridian. To keep L.H.A. within the range of 0° to 360° , add or subtract multiples of 360° .

5.1 Spherical Trigonometry

The use of spherical trigonometry has applications in surveying and navigation. It is the method used in astrogeodetic surveying, celestial navigation, and reduction of parametric latitude onto an auxiliary sphere for solving a geodesic line (Rolbiecki, 2024). The solution of the *PZS* triangle follows spherical trigonometry and the supplemental law of cosines and sines. We may substitute the spherical angles *PZS* in Fig. 7 with angles *A, B, C*, respectively, and substitute the complementary sides *ZD, CO— Φ , and CO— δ* with sides *a, b, c*, opposite of *A, B, and C*.



5.1.1 Spherical law of cosines

$$570 \quad \cos(a) = \cos(A) \sin(b) \sin(c) + \cos(b) \cos(c) \quad (6)$$

We may apply this similarly to the other two sides as well. Sides A , B , & C are central angles and the variables, a , b , & c are their opposite sides, respectively, whose lengths are represented in arc degrees.

5.1.2 Supplemental law of cosines

$$\cos(A) = \cos(a) \sin(B) \sin(C) - \cos(B) \cos(C) \quad (7)$$

575 If we want to solve unknown side a , and know the internal angles A , B , & C :

$$\cos a = \frac{\cos(A) + \cos(B) \cos(C)}{\sin(B) \sin(C)} \quad (8)$$

5.1.3 Spherical law of sines

$$\frac{\sin(a)}{\sin(A)} = \frac{\sin(b)}{\sin(B)} = \frac{\sin(c)}{\sin(C)} \quad (9)$$

To solve for unknown variable $\sin(a)$, invert and cross multiply:

$$580 \quad \sin(a) = \frac{\sin(A) \sin(b)}{\sin(B)} \quad (10)$$

Conversely,

$$\frac{\sin(A)}{\sin(a)} = \frac{\sin(B)}{\sin(b)} = \frac{\sin(C)}{\sin(c)} \quad (11)$$

6 Observational Procedures for Obtaining an Astronomic Azimuth of Practical Significance

The astronomic azimuth of stars and solar system bodies in the northern hemisphere is discussed below using the equatorial
 585 star Sirius (*α Canis majoris*) as an example. For this practical example, procedures follow specifications for ordinary
 astronomical observations (not for First Order geodetic accuracy) using the *Method of Azimuth Determination at any Hour
 Angle* (Thomson, 1981; Elgin et al., 2001; Lambrou and Pantazis, 2008) for measuring horizontal angles to the heavenly body.
 It is assumed the surveyor is well versed in operating an optical theodolite or total station and procedures to eliminate parallax
 when sighting through the telescope. Observations of both circumpolar and equatorial stars may be done using a theodolite or
 590 electronic total station set up over a survey control station with a back-sight survey reference station. To obtain the appropriate
 accuracy, either instrument must be capable of measuring to 1 arcsec or less in both horizontal and vertical circles and taking
 a station angle reading by pointing the telescope to the celestial body in direct (Face I) and reverse (Face II) to obtain a mean
 station angle and t -angle with their corresponding UT1 of observation; accurate stopwatch and access to the broadcast Radio
 Time Signal; and barometer and thermometer for determining the Index of Refraction if the observer is measuring the zenith
 595 distance of a heavenly body. This procedure will be discussed in Section 7.6 and Eq. 22.

The example astronomic observation took place on the night of May 2, 1998, at Camp McCain, Mississippi (United States).
 The theodolite used was a Spectra Physics Geodolite Constructor™ electronic total station. The station used to set up the
 theodolite was a bronze disk benchmark set in concrete that was established in World War 2 and was stamped “228.56” on the
 top of the disk and reported to be the elevation in feet (69.665 meters) above sea level. It was requested that the survey mark



600 was to be used as a declination station for use in the orientation of field artillery and other indirect fire control procedures on
 the Camp McCain Military Reservation. The assumed position of the declination station was taken from a Magellan Systems
 GPS 3000™ receiver configured to the North American Datum of 1927 (NAD27) horizontal datum.

6.1 Establishing the Declination Station

On the ending evening twilight of May1, 1998, the theodolite was set up over the declination station and back sighted a
 605 distant traverse station designated as RM1, an iron rebar set on the north edge of an asphalt road running west-east along the
 north margin of the Camp McCain military parachute drop zone, a distance of 3 536.72 ft (1 077.994 m) from the declination
 stations.

The backsight to RM1 was designated to be the astronomic azimuth reference for the declination station. At approximately
 01 h 30 m UTC, the star Sirius was sighted, and observations took place from 01 h 36 m to 01 h 45 m UTC. From the backsight,
 610 the instrument was pointed to Sirius and a stopwatch synchronized to the WWV radio time signal was started. The instant the
 star crossed the vertical reticle in the instrument's telescope, UTC was recorded along with the horizontal angle from the
 backsight to Sirius, and a correction for DUT1 was made to obtain UT1. Three successive readings with the telescope in its
 direct (Face I) position, and three successive readings with the telescope reversed (Face II) were made to obtain the mean
 station angle. The mean of the six horizontal angles as well as a mean of the six UT1 recordings was obtained to solve the
 615 astronomic azimuth of RM1 (see Table 1). With the azimuth of the star Sirius accurately established, the astronomic azimuth
 from the Declination station to RM1 was determined.

**Table 1: Field notes for astronomic observation of Sirius at the Camp McCain, Mississippi declination station
 (33°41'50" N // 089°42'38" W) on May 2, 1998 (UTC).**

Instrument set up on: Declination Station		
Back sight Station: RM1		
Horizontal Reference Angle: 000°00'00" (mean of Direct & Reverse)		
Telescope Pointing (D- direct), (R-reverse)	Horizontal Angle to Sirius	Universal Time (UT1) corrected for DUT1 (-0.2s)
D	98°36'35"	01 ^h 36 ^m 02.8 ^s
D	98°49'51"	01 ^h 37 ^m 17.0 ^s
D	99°04'01"	01 ^h 38 ^m 36.7 ^s
R	279°35'25"	01 ^h 41 ^m 34.2 ^s
R	279°51'41"	01 ^h 43 ^m 06.8 ^s
R	280°03'01"	01 ^h 44 ^m 11.9 ^s
Mean	99°20'05.7"	01^h 40^m 08.2^s



620 6.2 Computations

Required are the G.H.A. and Declination of Sirius (*α Canis majoris*) May 2, 1998, UT1 01^h 40^m 08.2^s:

A. Greenwich Hour Angle and Declination of Sirius at 01^h 40^m 08.2^s (Apparent positions of stars, moon, sun and planets interpolated from the U.S. Nautical Almanac, year 1998):

625
$$\begin{aligned} \text{G.H.A.} &= 143^{\circ}32'54'' \\ \delta &= 16^{\circ}42'57'' \text{ S} \end{aligned}$$

B. Calculate Local Hour Angle (L.H.A.):

$$\text{G.H.A. } 143^{\circ}32'54'' - \text{W Longitude } 089^{\circ}42'38'' = 53^{\circ}50'16''$$

C. Calculate astronomic azimuth (AZ) of Sirius at 01^h 40^m 08.2^s:

630
$$AZ = 54^{\circ}26'11'' \text{ from the Modified Hour-Angle formula (Lambrou and Pantazis, 2008):}$$

$$\tan^{-1} AZ = \frac{-\sin L.H.A.}{\cos \Phi \tan \delta - \sin \Phi \cos L.H.A.} \quad (12)$$

where Φ = station latitude; δ = star's declination

635 *Note:* If measuring eastern (+) or western (−) limbs of sun, moon and planets, apply the semidiameter correction in arcsec (Bowditch, 1802; Nassau, 1948; USNO, 2025):

$$\sin S = \frac{R}{\rho} \quad (13)$$

where R is the mean radius of the earth, ρ (Greek rho) is the mean geocentric distance to the sun, $H = 90^{\circ} - ZD$.

To obtain H without direct observation, we get

640
$$\sin^{-1}(H) = \sin \Phi \sin \delta + \cos \Phi \cos \delta \cos t \quad (14)$$

The Nautical Almanac provides sun values for S every tenth day; for the moon, S is given every day.

D. Calculate the astronomic azimuth of Sirius:

Table 2: Correction for astronomic azimuth from the AZ angle.

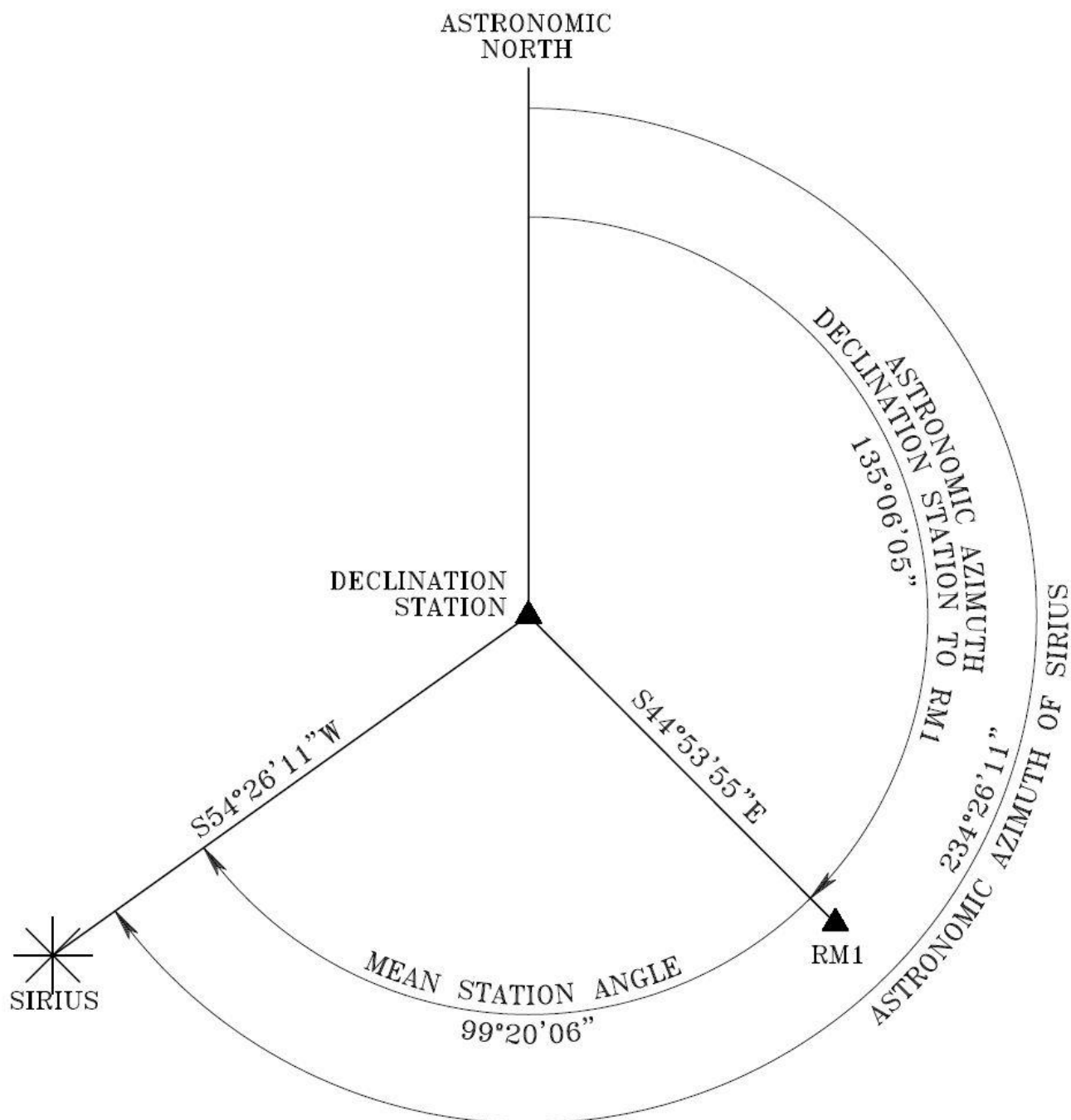
When LHA is:	AZ is (+)	AZ is (−)
0 - 180°	Add 180°	Add 360°
180 - 360°	Add 0°	Add 180°

645

$0^{\circ} < \text{LHA} < 180^{\circ}$; AZ is positive $\therefore$ add 180° to AZ:

$$= 54^{\circ}26'11'' + 180^{\circ} = 234^{\circ}26'11''$$

Plot azimuth using a Time Diagram (Fig. 10):



650

Figure 10: Time diagram drawn in the determination of the astronomic azimuth of Sirius from declination station to Reference Mark 1 (RM1).



7 Corrections Made to the Astronomic Azimuth for First Order Geodetic Accuracy

7.1 Astronomic Refraction

From the surface of the earth, the direction of light coming from a star is changed by refraction as it passes through the atmosphere. For practical purposes in this paper, refraction takes into account the different layers of density and water vapor through the medium known as the troposphere. Assuming the troposphere has layers of increasing density closer to the earth's surface, the path of light from the star changes direction as it passes through each layer. The effect known as *astronomic refraction* causes celestial objects to appear higher above the horizon than they really are (Nassau, 1948; Roelofs, 1950; Davis et al., 1966; Burnett and White, 1970; Thomson, 1981; Elgin et al., 2001).

Astronomical refraction affects zenith distance readings taken from the vertical circle of the theodolite where the determination of latitude or azimuth other than the Hour Angle Method is used. The correction for astronomic refraction (r) is given using Comstock's empirical formula (Nassau, 1948; Stebbins, 1938):

$$r = \frac{983b}{460 + t} \tan z' \quad (15)$$

where r is in arcsec, b is barometric pressure in inches, t is temperature in degrees Fahrenheit, and z' is the observed ZD . The correction, r is added to the observed zenith distance. The inverse of z' applies if the vertical circle only reads to h' (observed altitude), and the formula is rewritten using the function of $\cot h'$, whereby r is subtracted.

A comprehensive explanation of astronomic refraction as it relates to index of refraction affected by atmospheric temperature and pressure is given in C&GS (1947).

7.2 Level Correction for Inclination of Standing Axis

Instruments used in precise control surveys are not free from inclination of standing axis error and need to be adjusted so that the vertical circle reading of the star is orthogonal to the horizontal circle. Early astronomical theodolites used a striding level to measure this error and compensate for it (Nassau, 1948, C&GS, 1947). Later models of the Wild T2 theodolite left off the striding level and replaced it with a plate level with a sensitivity of 20 arcsec/2 mm graduations or with an optional 10 arcsec/2 mm plate level. Modern total stations have dual-axis compensation that automatically corrects the inclination error.

Assume that a Wild T2 theodolite is used; the inclination of standing axis error can be determined by reading the vertical circle with the automatic index by the following procedure outlined in the Wild T2 User Manual. With the instrument set up and levelled over the station, point to the star with the telescope in direct (Face I), take a ZD reading of the star. Lock the vertical circle clamp and rotate the instrument 90° to the right of and read the vertical circle (ZD_R). This pointing is at right angles to the star. Rotate the instrument 180° to and reread the vertical circle (ZD_L). The Component of Inclination (I) is expressed as:



$$I = \frac{ZD_R - ZD_L}{2} \quad (16)$$

7.3 Correction for Curvature of the Path of the Star

As the star moves across the sky between direct and reverse readings, it traces a curved instead of a straight path. The star's azimuth and t -angle (L.H.A.) changes as well, and the rate of change is directly related to the curved path and the time between each pointing. The effect is small, but a "second-order" curvature correction (C_c) in arcsec to the azimuth should be used (Thomson, 1981; Nassau, 1948; C&GS, 1947):

$$C_c = \tan AZ_0 \frac{1}{n} \sum_{i=1}^n \left(\frac{2 \sin^2 \frac{1}{2} \Delta t_i}{\sin 1''} \right) \quad (17)$$

where AZ_0 is the mean azimuth determined from all direct and reverse observations; n = the number of separate pointings; and $\Delta t_i = t_i - t_0$ (the difference between the L.H.A. of a single pointing and the mean L.H.A.).

7.4 Probable Error

Observations are subject to systematic errors and blunders. These can be mitigated by taking out these errors beforehand. Random errors occur due to observational noise and can be estimated by a statistical test. The assumption is that the mean of the observations falls under the probability of a normal distribution. The probable error (e) of a single observation set is (C&GS, 1947; described as mean error by Roelofs, 1950, p. 142):

$$e = \sqrt{\frac{\sum v^2}{n-1}} \quad (18)$$

where v is the residual from the mean and n is the total number of observation sets taken.

The probable error of the result (e_R) of the total number of observations is:

$$e_R = \sqrt{\frac{\sum v^2}{n(n-1)}} \quad (19)$$

7.5 Aberration of Starlight

The earth is not stationary in space, but in motion along its ecliptic path around the sun. Light from a distant object such as stars takes time to reach the observer. Due to the velocity and motion of the earth with the observer on its surface and the finite speed of light from the star to the observer at $\approx 300\,000 \text{ km} \cdot \text{s}^{-1}$ (Roelofs, 1950), the apparent path of light becomes displaced, causing the observer to tilt the telescope in the direction of the motion of the earth in order to align the telescope to the star. This phenomenon is known as stellar aberration (Fig. 11) and is comprised of three components called secular



aberration, annual aberration, and diurnal aberration (Nassau, 1948; Roelofs, 1950; Burnett and White, 1970; Thomson, 1981; USNO, 2005). Secular aberration is associated with the uniform motion of our solar system in space and disregarded in topocentric and geocentric positions. Annual aberration is an effect resulting from the orbit of the earth around the sun and accounted for in the apparent places of stars in published ephemerides.

Diurnal aberration (dS) is due to the daily rotation of the earth around its axis. It has a small but measurable effect and needs to be accounted for in precise astronomic observations for azimuth (Nassau, 1948; C&GS, 1947). In Fig. 11, let O be the telescope's objective and E the eye of the observer. OE is the telescope pointed in the direction of the motion of the observer towards the ray of light (R) of the star. EE' is the elapsed time and distance of the motion of the observer when R passes through the telescope and reaches E' . S is the required angle of the telescope to the star and S' is the apparent angle formed by the motion of the observer and the displacement of the light to E . Diurnal aberration is a function of the latitude of the observer, and the correction ($S - S' = dS = -0''.319 \cos \Phi \sin S$) becomes dZ , the diurnal correction in arcsec to the azimuth angle, AZ (Nassau, 1948; Thomson, 1981):

$$dZ = \pm 0''.319 \frac{\cos AZ \cos \Phi}{\cos h} \quad (20)$$

where AZ is the azimuth of the star, Φ is the astronomical latitude of the observer, h is the star's observed vertical angle above the horizon ($h = 90^\circ - ZD$)

The correction for dZ is negative for stars observed at upper culmination and positive for stars observed at lower culmination (C&GS, 1947).

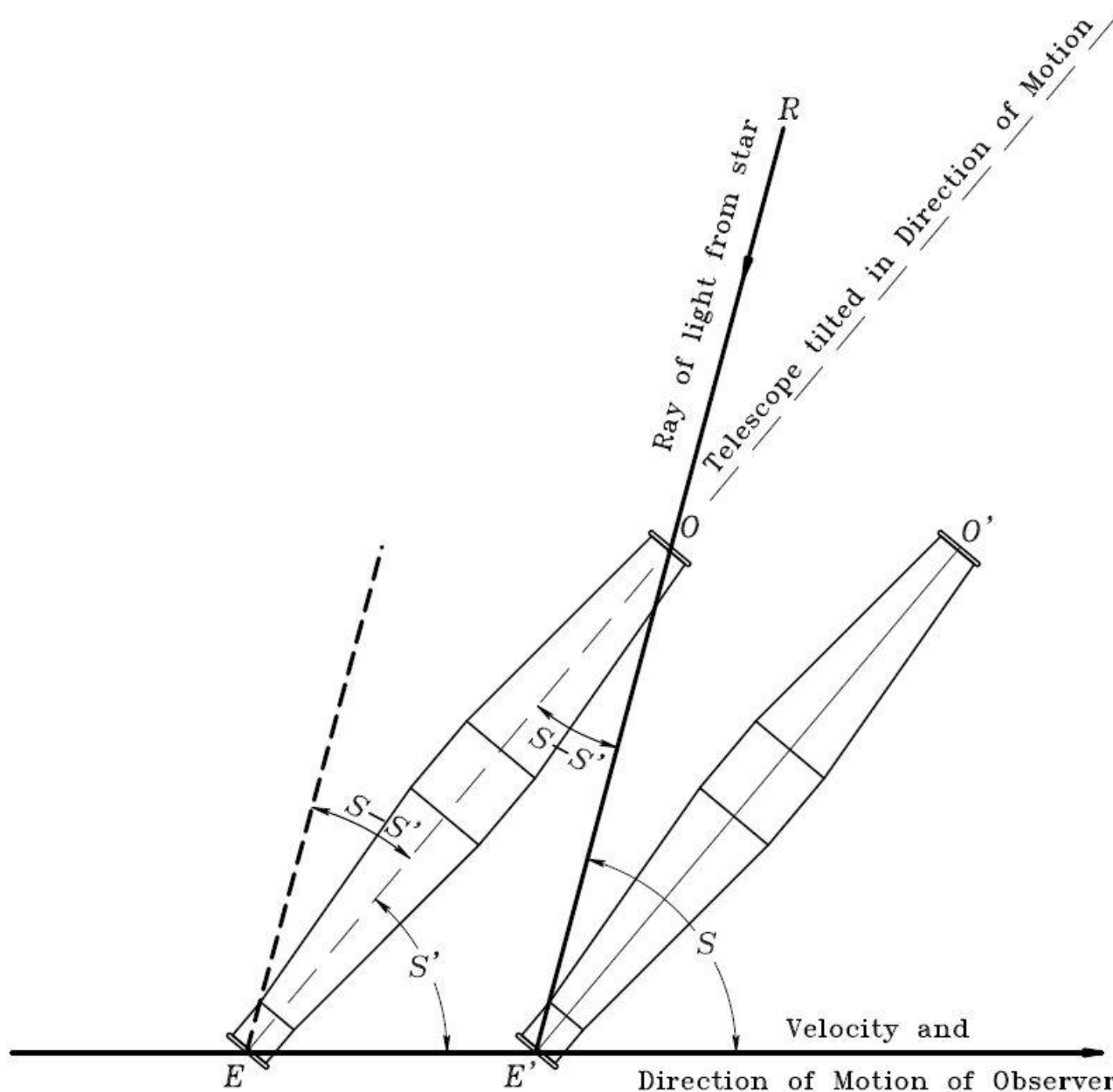


Figure 11: Author's rendering of Nassau's stellar aberration seen in the velocity and direction of motion of the observer (Nassau, 1948).

7.6 Horizontal Parallax of Solar System Bodies

In the previous discussion of the celestial sphere in Section 2.2, the distance of stars from the center of the earth is considered to infinity, and a parallax correction (p) from a topocentric position is not needed (Nassau, 1948). However, the distance between the earth and solar system bodies is small—relative to a star's distance—and a horizontal parallax correction (π) is



745 required (Nassau, 1948; Roelofs, 1950; Burnett and White, 1970; Thomson, 1981; Elgin et al., 2001; Mignard, 2004; see Fig.
 12) if the azimuth is determined from measuring the zenith distance, ZD (vertical measured angle) of the solar system body
 when time is unknown (refer back to Fig. 7 & 8):

$$\cos^{-1}AZ = \left(\frac{\sin \delta - \sin Lat \times \sin h}{\cos Lat \times \cos h} \right) \quad (21)$$

750 where Lat is the assumed latitude of the observer, δ is the declination of the solar system body, and h is $90^\circ - ZD$

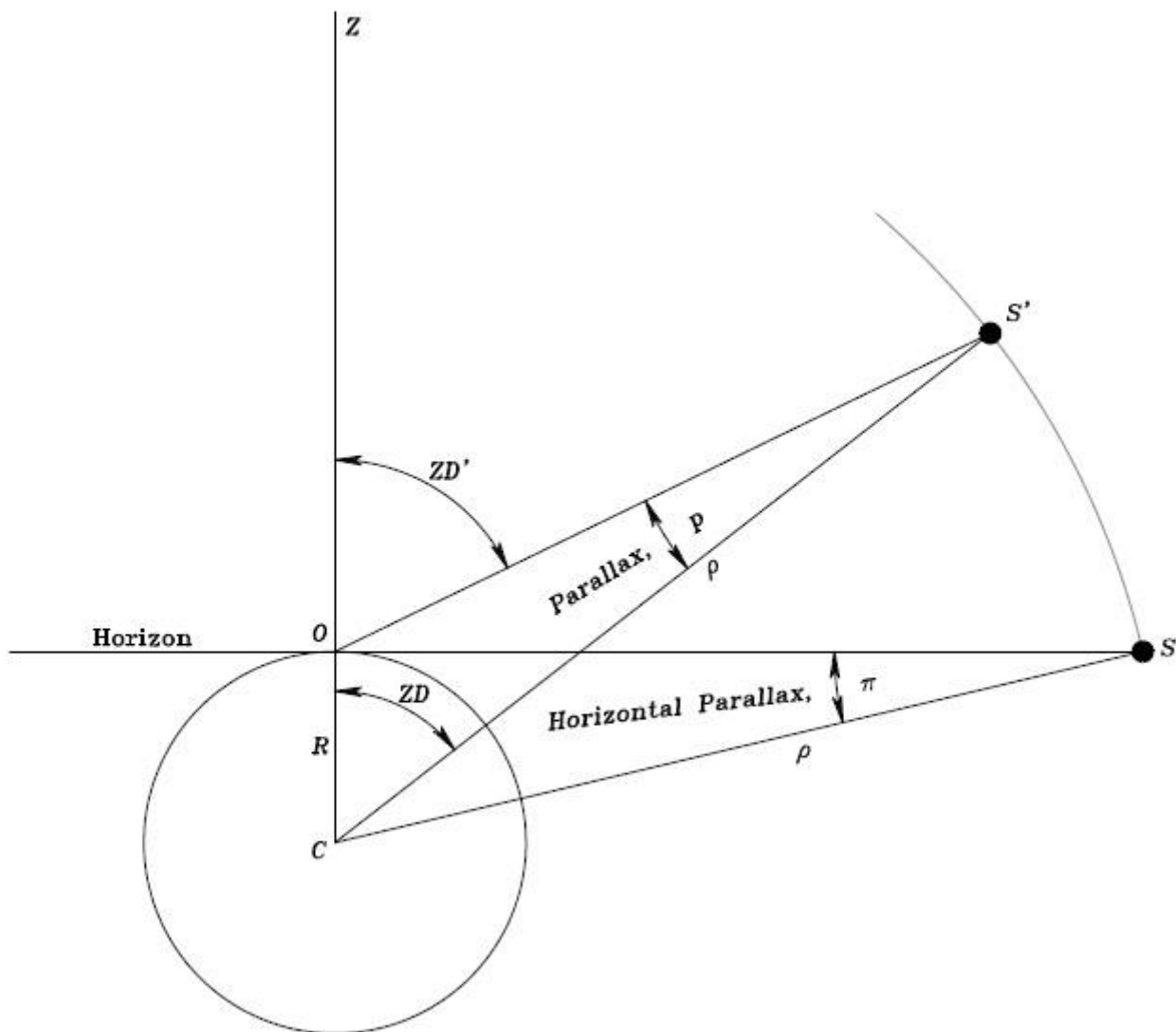


Figure 12: Geocentric parallax (p) and horizontal parallax (π).



755 For practical purposes of spherical geometry, consider the earth as a sphere, where the mean radius, R of the earth at the equator is 6 371.008 8 km and ρ is the sun's distance to the center of the earth (149 597 871 km); then from $OS'C$ in Fig. 12 we have

$$\sin \pi = \frac{R}{\rho} \quad (22)$$

760 Where π is the horizontal parallax, R is the mean radius of the earth, ρ is the mean geocentric distance to the sun
 The parallax p of the sun at any observed zenith distance ZD' (Nassau 1948) is:

$$\sin p = \sin \pi \sin ZD' \quad (23)$$

765 In the case of a solar observation, the horizontal parallax is $< 8.8 \text{ arcsec} >$; hence the parallax correction will not exceed $08''.8$. The values may be found for any given day in the Nautical Almanac and American Ephemeris or by the following equation (Nassau 1948):

$$ZD = ZD' - 8''.8 \sin ZD' \quad (24)$$

770

where p is always subtracted from ZD'

If the observer is observing the altitude h using a sextant or transit instrument, we have

$$775 \quad h = h' + 8''.8 \cos h' \quad (25)$$

In comparison, the horizontal parallax of Jupiter and Saturn due to their vast distance from earth is $< 1 \text{ arcsec}$, and π can be disregarded due to observational noise. Venus, being close to the sun, has a horizontal parallax of around 36 arcsec (Mignard, 2004). The Moon being much closer to the earth with a mean distance to the center of the earth of 384 400 km, having a
 780 horizontal parallax of $\approx 1^\circ$, which can be obtained by

$$\sin p_m = \sin \pi_m \sin ZD' \quad (26)$$

and if using the observed altitude, h

785

$$p_m = \pi_m \cos h' \quad (27)$$

As seen in the additional horizontal parallax corrections required for solar system bodies, the easiest (and preferred) method of determining astronomical azimuth is the Azimuth Determination at any Hour Angle formula (Eq. 12; Lambrou and Pantazis, 2008).

8 Laplace Correction in Azimuths

For astrogeodetic orientation, the Laplace correction is required (Lambert, 1964; Featherstone, 1999; Soler et al., 2014).³ Due to the different coordinate systems between the astronomic and geodetic reference frames (refer to Fig. 2 & 3, respectively), the two deflection components, ξ & η are included in the computation of the Laplace correction in order to convert an astronomic azimuth between two survey control stations to geodetic azimuth. The Laplace correction takes on two forms (Soler et al., 2014): Complete (Extended) and Simplified (horizontal Laplace).

The Extended Laplace correction is expressed by the addition of the following two terms:

$$\eta \tan \varphi \text{ (First Term)} \quad (28)$$

$$(\xi \sin AZ - \eta \cot AZ) \cos ZD \text{ (Second Term)} \quad (29)$$

where AZ is the computed astronomic azimuth between the two survey control stations and ZD is the zenith distance to the reference (back sight) station.

The Second Term is used if ZD departs significantly from 90° sighted to the reference station. In most instances, the First Term is only used because the reference station is at sufficient distance to where the vertical circle of the theodolite is sighted nearly horizontal to it; hence, the “horizontal Laplace” correction. By inspection of the algebraic sign, the computed Laplace correction is added to the clockwise astronomic azimuth, producing a geodetic (Laplace) azimuth.

9 Convergence of the Meridians

Recalling Section 5, it should be noted that the sum of three angles in an oblique spherical triangle is always greater than 180° , but less than 540° : thus, $180^\circ < A + B + C < 540^\circ$. The positive quantity of $A + B + C - 180^\circ$ is known as *spherical excess*. By the application of the Legendre theorem, one third of the spherical excess is subtracted from each angle to get 180° (C&GS, 1911). This was especially important in the triangulation of arcs connected to geodetic stations across the country.

To illustrate this, imagine a long line between Station A and Station B, and an intermittent point is staked in the ground on-line between A & B. The theodolite is brought forward from A and set up on the intermittent point set on-line. Assuming the theodolite is in adjustment and set up correctly, a backsight is taken on Station A and the horizontal circle is clamped. Plunge the telescope and sight Station B. The vertical cross hair of the theodolite will not bisect Station B because of the curve of

³ Named after Pierre-Simon Laplace (1749—1827), a leading physicist and mathematician of his day.



alignment along a normal section curve on the ellipsoid (Deakin 2009a, p. 2). When laying out a true meridian, it is on a great
820 circle. The meridian is the only great circle where a forward and back azimuth are exactly 180° apart.

In the United States, all 50 states have adopted a state plane coordinate system having a low-distortion (1: 10 000) mapping
projection using a Lambert conformal conic projection or a transverse Mercator projection. The state of Alaska, covering 10
state plane zones and extending from its panhandle west into the Aleutian Islands, uses the Lambert, transverse Mercator, and
oblique Mercator projections, the latter in Zone 1 along the west Pacific coast in its southwest panhandle region, and is the
825 only state in the United States to use this projection. The Lambert conformal conic projection is used for states having a greater
east-west area versus the transverse Mercator projection for states having a greater north-south area of coverage. The
advantage of using the oblique Mercator projection is to align a mapping projection to cover areas with an oblique orientation.
All state plane projections have false northings and false eastings so that the grid coordinate values are never a negative value.

For example, the State of Texas has adopted the Texas State Plane Coordinate System using the Lambert conformal conic
830 projection that covers five zones: Texas North, Texas North Central, Texas Central, Texas South Central, and Texas South. In
each zone, there is a central meridian and two standard parallels, a latitude of origin, and false northings and eastings (Fig. 13).

When using a mapping projection such as the Lambert conformal conic or transverse Mercator, the survey represented on
the projection takes on a plane surface tangent to a point on the curved surface of the earth. The “grid” north lines on the
projection remain parallel and never converge as do the meridians of longitude on a spherical or ellipsoid surface. The
835 convergence angle is known as the “mapping angle” (θ).

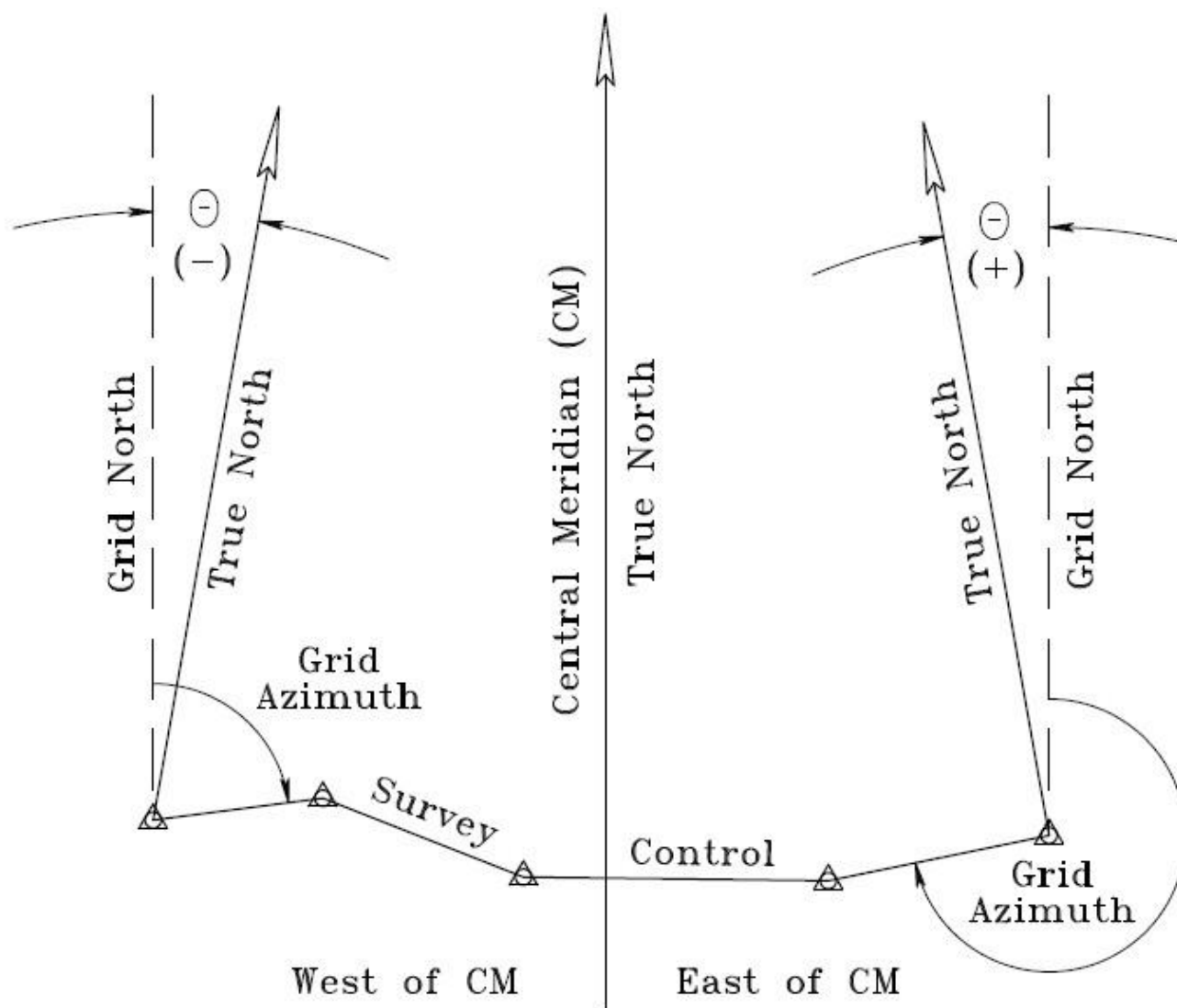


Figure 13: Convergence of the meridians on opposite sides of the central meridian. True (geodetic) azimuth = grid azimuth $\pm \theta$.

840

A good rule of thumb taught in military land navigation to convert grid-to-true, magnetic-to-true; true-to-grid; or true-to-magnetic is by using the acronym LARS (Left-Add Right-Subtract; Fig. 14).

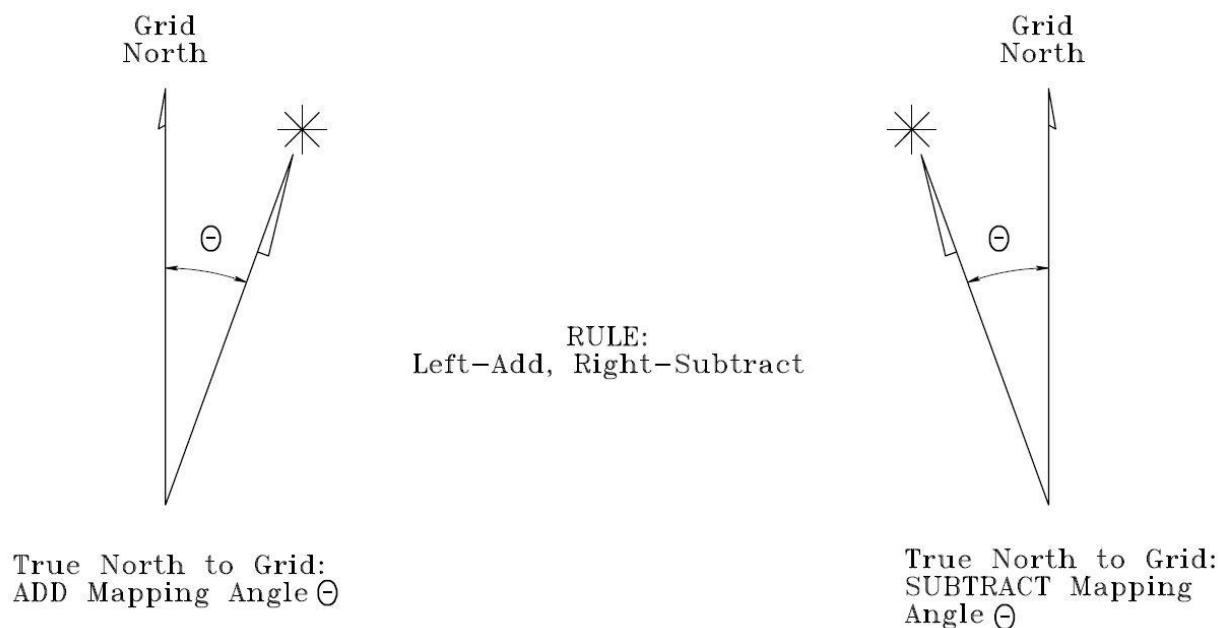


Figure 14: The Left-Add, Right-Subtract method.

845 In high-order surveys extending over four miles east-west, a “second-term” correction or arc-to-chord ($t-T$) is applied. By
 applying the convergence or mapping angle correction (θ) of any map projection and the $t-T$ angle to the geodetic azimuth, the
 grid azimuth (t) is obtained. For a comparison of the geodetic and grid azimuth of a line between two survey control stations
 A & B, see Fig. 15 and Table 3. Note that T represents the projected geodetic azimuth based on the curvature of the earth and
 represents the true observed angle between any two geodetic azimuths (see Stem, 1989 for computations of θ and $t-T$).⁴

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⁴ Stem, J.E. (1989). “State Plane Coordinate System of 1983.” National Oceanographic Atmospheric Administration Manual
 NOS NGS 5. Rockville, MD, p. 12-50.

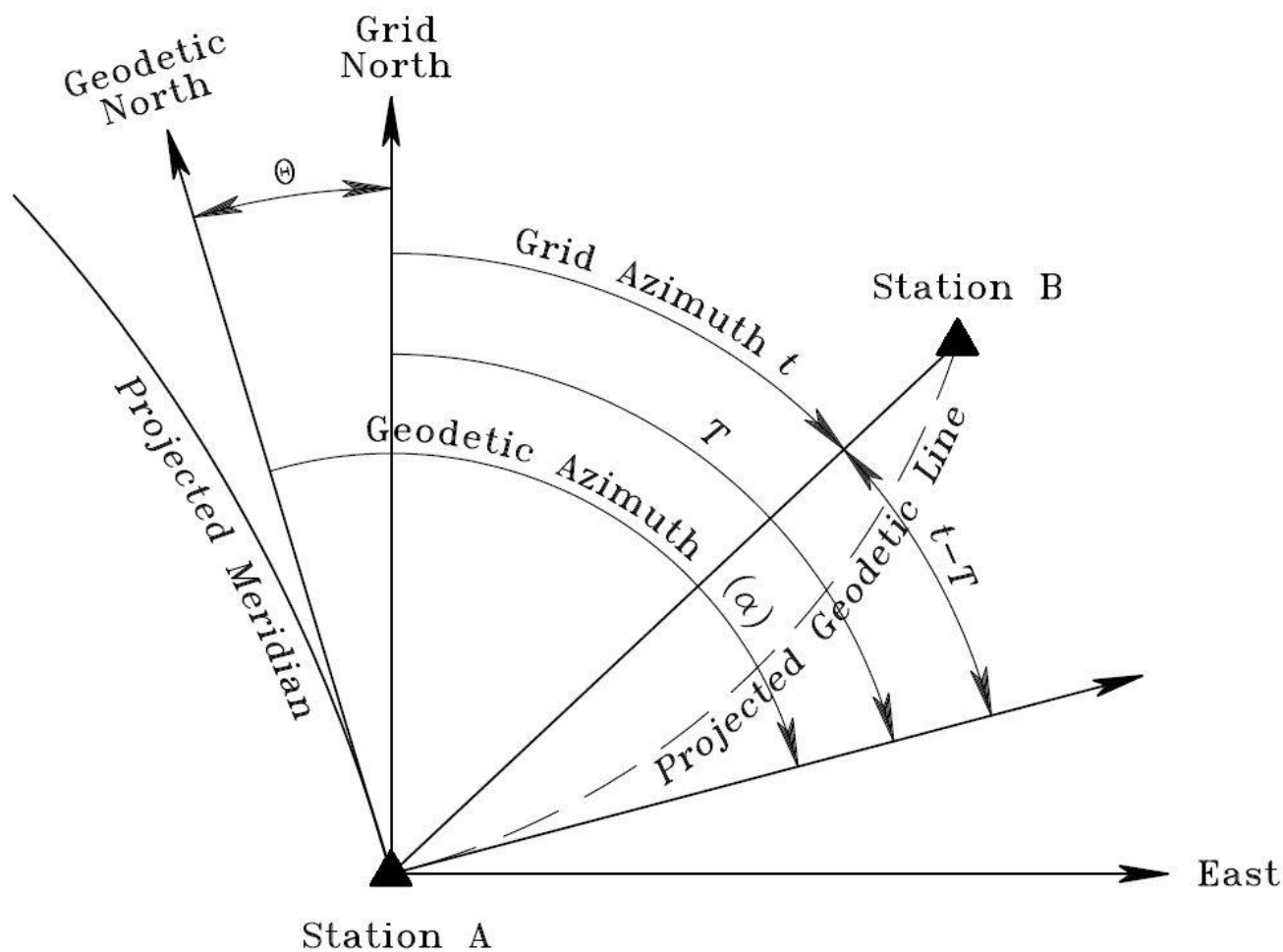


Figure 15: Converting geodetic azimuth (α) to grid azimuth (t) from station azimuth A-B by applying the $t-T$ correction and θ .

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Table 3: Inverse between the found southeast corner of Wise County (SECWC) and the northwest corner of Tarrant County (NWCTC), Texas in 2018 by the author. The distance between these two points is 8.43 miles (13.567 km) and the difference between to geodetic forward and back azimuths is 180°04'45". Even at this distance, the $t-T$ correction is negligible. However, you will note by the convergence angles of the two points, this site is close to the state plane projection central meridian. The effects of $t-T$ are amplified the further east or west the survey is from the central meridian.

860



From	To	Grid Azimuth	Grid Distance (ft)	Δ Elevation (ft)	
SECWC	NWCTC	270°30'20"	44 518.766	-46.469	
From: SECWC					
Grid		Geodetic (U.S. State Plane 1983)		Cartesian (WGS-84)	
Northing:	7 045 508.283 ft	Latitude:	N32°59'30.212 32"	X:	-2 262 408.265 ft
Easting:	2 306 205.277 ft	Longitude:	W097°23'54.666 00"	Y:	-17 423 107.021 ft
Elevation:	809.321 ft	Height:	717.849 ft	Z:	11 329 728.422 ft
Convergence:	0°36'02.671 554"				
Inverse:					
Grid Azimuth:	270°30'20"	NS Forward Azimuth:	271°06'23"	Δ X:	-44 069.369 ft
Grid Distance:	44 518.766 ft	NS Back Azimuth:	91°01'38"	Δ Y:	6 274.623 ft
Δ Elevation:	-46.469	Ellipsoid Distance:	44 517.360 ft	Δ Z:	669.327 ft
<i>t-T</i> Correction:	0°00'00.226 070"	Ground Distance:	44 518.835 ft		
Height Scale Factor:	0.999 966 864 3	Δ Height:	-47.508 ft		
Grid Scale Factor:	1.000 031 597 5				
Combined Factor:	0.999 998 460 7				
To: NWCTC					
Northing:	7 045 901.165 ft	Latitude:	N32°59'38.412 75"	X:	-2 306 477.634 ft
Easting:	2 261 688.244 ft	Longitude:	W097°32'37.233 57"	Y:	-17 416 832.398 ft
Elevation:	762.852 ft	Height:	670.341 ft	Z:	11 330 397.749 ft.
Convergence:	0°31'17.665 577"				



870 southern hemisphere, the apparent motion is clockwise (Fig. 16).

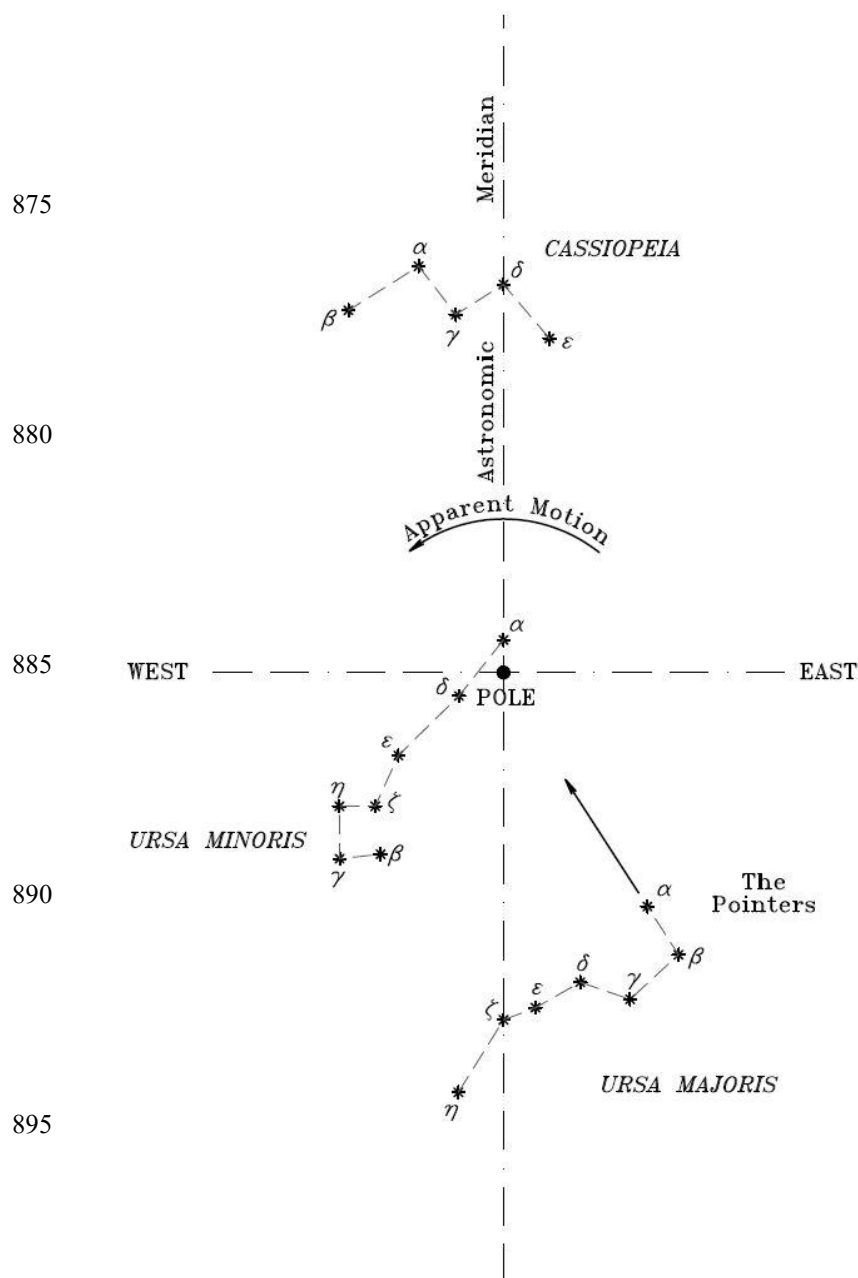


Figure 16: Upper culmination of Polaris (α *Ursa minoris*) as it transits the local astronomic meridian.



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Circumpolar stars provide for observing their transits across the observer's local astronomic meridian and allow a relatively easy way of observation for determining astronomic latitude and longitude at *upper* and *lower culmination*. These transits occur twice each day, 12 hours apart. The observer may refer to an ephemeris or nautical almanac to determine the approximate times of these culminations and point the instrument's telescope on the local astronomic meridian and wait for the instant of the star's greatest altitude to occur. The downside of this method is that the observer gets only one chance to observe one star transiting the local astronomic meridian. Therefore, the observer shall select several stars over the night's observation. However, the observer may experience cloudy nights where some of the stars are obscured. The method of determining longitude at upper and lower culmination for precise astronomic surveying is discussed in detail in the "*Manual of Geodetic Astronomy: Determination of Longitude, Latitude, and Azimuth.*" (C&GS 1947).

910

Using circumpolar stars in determining astronomic azimuth are generally preferred as they are easy to identify in the night sky. In the northern hemisphere, the star Polaris (*α Ursa minoris*) is the single most popular star for determining astronomic azimuth at *eastern* and *western elongation*, which occur twice each day, 12 hours apart (Fig. 17). Again, the observer may consult an ephemeris or nautical almanac to determine the approximate times of these elongations and set the instrument in the manner described above in preparation to observe the star. For ordinary surveys below First Order, the observation of Polaris is easy, and its computed azimuth is *subtracted* when observed at eastern culmination and *added* when observed at western elongation.

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For stars seen from locations in the northern and southern hemispheres, see Fig. 18 & Fig. 19. These include circumpolar stars, and stars that rise and set over the horizon are called equatorial stars.

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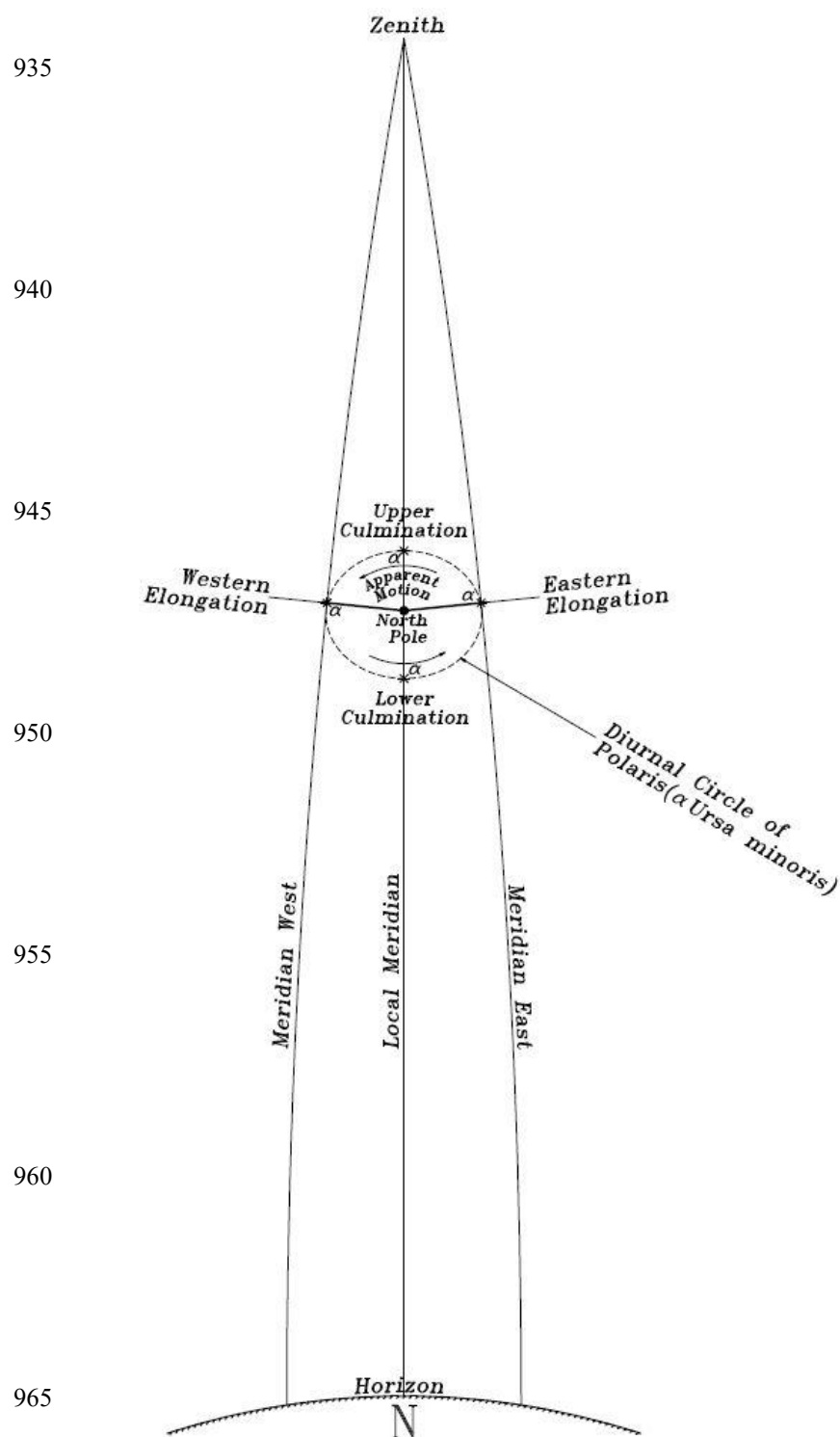
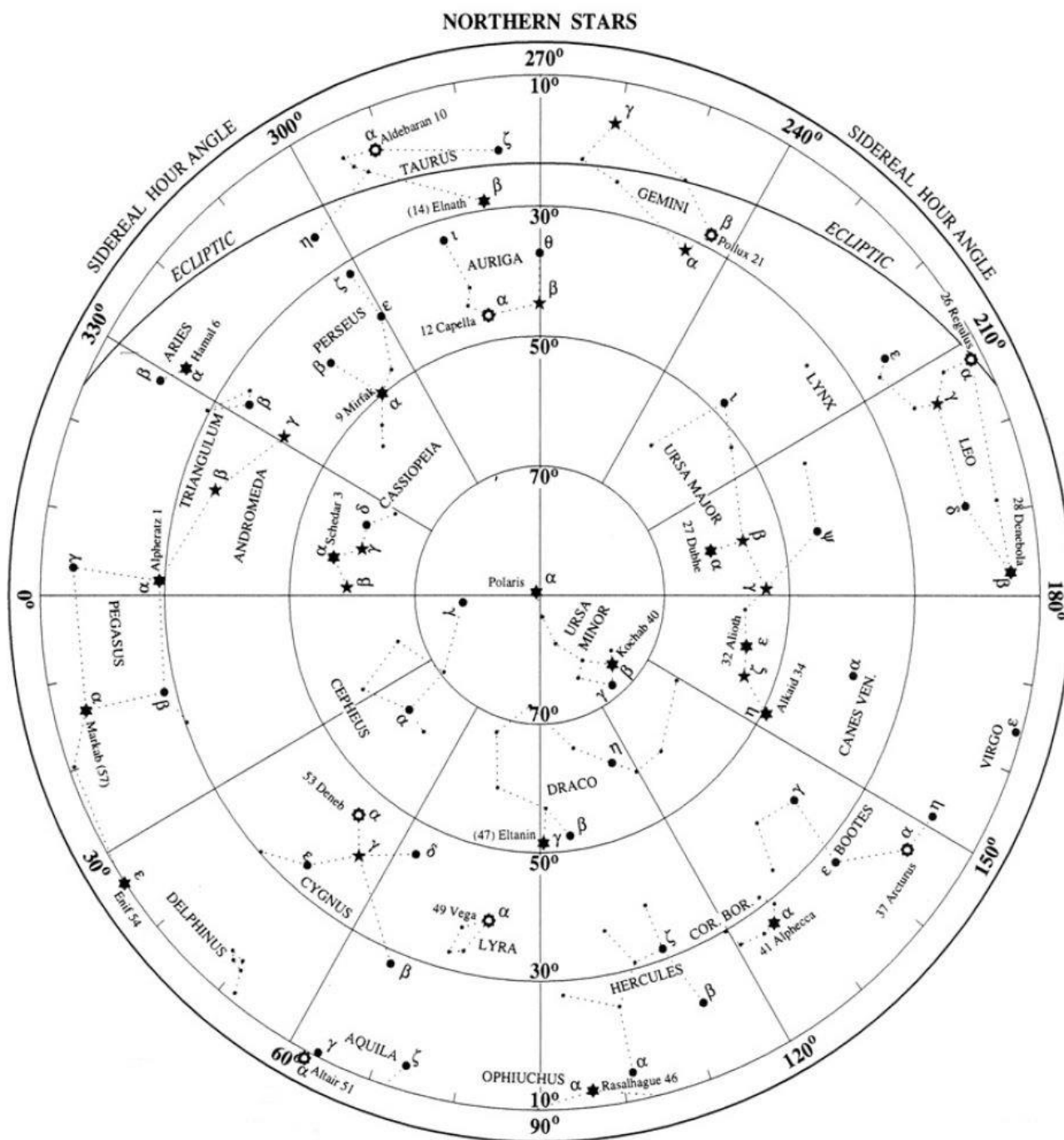


Figure 17: The diurnal circle of a circumpolar star at its culminations and elongations.



970 **Figure 18: Polar projection map of the navigational stars seen in the northern hemisphere (Source: U.S. Naval Observatory).**

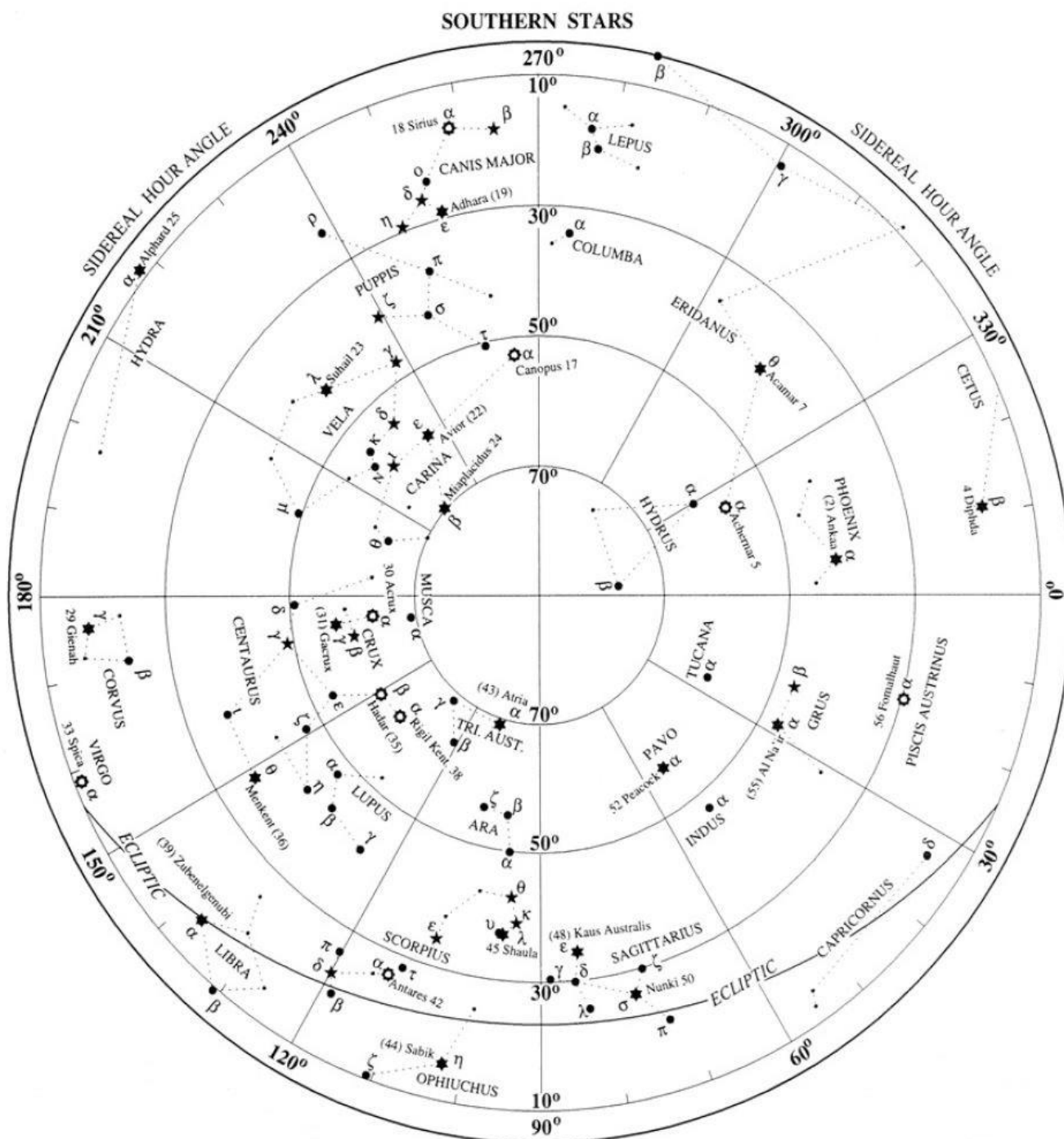


Figure 19: Polar projection map of the navigational stars seen in the southern hemisphere (Source: U.S. Naval Observatory).



10. Conclusion

This monograph is a study of geodetic astronomy, focusing on historic, classical astrogeodetic methods that preceded modern GNSS surveying. It analyzes key terrestrial coordinate systems, comparing earth-centered, earth-fixed, astronomic, and geodetic frameworks. The importance of precise astronomical time scales for accurate azimuth determination is emphasized. The paper examines the relationship between local astronomic and geodetic coordinate systems, particularly the impact of gravity on the geoid and the resulting misalignment of the two systems. It discusses the deflection of the vertical and the necessity of transforming astronomic coordinates to geodetic coordinates before computing grid azimuths. An example of a night field observation illustrated the applications required in the determination of an astronomic azimuth for a station angle. The discussion addresses corrections for astronomic observations, including stellar aberration, apparent curvature of celestial paths, and instrument error with optical theodolites. The concept of meridian convergence is introduced to explain the curvature of the earth's surface and its effect on establishing straight survey lines over long distances. The paper concludes with computations for the Laplace correction, which are required to convert astronomic azimuths to geodetic and grid azimuths for survey engineering projects.

In the era of satellite positioning technology, astrogeodetic orientation remains vital for providing an independent, high-accuracy, physical reference for geodetic networks and gravity field models. While Global Navigation Satellite Systems (GNSS) offer rapid, real-time positioning, they are susceptible to environmental interference and intentional jamming and spoofing. Astrogeodetic methods complement and validate GNSS by aligning coordinate frames with the Earth's true rotational axis and gravity field.

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