Author Response CC2 - Michał Michalak

Curlew 1.0 - Spatio-temporal geological modelling with neural fields in python

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Dear Michał Michalak,

We thank you for your insightful community comment on our manuscript, and are pleased that you like our work. Please find the responses to your queries attached below. The green text refers to our reply whereas the blue text refers to the updates in the manuscript.

Kindest regards,
Akshay Kamath (on behalf of the authors)

In my opinion, this is a well-structured manuscript. In this "unsolicited" community comment, I have only a couple of specific questions to better understand differences with existing approaches: What is the minimum data to run your software? For example, in GemPy (co-kriging) one needs surface points and orientation data. Please specify the minimum data categories and data amount in the manuscript.

Within *curlew* we have multiple kinds of losses. At the very minimum, one could (hypothetically) run the model without any local or relational constraints (i.e., data points) and just fit to global losses to get a random (but geologically plausible) geometry. If one adds a single gradient, orientation, value, or relational constraint, then the model will locally fit to that (with the extent of continuation of the solution depending on the length scales provided by the user).

In your manuscript, you write about "bubbles". Can the following effect in the GemPy tutorial (1:31:57) be described as "bubbles"? Source: https://www.youtube.com/watch?v=1oS6xTJkRwo Could you provide a figure or a reference where "bubbles" are discussed in scientific literature?

Yes, the effect shown in the GemPy tutorial can indeed be described as "bubbles" or similar interpolation artifacts. At the referenced timestamp, the model displays highly irregular, "bumpy" surfaces where the topo layer and formation layers are being forced into the same series.

The presenter explains that these artifacts occur because the interpolator is attempting to find surfaces that pass through conflicting data points (topography vs. formation) without sufficient constraints. In the manuscript (at L100) we have also now explained why curvature minimising interpolators do not necessarily enforce the “no local extrema” condition that is necessary for having implicit fields that honor the rules of stratigraphic deposition and topology.

As for discussion in literature, "bubbles" and related topology errors are discussed in several key papers that focus on the constraints and quality of implicit models:

1) Wellmann and Caumon (2018) discuss "geological consistency" and the types of artifacts (like internal loops or "bubbles") that occur when implicit functions are poorly constrained.
2) Hillier et al. (2023): GeoINR explicitly addresses the reduction of "modeling artifacts" through geometrical initialization and loss functions designed to prevent unrealistic isosurfaces.

In the above-referenced GemPy tutorial, there is a claim that adding more orientations improves the geological model in that it avoids some negative effects (maybe "bubbles"?). The proposed solution, as far as I understand, calculates a normal vector from three surface points. Could you compare this approach with your approach in the manuscript?

Adding more orientation points would constrain the implicit field better by not allowing it to curl in on itself (giving rise to closed isosurfaces) and is a well known way of solving erroneous interpolation artefacts. In cases where such additional data are not available/accessible, our approach resolves this by looking at the normalised gradients of our field. If there is a local minima/maxima, the gradient of the field has to be exactly zero at that location. This further implies a very high absolute divergence of the gradient field (since the point acts as a source/sink). By penalising this divergence (i.e. our monotonicity loss) we indirectly enforce the underlying implicit scalar field to be bubble-free (even in the absence of any data).

Line 414: " This contrasts against established methods such as co-kriging and RBF, which scale poorly with data volume (typically with cubic complexity in the number of data points)," - do you mean the need of inverting matrices in geostatistical methods? Please provide a reference.

Yes, the cubic complexity mentioned refers specifically to the inversion of the N×N covariance or kernel matrix required to solve the kriging or RBF systems. We have cited several references in the same line as highlighted by the commenter. We have added a reference for Rasmussen and Williams, 2006, which refers to this computational bottleneck. We have updated the sentence (at L414) to specifically refer to inversion of the covariance matrix.

“This contrasts against established methods such as co-kriging and RBF, which scale poorly with increasing data volume. Specifically, the requirement to invert a dense $N \times N$ covariance or interpolation matrix incurs a cubic computational cost ($O(N^3)$), making these methods inefficient without numerical simplifications like compactly supported kernels, fast multipole methods, or domain decomposition (Rasmussen & Williams, 2006; Cavoretto et al., 2016; Beatson et al., 2001; Wendland, 1995).”