Response to Reviewer 2 (Anonymous)

March 2026

Manuscript: "Quantifying Resilience in Non-Autonomous and Stochastic Earth System Dynamics with Application to Glacial–Interglacial Cycles"

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We thank the Reviewer for their thorough and constructive review. Below we respond to each comment in turn.

> The modelled ice volume anomaly is normalised to zero at preindustrial level and unity at the last glacial maximum. Then the parameters of tolerance, etc. are set to proportional values to serve this specific dataset. I think if the authors like to achieve generality with the novel measures they introduce, they need to scale them with parameters of an arbitrary time series that may be considered in future. This generality would help adoption of the measures in the research community. What the authors demonstrate in Fig. 2 is the choice of the parameter value based on the fluctuations of the data. Can this be derived in general terms based on the properties of the input data?

We thank the reviewer for this observation. The measures introduced here are intended as a proof-of-concept demonstration within the specific model framework by Talento and Ganopolski (2021), where ice volume is normalised to [0,1]. We agree that generalising RAR and return time to arbitrarily scaled variables would require expressing the tolerance parameters relative to some property of the input data (e.g. its range or variance), and we will flag this explicitly as a direction for future work in the revised manuscript.

> I disagree with the statement (lines 210-213) that what rapidly happens during deglaciations has anything to do with resilience. Resilience means stability of a system state, when the system trajectory reliably fluctuates inside sufficiently deep potential well. Deglaciation is not that kind of behaviour; rather it is a transition from one state to another. If you drop one or two or three stones from a tower, they all will do something very similar, but this will not be resilience. I understand that the authors would like to interpret resilience as "the capacity of the ice-climate-carbon system to remain close to its characteristic largescale path in face of external shocks" (lines 35-36). But if the planet suddenly becomes hot, massive ice melting is alike a falling stone, and there are no less or more "resilient" alternatives to that.

We agree that, under the classical attractor/potential-well notion of resilience, deglaciations are transitions between regimes and should not be interpreted as "stability of a state". Our use of the term instead follows a *path-resilience* framing for a forced, non-autonomous system, where the reference is a time-dependent trajectory under the same external forcing. In this sense, the relevant diagnostic is whether perturbed realisations stay close to, or return toward, that reference path. This could also be phrased as the resilience or stability of the underlying pullback attractor. We will make this clearer in the manuscript and, additionally revise the wording around lines 210–213 by replacing

"More indicative of genuine resilience are the increases in RAR ... The convergence ... points to enhanced dynamical stability intrinsic to the system ..."

with

"More indicative of path resilience (in our sense) are the increases in RAR during deglaciations, when the lower-bound constraint is inactive. Here, despite rapid deterministic change along the glacial-cycle trajectory, the ensemble shows contraction toward the reference path (high RAR), indicating an attracting segment of the forced trajectory rather than stability of a fixed climate state."

> I think what the resilience function offers in Fig. 3 is a clear marking of deglaciation episodes (except N13) under the specific stochastic perturbation, which is interesting. The model was perturbed with a minor stochastic term, thus

In this paper, resilience is defined operationally as the tendency of perturbed trajectories to remain close to or return toward a reference trajectory under identical forcing. Within this framework, Fig. 3 indicates that deglaciations act as convergence phases for small perturbations. We emphasise, however, that this does not imply resilience of a stable state, but rather enhanced path-wise stability of the forced trajectory.

RAR indicates that during 420–400 kyr 60% of ensemble members lie within the tolerance band around the reference trajectory. In panel (a), those trajectories are visually obscured by the reference line, while the remaining members show a wider spread, which can make the ensemble appear more discordant than the tolerance-based count implies.

Thank you for pointing this out. We agree that the caption of Fig. 4 should state that the simulations ran for an additional 1000 kyr and will amend this in the manuscript.

Further Comments

We agree that the reviewer-suggested alternative is more precise, but it is also relatively long. Instead we suggest "pulse-perturbation ensemble".

We agree that RAR is time-dependent in our analysis (i.e. $\mathrm{RAR}(t)$). While the term "ratio" can sometimes suggest a single scalar value, it is also commonly used for quantities evaluated at each time step and analysed as a function of time. To avoid any ambiguity, we will clarify this explicitly in the manuscript by consistently writing $\mathrm{RAR}(t)$ and referring to it as the time-dependent RAR (including in figures and captions). We therefore prefer to retain the name Reference Adherence Ratio (RAR) rather than introduce the alternative term "RARF".

This is a small but interesting feature. Since the model tends to go to full ice collapse once melting starts (through the M_v term in Eq. (1)), longer glacial cycles become less common as the noise level increases (higher σ). Around ($\sigma = 0.01$), the ~100 kyr peak is already starting to break down. While we have not analysed this feature in detail, we hypothesise that the weak bump around ~80 kyr is likely just the ~100 kyr peak shifting/broadening as the cycle lengths shorten under stronger perturbations.

We agree and will replace

"To quantify how strongly the ensemble trajectories cluster around the reference path"

with

"To quantify how strongly the simulated ensemble trajectories cluster around the simulated reference path"

> The preprint by Lucarini and Chekroun has already been published: https://www.nature.com/articles/s42254-023-00650-8

Thank you, we will update the reference.

Minor comments

> LaTeX typesetting of text after formulas in display mode: the authors apparently leave a blank line, which leads to indentation of the next line, although it continues the sentence started by the equation. This happens in lines 102,111,115,126,172,199. This issue will disappear if the authors remove the blank lines after the display formulas preceding these lines.

Thank you, we will fix this.

> Line 24: I am not sure that "revitalise" is the right term for a planetary resilience. "Reinforce" may be a better term.

We thank the reviewer for the suggestion. We will amend the manuscript: "Such knowledge is crucial for developing strategies to regenerate ~~and revitalise~~ Earth system resilience".

> Line 117, paleo-reconstruction of what? (state the variable).

Thank you for pointing this out. We will update line 117 to state

"Figure 1 compares the simulated ice-volume anomaly with the **palaeo-reconstructed (inverted sea-level–based) normalised global ice-volume anomaly** from Spratt and Lisiecki (2016), along with the imposed orbital forcing and frequency spectra"

and the caption of Figure 1:

"Model validation against a palaeo-reconstruction **of the (normalised) global ice-volume anomaly (derived from inverted sea level)** over the past 800 kyr."

> Line 175, "We apply this" - what this?

We will update line 175 to state "We apply **this stochastic differential equation framework** to the ice-volume equation, which is the only differential equation in our model."

> In Fig.2, vertical axes are missing

Thank you. We will add the missing y-axis labels to Fig. 2.

> Line 316, I think system state cannot be "novel"; I would call them "previously unsampled". Or simply "new"

Will replace by "new".

> Line 370 , "RAR sensitivity to the tolerance value"

We will amend the section title as suggested to "Appendix B: RAR sensitivity to the tolerance value".

References:

Talento, Stefanie, and Andrey Ganopolski. "Reduced-complexity model for the impact of anthropogenic CO2 emissions on future glacial cycles." *Earth System Dynamics* 12.4 (2021): 1275-1293.