

1 Streamflow elasticity as a function of aridity

2
3 Vazken Andréassian^{*,1}, Guilherme M. Guimarães¹, Julien Lerat², Alban de Lavenne¹

4
5 ¹ Université Paris-Saclay, INRAE, HYCAR Research Unit, Antony, France

6 ² CSIRO, Canberra, Australia

7 ^{*}Corresponding author, vazken.andreassian@inrae.fr

8 9 Abstract

10 Relating variations in annual streamflow to a climate anomaly, commonly referred to
11 as *streamflow elasticity to climate*, is central for a rapid assessment of the impact of
12 climate change on water resources. This elasticity is classically estimated via a multiple
13 linear regression between anomalies in streamflow and climate variables. However,
14 this approach does not explicitly account for the fact that elasticity depends on aridity
15 as suggested by “Budyko-type” water balance formulas. Using a large dataset of 4,122
16 catchments from four continents, we first verify empirically the link between elasticity
17 and aridity. Then, we propose a method to constrain elasticity coefficients with
18 derivatives from a “Budyko-type” water balance formula, that allows introducing an
19 explicit dependency between elasticity and aridity. We show that adding this
20 dependency produces a regionalized elasticity formula with physically-realistic
21 elasticity coefficients.

22
23 **Keywords:** elasticity, sensitivity, aridity index, humidity index, Schreiber formula,
24 Oldekop formula, Turc-Mezentsev formula, Bagrov formula, Tixeront-Fu formula,
25 Budyko hypothesis, hierarchical linear model

26 Notations

27 This study uses three hydrological fluxes: precipitation (P_n), streamflow (Q_n), and
28 potential evaporation (E_{0n}). All fluxes are computed at the catchment scale as annual
29 sums, expressed in millimeters per year. The subscript n refers to a specific
30 hydrological year. For the Northern Hemisphere, the hydrological year spans from 1st

31 October of year $n - 1$ to 30th September of year n . For the Southern Hemisphere, it
32 spans from 1st April of year n to 31st March of year $n + 1$. Thus, Q_n , represents the
33 streamflow for the hydrological year n . Long-term mean values are denoted by an
34 overbar (e.g., \bar{Q}). Annual anomalies, denoted by Δ , are computed as the difference
35 between the annual value and the long-term mean. For example, the streamflow
36 anomaly is calculated as $\Delta Q_n = Q_n - \bar{Q}$. This is also applied to precipitation ($\Delta P_n = P_n -$
37 \bar{P}) and potential evaporation ($\Delta E_{0n} = E_{0n} - \bar{E}_0$).
38 Additionally, we also define a combined flux, Λ_n (see Eq. 2), which reflects the
39 synchronicity of precipitation and potential evaporation. This is also expressed in
40 millimeters per year, and its anomalies are computed as $\Delta \Lambda_n = \Lambda_n - \bar{\Lambda}$.
41 The *aridity index*, φ , corresponds to the ratio \bar{E}_0/\bar{P} , while the inverse ratio \bar{P}/\bar{E}_0
42 corresponds to the *humidity index*.

43 1 Introduction

44 1.1 About streamflow elasticity

45 The climate elasticity of streamflow (Schaaque and Liu, 1989; Dooge et al., 1999;
46 Sankarasubramanian et al., 2001) describes the sensitivity of streamflow to changes
47 in a climate variable. Elasticity is classically derived from the following regression:

$$\Delta Q_n = e_{Q/P} \Delta P_n + e_{Q/E_0} \Delta E_{0n} \quad \text{Eq. 1}$$

48 where: $e_{Q/P}$ denotes the precipitation elasticity of streamflow, e_{Q/E_0} denotes the
49 potential evaporation elasticity of streamflow; both coefficients are dimensionless. Note
50 that elasticity is defined here in absolute terms, i.e. as the sensitivity between quantities
51 of the same dimension (ΔQ , ΔP and ΔE_0 are all in mm/y) following Andréassian et al.
52 (2016).

53 Andréassian et al. (2025) recently proposed to enrich the traditional computation given
54 in Eq. 1, to account for the seasonal time shift between precipitation and potential
55 evaporation, because of its decisive impact on catchment water yield (see e.g. Pardé,
56 1933; Coutagne and de Martonne, 1934; Thornthwaite, 1948; Milly, 1994; Yokoo et al.,
57 2008; Roderick and Farquhar, 2011; de Lavenne & Andréassian, 2018; Feng et al.,
58 2019). We compute the synchronous amount of precipitation and potential evaporation
59 Λ , using monthly data as in Eq. 2:

$$\Lambda_n = \frac{\sum_{m=1}^{12} \min(P_{m,n}, E_{0m,n})}{\sqrt{\bar{P}_n * E_{0n}}} * \bar{P} \quad \text{Eq. 2}$$

60 Where index m stands for the month. The dimension of Λ_n is mm/y and it represents
 61 the annual precipitation volume most easily accessible to evaporation. For two years
 62 with the same annual amounts of precipitation and potential evaporation, Λ will be
 63 higher when they are synchronous, and lower when they are out of phase (for more
 64 details, please refer to Andréassian et al., 2025). With this new term, the regression in
 65 Eq. 1 becomes:

$$\Delta Q_n = e_{Q/P} \Delta P_n + e_{Q/E_0} \Delta E_{0n} + e_{Q/\Lambda} \Delta \Lambda_n \quad \text{Eq. 3}$$

66 1.2 Using aridity to estimate streamflow elasticity

67 The link between streamflow elasticity and catchment aridity is a well-established
 68 concept in hydrology, an idea that can be traced back to Oldekop (1911) and his
 69 followers, including Budyko (1948), Bagrov (1953) and Mezentsev (1955). Many
 70 ‘modern’ hydrologists such as Dooge (1992) and Dooge et al. (1999) discussed the
 71 form that aridity-dependent streamflow formulas could take. This dependency was
 72 emphasized by Koster and Suarez (1999), who write that “*the partitioning of a*
 73 *precipitation anomaly into evaporation and runoff anomalies is a simple function of the*
 74 *dryness index*”, as well as by Sankarasubramanian et al. (2001) who argue that
 75 empirical elasticity estimates would only follow the direction shown by the Budyko-type
 76 formulas for the very humid regions of the US, while Arora (2002) concludes that “*the*
 77 *use of aridity index provides a straight-forward method to obtain a first order estimate*
 78 *of the effect of climate change on annual runoff*”. Chiew (2006) shows the dependency
 79 of streamflow elasticity on aridity, Renner et al. (2012) stress that the elasticity of
 80 streamflow “*is largely dependent on [...] the aridity of the climate*” and Roderick and
 81 Farquhar (2011) underline that “*the response of runoff to changes in the main driving*
 82 *variables is not constant but depends on the overall climatic dryness*”.

83 More recently, the concept has been applied at a global scale, with Berghuijs et al.
 84 (2017) who use the elasticity pattern provided by the Tixeront-Fu formula to propose a
 85 world map of aridity-dependent streamflow elasticities, Zhang et al. (2022) discuss the
 86 impact of aridity on the sensitivity of the elasticity coefficient to the aggregation time
 87 step, and Anderson et al. (2024) extends the computation of elasticity to different flow

88 quantiles, and show that aridity impacts the shape of the curve relating the different
89 elasticity quantiles.

90 However, Addor et al. (2018), using random forests to explain (among others) the
91 precipitation elasticity of streamflow, concluded that signatures of “hydrological
92 dynamics are poorly predicted by aridity alone, or even by a combination of several
93 climatic indices”.

94 **1.3 Local vs class estimation of elasticity**

95 To estimate the climate elasticity of streamflow at regional or national scales, making
96 the dependency of streamflow elasticity on aridity explicit can constrain the estimation
97 of elasticity coefficients and increase their physical realism.

98 For a given catchment with a sufficiently long series of annual observations, streamflow
99 elasticity can be computed *locally* by linear regression (Andréassian et al., 2016).
100 However, for ungauged catchments, local estimation of elasticity coefficients is no
101 longer possible. Instead, a *class*-elasticity can be estimated by combining all available
102 records in a region. The estimation by class has both advantages and drawbacks.
103 While this approach improves the statistical significance of elasticity coefficients, which
104 can have high uncertainty when estimated locally (especially for potential evaporation),
105 it also requires combining data from catchments with different aridity indices. This
106 presents a challenge, precisely because we know that aridity and elasticity are linked.
107 Methods to estimate local- and class-elasticity are detailed in section 2.

108 **1.4 Formulas relating streamflow elasticity to aridity**

109 We mentioned above the seminal work of the hydrologists who, following Oldekop
110 (1911), developed various mathematical formulas to represent catchment water
111 balance. These studies established simple water balance formulas from which a
112 “theoretical” elasticity of streamflow can be derived as their partial derivatives. In Table
113 1, we present four long-term water balance formulas that can be used to provide these
114 theoretical elasticity estimates. The Schreiber and Oldekop formulas are parameter-
115 free, while the Turc-Mezentsev and Tixeront-Fu formulas each have one parameter
116 (ω^1 and m , respectively). These last two formulas are equivalent when setting $m = \omega +$

¹ We use ω instead of the more commonly used “ n ” on purpose, to avoid confusion with the subscript n used for years.

117 0.72 (Yang et al., 2008; Andréassian and Sari, 2018), which explains why their curves
118 overlap in some of the later figures.

119 Table 1 also presents the partial derivatives for each formula, allowing to compute the
120 precipitation and the potential evaporation elasticities of streamflow. Unsurprisingly,
121 these formulas are all functions of the aridity index.

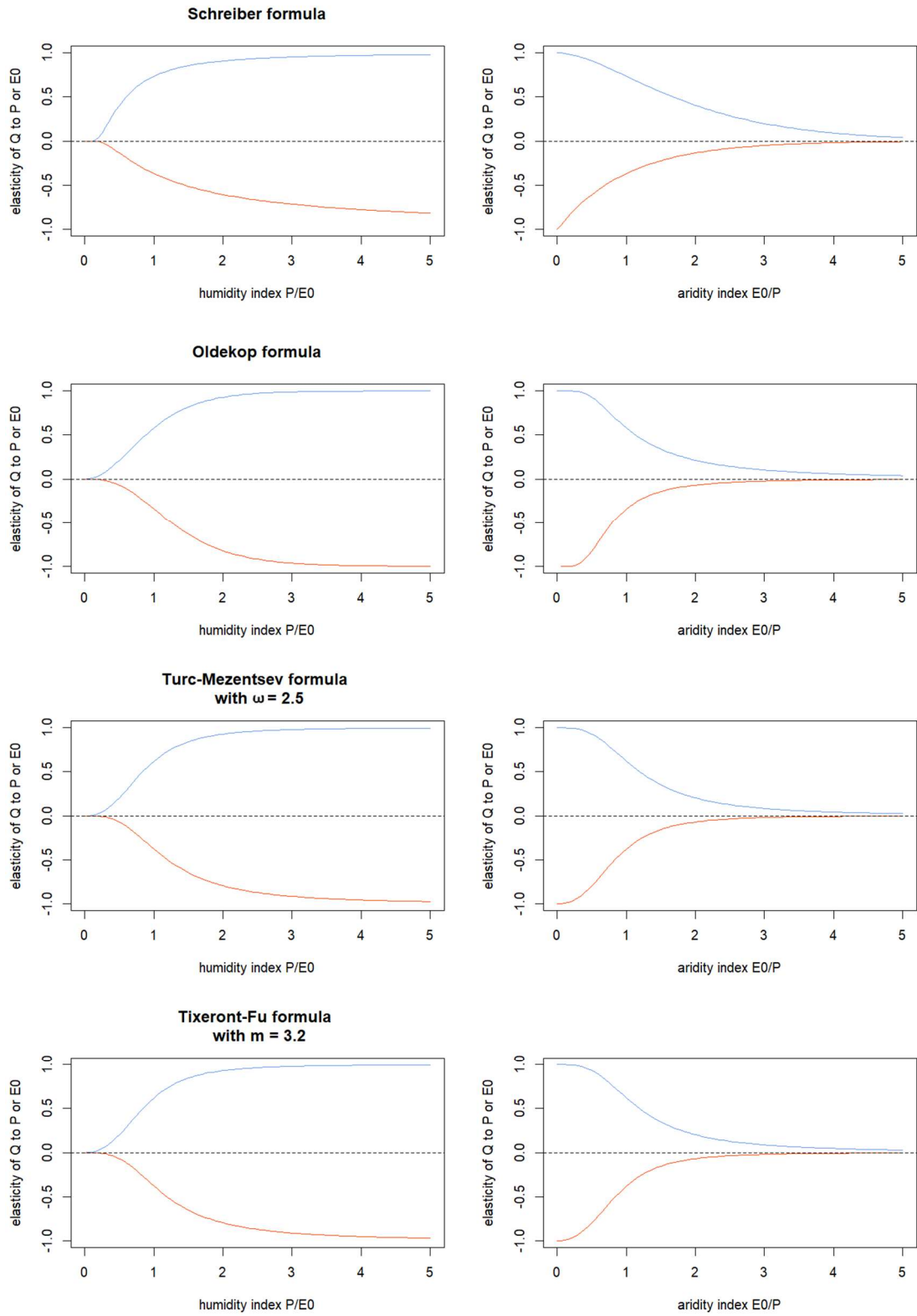
122

123 **Table 1. Common long-term water balance formulas and the associated**
 124 **elasticities (\bar{Q} – long-term average streamflow [mm/y], \bar{P} – long-term average**
 125 **precipitation [mm/y], \bar{E}_0 – long-term average reference evaporation [mm/y], $\varphi =$**
 126 **$\frac{\bar{E}_0}{\bar{P}}$ is the aridity index)**

Name	Formula	Precipitation elasticity $\frac{\partial Q}{\partial P}$	Potential evaporation elasticity $\frac{\partial Q}{\partial E_0}$
Schreiber (Oldekop, 1911)	$\bar{Q} = \bar{P} \cdot \exp\left(-\frac{\bar{E}_0}{\bar{P}}\right)$ Eq. 4	$e_{Q/P} = (1 + \varphi)e^{-\varphi}$ Eq. 5	$e_{Q/E_0} = -e^{-\varphi}$ Eq. 6
Oldekop (Oldekop, 1911)	$\bar{Q} = \bar{P} - \bar{E}_0 \cdot \tanh\left(\frac{\bar{P}}{\bar{E}_0}\right)$ Eq. 7	$e_{Q/P} = \tanh^2\left(\frac{1}{\varphi}\right)$ Eq. 8	$e_{Q/E_0} = -\tanh\left(\frac{1}{\varphi}\right) + \frac{1}{\varphi} \left[1 - \tanh^2\left(\frac{1}{\varphi}\right)\right]$ Eq. 9
Turc-Mezentsev (Turc, 1954; Mezentsev, 1955)	$\bar{Q} = \bar{P} - [\bar{P}^{-\omega} + \bar{E}_0^{-\omega}]^{-\frac{1}{\omega}}$ with $\omega > 0$ Eq. 10	$e_{Q/P} = 1 - (1 + \varphi^{-\omega})^{-\frac{1}{\omega}-1}$ Eq. 11	$e_{Q/E_0} = -(1 + \varphi^\omega)^{-\frac{1}{\omega}-1}$ Eq. 12
Tixeront-Fu (Tixeront, 1964; Fu, 1981)	$\bar{Q} = [\bar{P}^m + \bar{E}_0^{-m}]^{\frac{1}{m}} - \bar{E}_0$ with $m > 1$ Eq. 13	$e_{Q/P} = (1 + \varphi^m)^{\frac{1}{m}-1}$ Eq. 14	$e_{Q/E_0} = -1 + (1 + \varphi^{-m})^{\frac{1}{m}-1}$ Eq. 15

127
128
129
130
131
132
133
134
135
136
137
138
139
140

Figure 1 illustrates the similarities and differences among the formulas by showing their respective elasticity-aridity relationships. The embedded dependency on aridity is clearly visible, and we notice that the four formulas have distinct but similar shapes (with the difference between the Turc-Mezentsev and the Tixeront-Fu being negligible). Furthermore, the precipitation elasticity is bounded between 0 and 1, which means that one millimeter of additional precipitation will always result in less than one millimeter of additional streamflow. Similarly, the potential evaporation elasticity is bounded between 0 and -1, which means that one millimeter of additional potential evaporation will always result in a decrease of streamflow of less than one millimeter. These bounds represent a *physically-realistic* catchment response, in the sense that the yield (of the additional mm of precipitation or the additional mm of potential evaporation) must be comprised (in absolute value) between 0 and 100%.



141
 142
 143
 144

Figure 1: Theoretical relationships between streamflow elasticities and the humidity index (left panel) and the aridity index (right panel). Blue lines represent the precipitation elasticity of streamflow, and orange lines represent the potential evaporation elasticity of streamflow.

145 **1.5 Purpose of this paper**

146 This paper aims to verify empirically the fact that streamflow elasticity depends on
147 aridity, and to show how the theoretical pattern provided by the “Budyko-type” water
148 balance formulas can help constrain the estimation of elasticity coefficients, yielding
149 physically-coherent regionalized streamflow elasticities. We use for this purpose a
150 large dataset of catchments covering a wide variety of climates.

151 **2 Catchments and Method**

152 **2.1 Test catchments**

153 To ensure that our analysis was based the widest possible range of climates, we used
154 a set of 4,122 catchments, representing 162,005 station-years of data (average length
155 of catchment time series is 39 years). It includes catchments from Australia (Fowler et
156 al., 2024), Brazil (Almagro et al., 2021), Denmark (Liu et al., 2024), France (Delaigue
157 et al., 2024), Germany (Loritz et al., 2024), Sweden (de Lavenne et al., 2022),
158 Switzerland (Höge et al., 2023), the United Kingdom (Coxon et al., 2020) and the USA
159 (Addor et al., 2017). Because this dataset is exactly the same as the one used by
160 Andréassian et al. (2025), we refer the reader to this paper for the details of the
161 selection of the catchments from the original datasets. Let us just mention that we
162 excluded a few catchments with a long memory, for which the linear elasticity model
163 presented in Eq. 16 would not have been justified. Indeed, if a catchment has a
164 hydrogeology that provides it long memory, the elasticity cannot be expressed as a
165 function of the current year climate, but instead should be estimated by accounting for
166 as many previous years as necessary. The absence of interannual memory
167 guarantees the lack of autocorrelation in annual streamflow, which is an important
168 statistical assumption for OLS.

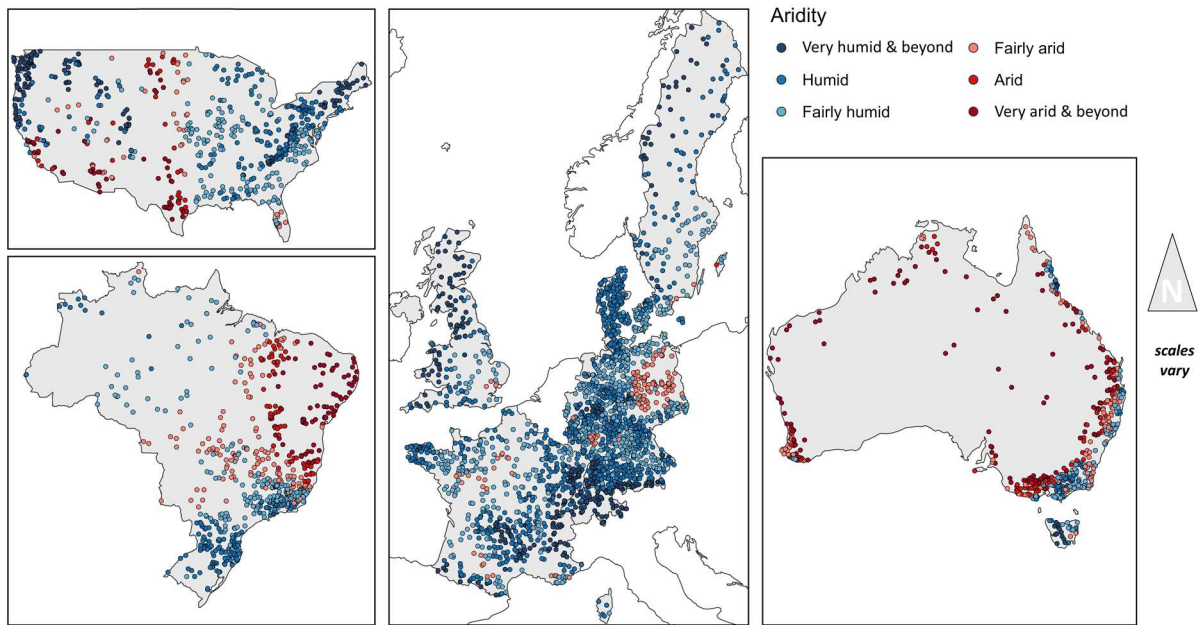
169 In our dataset, the aridity indices range from 0.1 to 6.3, with a first quartile of 0.6 and
170 a third quartile of 1.0. The mean and the median of the aridity index are both 0.8. To
171 assess the generality of the results, we will discuss them at the global scale and also
172 by aridity classes (as defined in Table 2).

173

174 **Table 2. Aridity classes used in this study (we only kept the classes counting more than 100**
 175 **catchments)**

Aridity class	Average aridity of the class	Number of catchments	Name
[0.25,0.50[0.39	484	Very humid
[0.50,0.75[0.64	1461	Humid
[0.75,1.00[0.85	1238	Fairly humid
[1.00,1.25[1.09	434	Fairly arid
[1.25,1.50[1.37	186	Arid
[1.50,1.75[1.61	109	Very arid

176



177

178 **Figure 2. location of the catchments studied and repartition by aridity classes**

179

180 **2.2 Computation of local elasticities**

181 Our reference method will consist in the local (i.e., catchment-specific) computation of
 182 streamflow elasticities using Eq. 16:

$$\Delta Q_n = e_{Q/P}^{loc} \Delta P_n + e_{Q/E_0}^{loc} \Delta E_{0_n} + e_{Q/\Lambda}^{loc} \Delta \Lambda_n \quad \text{Eq. 16}$$

183 Because the elasticity coefficients are obtained through linear regression, they are
 184 associated with statistical uncertainty, which we assess using the p-value. A
 185 significance threshold must be chosen, above which a coefficient is not considered
 186 statistically different from zero. For this paper, we use a conventional threshold of 0.05.
 187 With this local approach, a unique triplet of elasticities is computed for each of the
 188 4,122 catchments, and the goodness of fit for each regression is, by definition,
 189 maximized (hence our choice of the local calibration as reference).

190 Note that the correlation between the independent variables of the regression
 191 presented in Eq. 16 is rather limited: the correlations computed at the catchment scale
 192 are comprised for 90% of the cases in the range [-0.6,0.2] for $(\Delta P_n, \Delta E_{0n})$, in the range
 193 [-0.7,0.5] for $(\Delta P_n, \Delta \Lambda_n)$, and in the range [-0.5,0.4] for $\Delta E_{0n}, \Delta \Lambda_n)$,

194 **2.3 Computation of unique elasticities by aridity class and for the entire dataset**

195 We can also estimate a single triplet of elasticities for each different aridity class (as
 196 defined in Table 2) and we then use Eq. 17 for all the catchments of the given aridity
 197 class.

$$\Delta Q_n = e_{Q/P}^{cl} \Delta P_n + e_{Q/E_0}^{cl} \Delta E_{0n} + e_{Q/\Lambda}^{cl} \Delta \Lambda_n \quad \text{Eq. 17}$$

198 Estimating a single triplet of elasticities for each class allows investigating the
 199 dependency of elasticity to aridity. To calibrate the three parameters, we use a simple
 200 grid search algorithm, exploring the following intervals: [-0.1,1.1] for $e_{Q/P}^{cl}$, and [-1.1,0.1]
 201 for e_{Q/E_0}^{cl} and $e_{Q/\Lambda}^{cl}$, first with a coarse step of 0.1 and then a finer step of 0.01 around
 202 the optimum The objective function to be maximized is the bounded Nash-Sutcliffe
 203 Efficiency of Mathevet et al. (2006), which is first calculated for each catchment
 204 separately, and then averaged over the catchments belonging to the class and used
 205 as the objective to maximize (see Section 2.5).

206 For reference, we also compute a single triplet of elasticities at the global scale by
 207 pooling all 4,122 catchments together. By construction, this world-wide triplet yields
 208 the lowest mean efficiency.

209 **2.4 Computation of regionalized elasticities**

210 In the regionalized approach, we use the entire dataset to calibrate a single underlying
 211 model, similarly to the calculation of elasticities at the global scale. However, this
 212 method ultimately produces catchment-specific results. Each catchment has a distinct
 213 triplet of elasticities because the elasticities for precipitation ($e_{Q/P}^{reg}$) and potential
 214 evaporation (e_{Q/E_0}^{reg}) are modeled as functions of each catchment's aridity index (φ),
 215 given by Eq. 18. The regionalization formulas are adjusted by a shape parameter noted
 216 α :

217

$$\Delta Q_n = e_{Q/P}^{reg} \Delta P_n + e_{Q/E_0}^{reg} \Delta E_{0n} + e_{Q/\Lambda}^{reg} \Delta \Lambda_n \quad \text{Eq. 18}$$

$$e_{Q/P}^{reg} = f_P(\alpha_P, \varphi)$$

$$e_{Q/E_0}^{reg} = f_{E_0}(\alpha_{E_0}, \varphi)$$

$$e_{Q/\Lambda}^{reg} = \text{constant (does not depend on } \varphi)$$

218

219 There were several alternatives available for choosing the shape of functions f_P , and
 220 f_{E_0} , as well as for adjusting the shape parameters. For f_P and f_{E_0} we used the
 221 derivatives of the Oldekop formula (see Eq. 8 and Eq. 9). The variation range for these
 222 functions was constrained based on the results of the class calibration (Section 2.3).
 223 The synchronicity elasticity ($e_{Q/\Lambda}^{reg}$) was kept constant because no clear empirical
 224 relationship was observed when examining either the local or the class-calibrated
 225 elasticities.

226 Figure 3 illustrate the dependency of streamflow elasticities to aridity, which is apparent
 227 both with the locally- and the class-estimated values. To keep the number of adjusted
 228 parameters low, we adjusted only three parameters (α_P , α_{E_0} and $e_{Q/\Lambda}^{reg}$) for Eq. 18, the
 229 variation bounds were set up empirically once for all based on the results of the class
 230 calibration.

231 2.5 Model evaluation criterion

232 To evaluate the performance of the different elasticity models in simulating streamflow
 233 anomalies, we use the classical Nash and Sutcliffe (1970) efficiency criterion (NSE).
 234 The NSE is usually computed for each of the 4,122 catchments separately using Eq.
 235 19:

$$NSE = 1 - \frac{\sum_n (\Delta Q_n^{obs} - \Delta Q_n^{cal})^2}{\sum_n (\Delta Q_n^{obs} - \overline{\Delta Q^{obs}})^2} \quad \text{Eq. 19}$$

236 Because the NSE varies in the interval $] - \infty, 1]$, it is not recommended to compute an
 237 average over large sets (indeed, a few very low criteria values will impact the average
 238 criterion value). For this reason, we follow Mathevet et al. (2006) and use the bounded
 239 form (called “C2M” in the original paper) as in Eq. 20:

$$\text{Bounded NSE (C2M)} = \frac{NSE}{2 - NSE} \quad \text{Eq. 20}$$

240

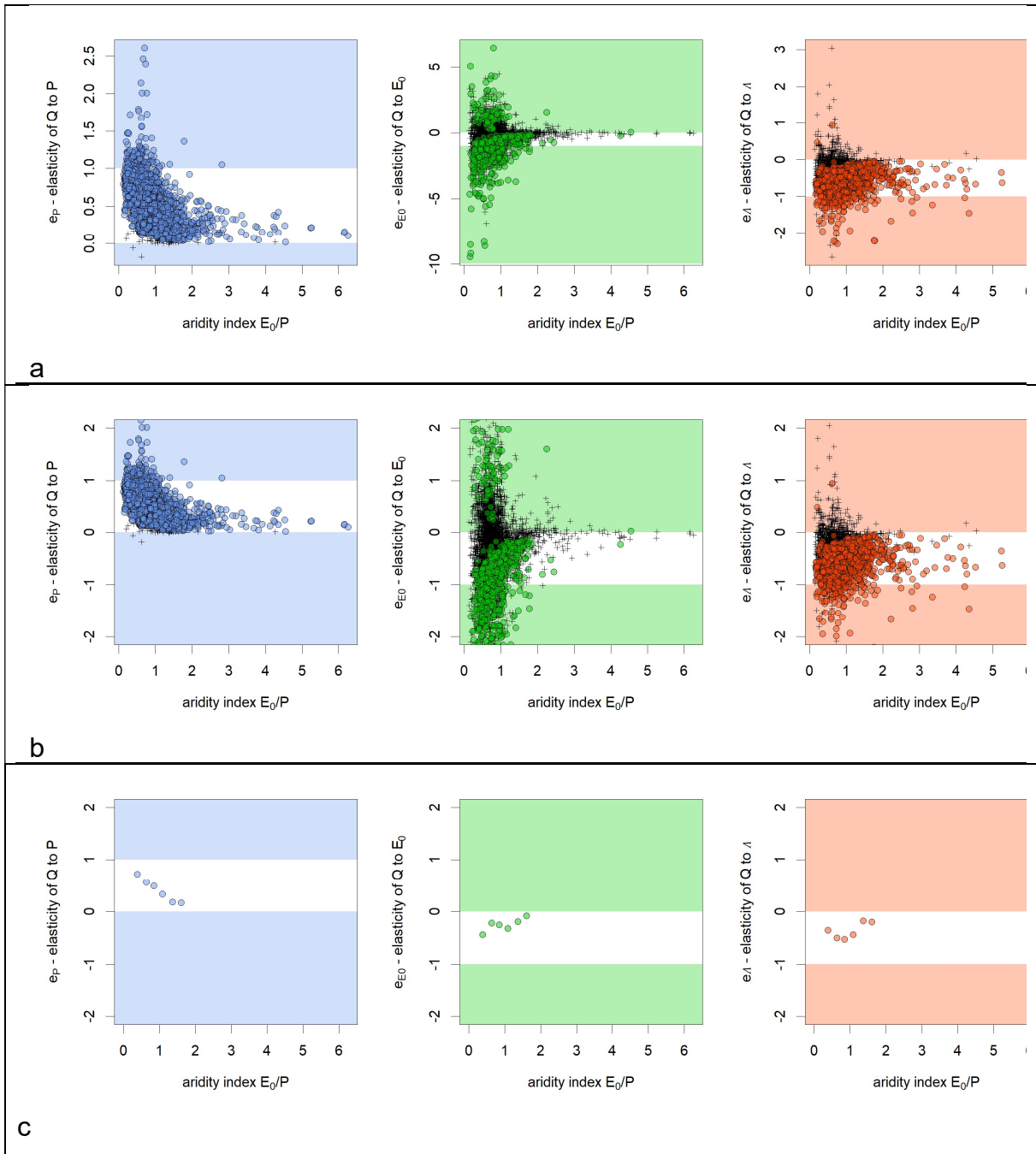
241 3 Results

242 3.1 Empirical verification of the dependency between locally-estimated 243 streamflow elasticities and aridity

244 We first computed the local streamflow elasticities for each catchment by linear
245 regression (Eq. 16). Choosing an (arbitrary) significance level equal to 0.05, our
246 dataset yielded 97% of the catchments with a significant $e_{Q/P}$ parameter, but only 23%
247 of the catchments with a significant e_{Q/E_0} parameter, and 64% of the catchments with
248 a significant $e_{Q/\Delta}$ parameter.

249 Figure 3 presents the link between aridity and the locally-estimated elasticity
250 coefficients. The results confirm the expected dependency between precipitation
251 elasticity and aridity, which was previously shown for the theoretical formulas in Figure
252 1. For the potential evaporation elasticity, a satisfying trend is visible but many
253 physically unrealistic elasticities show that additional constraints are required for this
254 term. Finally, the Δ -elasticity of streamflow (i.e. the streamflow elasticity towards the
255 synchronous amounts of precipitation and streamflow), shows no clear dependency
256 on the aridity index (but we did not expect any relationship).

257



259 **Figure 3. Relationship between the aridity index and locally-estimated climatic elasticities of**
 260 **streamflow, for precipitation elasticity (left), potential evaporation elasticity (middle),**
 261 **synchronicity elasticity (right). The white domain indicates the physically-plausible range (i.e.**
 262 **[0,1] for precipitation elasticity and [-1,0] for potential evaporation and synchronicity elasticities.**
 263 **a – (upper panel) locally calibrated elasticity coefficients, showing all catchments (elasticities**
 264 **coefficients non-significant at the 0.05 level are figured as crosses), b – (middle panel) same as**
 265 **a with a zoom on the [-2,2] range; c – (lower panel) class calibrated elasticity coefficients (from**
 266 **Table 3)**

267

268 **3.2 Results by aridity class**

269 We also calibrated the three elasticity coefficients to obtain a single triplet of values for
 270 each of the aridity classes as defined in sections 2.4 and 3.2. The resulting class-
 271 calibrated values are presented in Table 3. As reference, the performance of the local
 272 (catchment-specific) estimation is also provided (by construction, it represents the
 273 upper limit of performance).

274

275 **Table 3. Class-calibrated elasticity values for catchments grouped by the aridity index φ**

Aridity class	Number of catchments	Elasticity values			Performance expressed in mean bounded NSE for	
		$e_{Q/P}^{cl}$	e_{Q/E_0}^{cl}	$e_{Q/\Lambda}^{cl}$	Class approach (same elasticities for all catchments in the same class)	Reference approach (local i.e., catchment-specific estimation)
Very Humid $\varphi \in [0.25, 0.5[$	484	0.72	-0.44	-0.36	0.59	0.68
Humid $\varphi \in [0.5, 0.75[$	1461	0.56	-0.22	-0.50	0.46	0.57
Fairly Humid $\varphi \in [0.75, 1[$	1238	0.49	-0.25	-0.53	0.42	0.52
Fairly Arid $\varphi \in [1, 1.25[$	434	0.33	-0.32	-0.44	0.32	0.49
Arid $\varphi \in [1.25, 1.5[$	186	0.18	-0.19	-0.18	0.27	0.56
Very Arid $\varphi \in [1.5, 1.75[$	109	0.17	-0.08	-0.20	0.29	0.55
World	4,122	0.46	-0.19	-0.56	0.38	0.56

276

277 The numeric values in Table 3 confirm the tendency identified in Figure 3: the
 278 precipitation elasticity of streamflow shows a clear decreasing trend with increasing
 279 aridity, while the potential evaporation elasticity shows a symmetric increasing trend.
 280 The empirical range of variation observed in the class-calibrated results is narrower
 281 than the theoretical range from the water balance formulas: for $e_{Q/P}$, the observed
 282 range is [0.17, 0.72] compared to the theoretical [0, 1], and for e_{Q/E_0} , the range is [-0.44,
 283 -0.08] compared to the theoretical [-1, 0]. Finally, there is no clear trend identifiable for
 284 $e_{Q/\Lambda}$. A clear advantage of the class-based calibration approach is that all resulting
 285 elasticities values fall, without exception, within the physically-realistic ranges.

286 **3.3 Constraining the elasticity estimation with an aridity-dependent**
 287 **formulation: test for the entire dataset**

288 The observed link between the aridity index and the local elasticity estimates
 289 suggested us to test the solution presented in section 2.4, using a “regionalized”
 290 estimation of the elasticities of streamflow. This approach makes use of the identified
 291 pattern to enforce physical coherence across the entire dataset. To parameterize this
 292 relationship, we adapted the partial derivative of the parameter-free Oldekop formula
 293 (Table 1). We constrained the output of the Oldekop formulas to the empirical range
 294 observed in the class-based calibration (Table 3), offsetting the range for e_P to [0.15,
 295 0.75], and for e_{E_0} to [-0.45, -0.10]. Thus, the regionalized elasticities are calculated as:
 296

$$e_{Q/P}^{reg} = 0.15 + |0.75 - 0.15| * f_{P-oldekop}(\alpha_P, \varphi) \quad \text{Eq. 21}$$

where $f_{P-oldekop}$ is given by Eq. 8

$$e_{Q/E_0}^{reg} = -0.10 + |-0.45 + 0.10| * f_{E_0-oldekop}(\alpha_{E_0}, \varphi) \quad \text{Eq. 22}$$

where $f_{E_0-oldekop}$ is given by Eq. 9

297 Note that the restricted ranges remain within the physically-realistic limits.

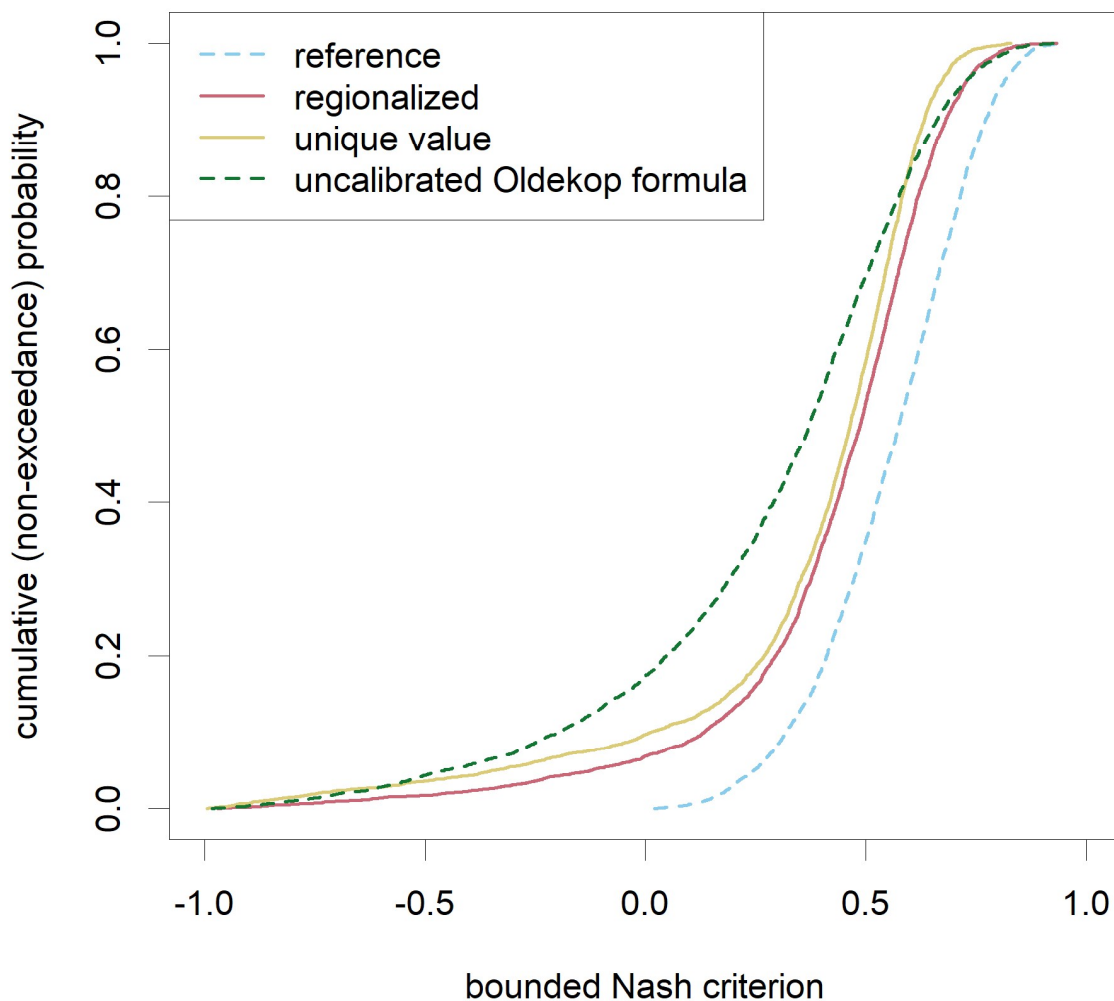
298
 299 We can now compare the performance of three modeling approaches: the “upper
 300 reference” where elasticities are calibrated locally at the catchment scale, the
 301 regionalized approach, and a “lower reference” with elasticities calibrated at global
 302 scale. While the upper reference requires the estimation of 12,366 parameters (3
 303 elasticities for 4,122 catchments), the latter two require only 3 parameters each. The
 304 corresponding results are presented below in Table 4 and Figure 4.

305
 306 **Table 4. Results of the application of the regionalized approach to all the catchments of our**
 307 **dataset (4,122): the performance is compared to a upper reference (with locally calibrated**
 308 **elasticity values) and a lower reference (with a unique value calibrated for all the catchments in**
 309 **the world)**

Performance expressed in mean bounded NSE for		
Upper reference approach: local, i.e. catchment-based estimation	Regionalized approach: elasticities function of each catchment's aridity index	Lower reference approach: same elasticities for all catchments
0.56	0.43	0.38

310

311 There is a clear advantage for taking into account the aridity in the regionalized
 312 formula. This approach covers 28% of the performance gap between the lower and
 313 upper references, while using only three parameters. In addition, all elasticity
 314 parameters remain within the physically-realistic range. The proposed parametrization
 315 is therefore successful from both explanatory and predictive point of views, which is a
 316 clear advantage (Andréassian, 2023).



317
 318 **Figure 4.** Distribution of the performances of the options compared in our paper, with the
 319 addition of the uncalibrated theoretical formulation derived from the Oldekop formula. The
 320 (unreachable) upper reference at the extreme right is followed by our regionalized solution,
 321 which has a better performance than an elasticity formula with a unique value (that would be
 322 independent from aridity) or than the elasticities derived from the (uncalibrated) Oldekop
 323 formula.

325 **4 Discussion**

326 In this paper, our aim was two-fold: (i) to empirically verify that at the catchment scale,
327 streamflow elasticity and climate aridity are linked, and (ii) to propose an aridity-
328 dependent parameterization allowing for the quantification of elasticity.

329 **4.1 The need for an empirical verification**

330 Because of the present popularity of Budyko's framework and its associated theoretical
331 formulas (Table 1), an empirical verification of the elasticity-aridity link might appear
332 superfluous. However, applying these theoretical formulas, such as the Oldekop
333 derivative, relies on a "space-for-time-trade" assumption. This consideration assumes
334 that a model validated across different spatial locations will also be valid for those
335 locations for different time periods (see Peel and Blöschl, 2011, and Singh et al., 2011).
336 Berghuijs and Woods (2016) have warned that this trade requires validation, and
337 Berghuijs et al. (2020) stress that although the Budyko-type curves have been used to
338 predict the evolution of catchments in response to climatic changes, they originate from
339 *"observations of spatial differences in long-term water balances, and not from*
340 *observations or theory of how individual catchments respond to aridity changes"*. Thus,
341 we argue that the elasticity-aridity link cannot be taken for granted and requires
342 empirical verification, especially given the mixed results reported by Oudin and
343 Lalonde (2023), who tested the classical space-time trading when parametrizing a land
344 use dependent hydrological model, which failed to efficiently predict the direction and
345 magnitude of hydrological changes after land use conversions.

346 **4.2 An aridity-dependent parameterization that uses the shape of the Oldekop** 347 **formula**

348 Regarding our parameterization, our results confirm the general shape of the elasticity-
349 aridity relationship given by the Oldekop formula, but they use a narrower range of
350 variation than the theoretical one. Our work is therefore only partially coherent with the
351 theoretical Budyko-type formulas, which appear to provide a wider range of elasticity
352 values than our empirical data support. This should not be a surprise to hydrologists
353 who know in particular how precipitation intensities impact the hydrological response

354 of arid catchments. What is remarkable, however, is that the intuition of Schreiber
 355 (1904) and Oldekop (1911), embedded in formulas of elegant simplicity, remain so
 356 useful in the 21st century. We agree on this point with Zhang and Brutsaert (2021) who
 357 suggested that the “Budyko hypothesis” could justifiably have been named after
 358 Schreiber and Oldekop, who, with so little data and only slide rules, were able to
 359 imagine tools still in use today.

360 Concerning the difference between the elasticities derived from the theoretical Budyko-
 361 type formulas and the empirical class-calibrated values, we can suggest two
 362 explanations: first, a Budyko-type formula will always remain a conjecture (an elegant,
 363 mathematically relevant one but nevertheless, still a conjecture); second, Gnann et al.
 364 (2026) have shown that realistic observational noise will introduce systematic
 365 departures from the theoretical optimum.

366 5 Conclusion

367 5.1 Summary

368 In this paper, we investigated the dependency between streamflow elasticity and aridity
 369 using a large dataset of 4,122 catchments across Europe, Australia, North America
 370 and South America. Our analysis confirmed the well-established dependency between
 371 elasticity and aridity and showed that the shape of this dependency can be effectively
 372 reproduced by existing theoretical formulas. We further demonstrated that these
 373 theoretical formulas can be used to guide the regionalization process, producing a
 374 regionalized aridity-dependent estimate of streamflow elasticity for each catchment,
 375 based on a parsimonious parameterization. The proposed solution, based on the
 376 Oldekop formula, is summarized in Table 5 below and illustrated in Figure 5.

377
 378 **Table 5: Summary of the proposed aridity-accounting regionalized formulas for computing the**
 379 **precipitation and potential evaporation elasticities of streamflow. ΔQ , ΔP , ΔE_0 and $\Delta \Lambda$ are the**
 380 **annual streamflow, precipitation, potential evaporation and synchronicity anomalies,**
 381 **respectively [$mm\ year^{-1}$]. The nondimensional aridity index ($\varphi = \overline{E_0}/\overline{P}$) is computed as a long-**
 382 **term average. $f_P(\varphi)$ and $f_{E_0}(\varphi)$ are borrowed from the Oldekop formula (see Table 1)**

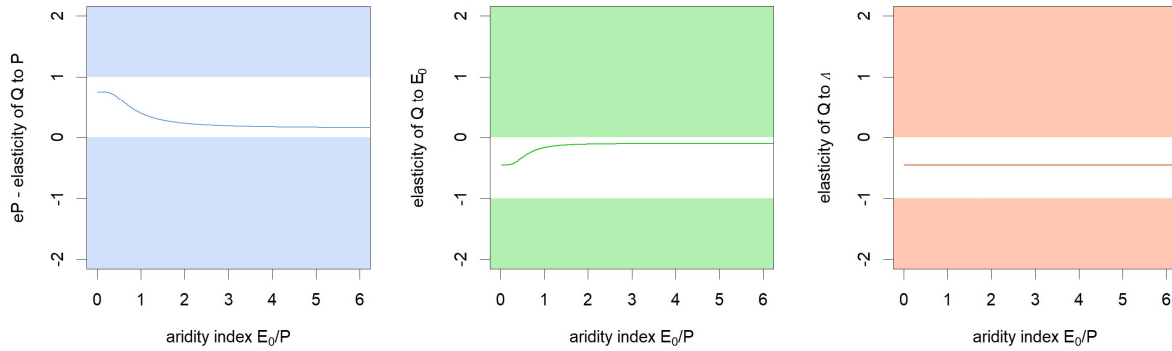
$\Delta Q = e_{Q/P}\Delta P + e_{Q/E_0}\Delta E_0 + e_{Q/\Lambda}\Delta \Lambda$ $e_{Q/P} = 0.15 + 0.6 * f_P(\varphi)$ $f_P(\varphi) = \tanh^2\left(\frac{1}{1.32 * \varphi}\right)$
--

$$e_{Q/E_0} = -0.10 + 0.35 * f_{E_0}(\varphi)$$

$$f_{E_0}(\varphi) = -\tanh\left(\frac{1}{1.43 * \varphi}\right) + \frac{1}{1.43 * \varphi} \left[1 - \tanh^2\left(\frac{1}{1.43 * \varphi}\right)\right]$$

$$e_{Q/\Delta} = -0.47$$

383



384

385 **Figure 5. Regionalized relationships (from the equations in Table 5) for the climatic elasticities**
 386 **of streamflow as a function of the aridity index: precipitation elasticity (left), potential**
 387 **evaporation elasticity (middle), synchronicity elasticity (right). The white domain indicates the**
 388 **physically-plausible range, i.e. [0,1] for precipitation elasticity and [-1,0] for potential evaporation**
 389 **and synchronicity elasticities.**

390

391 5.2 Limitations and perspectives

392 Because our work was empirical, and even if it is based on a very large set of real-
 393 world data, it will remain provisory, until improved by others. It is important to note three
 394 limitations in our study. First, the relationships in Table 5 were developed on
 395 catchments with limited interannual memory (in the sense of de Lavenne et al., 2022):
 396 this excludes those catchments for which Eq. 3 would not be warranted to estimate
 397 streamflow elasticity, since additional independent variables expressing the climatic
 398 anomalies of the previous years would have been required. This could have been done
 399 following the work of de Lavenne et al. (2022), or of Pelletier and Andréassian (2020),
 400 but we preferred to keep the elasticities' estimation as simple as possible. Second,
 401 aridity was computed using the Oudin et al. (2005) formula for potential evaporation,
 402 and the use of other formulas might require a recalibration of the model parameters.
 403 Third, although we do believe that aridity is the first-order driver of elasticity at the
 404 global scale, it is not the only one, and our regional model is clearly only a first step in
 405 the search for physical explanations.

406 **6 Acknowledgements**

407 The authors would like to acknowledge the many individuals that worked to make
408 available the hydrological datasets used in this paper. Special thanks are due to
409 Charles Perrin and Matteo Rosales for their suggestions, as well as to Manuela
410 Brunner, Bailey Anderson and Maik Renner for their reviews.

411 **7 Funding**

412 This research has been funded in part by the Agence Nationale de la Recherche
413 (projects CIPRHES ANR-20-CE04-0009 and DRHYM ANR-22-CE56-0007).

414 **8 Author contributions**

415 VA: conceptualization and writing, GMG: computations, figures, discussion, writing
416 (review and editing), AL: computations, discussion JL: discussion, writing (review and
417 editing).

418 **9 Code/Data availability**

419 The data are available from the original sources listed in section 2.1. The code allowing
420 to perform linear regressions is a standard function in the R programming language.

421 **10 Competing interests**

422 The authors have no competing interests.

423 **11 References**

- 424 Addor, N., Nearing, G., Prieto, C., Newman, A. J., Le Vine, N., & Clark, M. P.: A
425 Ranking of Hydrological Signatures Based on Their Predictability in Space. *Water*
426 *Resour Res*, 54, 8792–8812, <https://doi.org/10.1029/2018WR022606>, 2018.
- 427 Addor, N., Newman, A.J., Mizukami, N., and Clark, M.P.: The CAMELS data set:
428 catchment attributes and meteorology for large-sample studies, *Hydrol. Earth*
429 *Syst. Sci.*, 21, 5293–5313, <https://dx.doi.org/10.5194/hess-21-5293-2017>, 2017.

430 Almagro, A., Oliveira, P.T.S., Alves Meira Neta, A., Roy, T., and Troch, P.: CABra: a
431 novel large-sample dataset for Brazilian catchments, *Hydrol. Earth Syst. Sci.*, 25,
432 3105–3135, <https://doi.org/10.5194/hess-25-3105-2021>, 2021.

433 Anderson, B.J., Brunner, M.I., Slater, L.J., and Dadson, S.J.: Elasticity curves describe
434 streamflow sensitivity to precipitation across the entire flow distribution. *Hydrol.*
435 *Earth Syst. Sci.*, 28, 1567–1583, <https://doi.org/10.5194/hess-28-1567-2024>,
436 2024.

437 Andréassian, V., Guimarães, G.M., de Lavenne, A., and Lerat, J.: Time shift between
438 precipitation and evaporation has more impact on annual streamflow variability
439 than the elasticity of potential evaporation, *Hydrol. Earth Syst. Sci.*, 29, 5477–
440 5491, <https://doi.org/10.5194/hess-29-5477-2025>, 2025.

441 Andréassian, V.: On the (im)possible validation of hydrogeological models. *C. R.*
442 *Géosci*, 355 (S1), 337-345, <https://doi.org/10.5802/crgeos.142>, 2023.

443 Andréassian, V. and Sari, T.: Technical Note: On the puzzling similarity of two water
444 balance formulas – Turc-Mezentsev vs Tixeront-Fu. *Hydrol. Earth Syst. Sci.*, 23:
445 2339-2350, <https://dx.doi.org/10.5194/hess-23-2339-2019>, 2019.

446 Andréassian, V., Coron, L., Lerat, J., and le Moine, N.: Climate elasticity of streamflow
447 revisited – an elasticity index based on long-term hydrometeorological records,
448 *Hydrol. Earth Syst. Sci.*, 20, 4503–4524, [https://dx.doi.org/10.5194/hess-20-](https://dx.doi.org/10.5194/hess-20-4503-2016)
449 [4503-2016](https://dx.doi.org/10.5194/hess-20-4503-2016), 2016.

450 Arora, V.K.: The use of the aridity index to assess climate change effect on annual
451 runoff. *J. Hydrol.*, 265, 164-177, [https://doi.org/10.1016/S0022-1694\(02\)00101-](https://doi.org/10.1016/S0022-1694(02)00101-4)
452 [4](https://doi.org/10.1016/S0022-1694(02)00101-4), 2002.

453 Bagrov, N.: On long-term average of evapotranspiration from land surface (О среднем
454 многолетнем испарении с поверхности суши), *Meteorologia i Gidrologia -*
455 *Метеорология и Гидрология*, 10, 20-25, 1953.

456 Berghuijs, W.R., Gnan, S.J., Woods, R.A.: Unanswered questions on the Budyko
457 framework. *Hydrol. Process.*, 34, 5699-5703. <https://doi.org/10.1002/hyp.13958>,
458 2020.

459 Berghuijs, W.R., Larsen, J.R., van Emmerik, T.H.M., and Woods, R.A.: A global
460 assessment of runoff sensitivity to changes in precipitation, potential evaporation,
461 and other factors. *Water Resour. Res.*, 53, 8475–8486.
462 <https://doi.org/10.1002/2017WR021593>, 2017.

463 Berghuijs, W., Woods, R.: Correspondence: Space-time asymmetry undermines water
464 yield assessment. *Nat Commun* 7, 11603,
465 <https://doi.org/10.1038/ncomms11603>, 2016.

466 Budyko, M. I. Evaporation under natural conditions, Israel Program for Scientific
467 Translations, Jerusalem, 130 pp., 1948 (1963).

468 Chiew, F.H.S.: Estimation of rainfall elasticity of streamflow in Australia, *Hydrol. Sci.*
469 *J.*, 51, 613–625, <https://doi.org/10.1623/hysj.51.4.613>, 2006.

470 Coutagne, A. and de Martonne, E.: De l'eau qui tombe à l'eau qui coule – évaporation
471 et déficit d'écoulement, IAHS Red Book series, 20, 97–128, 1934.

472 Coxon, G., Addor, N., Bloomfield, J. P., Freer, J., Fry, M., Hannaford, J., Howden,
473 N.J.K., Lane, R., Lewis, M., Robinson, E.L., Wagener, T., and Woods, R.:
474 CAMELS-GB: hydrometeorological time series and landscape attributes for 671
475 catchments in Great Britain, *Earth Syst. Sci. Data*, 12, 2459–2483,
476 <https://dx.doi.org/10.5194/essd-12-2459-2020>, 2020.

477 Delaigue, O., Guimarães, G.M., Brigode, P., Génot, B., Perrin, C., Soubeyroux, J.-M.,
478 Janet, B., Addor, N., and Andréassian, V.: CAMELS-FR dataset: a large-sample
479 hydroclimatic dataset for France to explore hydrological diversity and support
480 model benchmarking, *Earth Syst. Sci. Data*, 17, 1461–1479,
481 <https://doi.org/10.5194/essd-17-1461-2025>, 2025.

482 de Lavenne, A., Andréassian, V., Crochemore, L., Lindström, G., and Arheimer, B.:
483 Quantifying pluriannual hydrological memory with Catchment Forgetting Curves,
484 *Hydrol. Earth Syst. Sci.*, 26, 2715–2732, [https://doi.org/10.5194/hess-26-2715-](https://doi.org/10.5194/hess-26-2715-2022)
485 [2022](https://doi.org/10.5194/hess-26-2715-2022), 2022.

486 de Lavenne, A. and Andréassian, V.: Impact of climate seasonality on catchment yield:
487 a parameterization for commonly-used water balance formulas, *J. Hydrol.*, 558,
488 266–274, <https://dx.doi.org/10.1016/j.jhydrol.2018.01.009>, 2018.

489 Dooge, J.C.I.: Sensitivity of runoff to climate change: A Hortonian approach, *Bull. Am.*
490 *Meteorol. Soc.*, 73, 2013-2024, [https://doi.org/10.1175/1520-](https://doi.org/10.1175/1520-0477(1992)073<2013:SORTCC>2.0.CO;2)
491 [0477\(1992\)073<2013:SORTCC>2.0.CO;2](https://doi.org/10.1175/1520-0477(1992)073<2013:SORTCC>2.0.CO;2), 1992.

492 Dooge, J.C., Bruen, M., and Parmentier, B.: A simple model for estimating the
493 sensitivity of runoff to long-term changes in precipitation without a change in
494 vegetation. *Adv Water Resour*, 23, 153-163, [https://doi.org/10.1016/S0309-](https://doi.org/10.1016/S0309-1708(99)00019-6)
495 [1708\(99\)00019-6](https://doi.org/10.1016/S0309-1708(99)00019-6), 1999.

496 Feng, X. Thompson, S.E., Woods, R., and Porporato, I.: Quantifying asynchronicity of
497 precipitation and potential evapotranspiration in Mediterranean climates,
498 *Geophys. Res. Lett.*, <https://doi.org/10.1029/2019GL085653>, 2019.

499 Fowler, K.J.A., Zhang, Z., and Hou, X.: CAMELS-AUS v2: updated
500 hydrometeorological time series and landscape attributes for an enlarged set of
501 catchments in Australia, *Earth Syst. Sci. Data*, 17, 4079–4095,
502 <https://doi.org/10.5194/essd-17-4079-2025>, 2025.

503 Fu, B.: On the calculation of the evaporation from land surface (in Chinese),
504 *Atmospherica Sinica*, 5, 23-31, [https://doi.org/10.3878/j.issn.1006-](https://doi.org/10.3878/j.issn.1006-9895.1981.01.03)
505 [9895.1981.01.03](https://doi.org/10.3878/j.issn.1006-9895.1981.01.03), 1981.

506 Gnann, S., Anderson, B. J., and Weiler, M.: Uncertainty and non-stationarity of
507 empirical streamflow sensitivities, *EGUsphere* [preprint],
508 <https://doi.org/10.5194/egusphere-2025-4527>, 2025.

509 Höge, M., Kauzlaric, M., Siber, R., Schönenberger, U., Horton, P., Schwanbeck, J. and
510 Floriancic, M. G. and Viviroli, D. and Wilhelm, S. and Sikorska-Senoner, A.E.,
511 Addor, N., Brunner, M., Pool, S., Zappa, M. and Fenicia, F.: CAMELS-CH: hydro-
512 meteorological time series and landscape attributes for 331 catchments in
513 hydrologic Switzerland, *Earth Syst. Sci. Data*, 15, 5755–5784,
514 <https://doi.org/10.5194/essd-15-5755-2023>, 2023.

515 Koster, R.D., and Suarez, M.J.: A simple framework for examining the interannual
516 variability of land surface moisture fluxes, *J. Clim.*, 12, 1911–1917,
517 [https://doi.org/10.1175/1520-0442\(1999\)012<1911:ASFFET>2.0.CO;2](https://doi.org/10.1175/1520-0442(1999)012<1911:ASFFET>2.0.CO;2), 1999.

518 Loritz, R., Dolich, A., Acuña Espinoza, E., Ebeling, P., Guse, B., Götte, J., Hassler, S.
519 K., Hauffe, C., Heidbüchel, I., Kiesel, J., Mälicke, M., Müller-Thomy, H., Stölzle,
520 M., and Tarasova, L.: CAMELS-DE: hydro-meteorological time series and
521 attributes for 1555 catchments in Germany, *Earth Syst. Sci. Data*, 16, 5625–5642,
522 <https://doi.org/10.5194/essd-16-5625-2024>, 2024.

523 Liu, J., Koch, J., Stisen, S., Troldborg, L., Højberg, A. L., Thodsen, H., Hansen, M. F.
524 T., and Schneider, R. J. M.: CAMELS-DK: hydrometeorological time series and
525 landscape attributes for 3330 Danish catchments with streamflow observations
526 from 304 gauged stations, *Earth Syst. Sci. Data*, 17, 1551–1572,
527 <https://doi.org/10.5194/essd-17-1551-2025>, 2025.

528 Mathevet, T., Michel, C., Andréassian, V. and Perrin, C.: A bounded version of the
529 Nash-Sutcliffe criterion for better model assessment on large sets of basins. IAHS
530 Red Books Series, 307, 211-219, 2006.

531 Mezentsev, V.: Back to the computation of total evaporation (Ещё раз о расчете
532 среднего суммарного испарения), Meteorologia i Hidrologia - Метеорология и
533 Гидрология, 5, 24-26, 1955.

534 Milly, P.C.D.: Climate, interseasonal storage of soil water, and the annual water
535 balance, Adv. Water Resour., 17, 19–24, [https://doi.org/10.1016/0309-
536 1708\(94\)90020-5](https://doi.org/10.1016/0309-1708(94)90020-5), 1994.

537 Nash, J.E. and Sutcliffe, J.V.: River flow forecasting through conceptual models. Part
538 I - a discussion of principles. J. Hydrol., 10, 282-290.
539 [https://doi.org/10.1016/0022-1694\(70\)90255-6](https://doi.org/10.1016/0022-1694(70)90255-6), 1970.

540 Oldekor, E.: Evaporation from the surface of river basins (Испарение с поверхности
541 речных бассейнов), Collection of the Works of Students of the Meteorological
542 Observatory, University of Tartu (Jurjew, Dorpat), Tartu, Estonia. [https://ars.els-
543 cdn.com/content/image/1-s2.0-S0022169416300270-mmc1.pdf](https://ars.els-cdn.com/content/image/1-s2.0-S0022169416300270-mmc1.pdf), 1911.

544 Oudin, L., Hervieu, F., Michel, C., Perrin, C., Andréassian, V., Anctil, F. and Loumagne,
545 C.: Which potential evapotranspiration input for a rainfall-runoff model? Part 2 –
546 Towards a simple and efficient PE model for rainfall-runoff modelling. J. Hydrol.,
547 303: 290-306, <https://dx.doi.org/10.1016/j.jhydrol.2004.08.026> 2005.

548 Oudin, L., and Lalonde, M.: Pitfalls of space-time trading when parametrizing a land
549 use dependent hydrological model, C. R. Géosci., 355 (S1): 99-115,
550 <https://doi.org/10.5802/crgeos.146>, 2023.

551 Pardé, M.: L'abondance des cours d'eau, Revue de Géographie Alpine, 21 (3), 497–
552 542, https://www.persee.fr/doc/rga_0035-1121_1933_num_21_3_5370, 1933.

553 Peel, M. C. and Blöschl, G.: Hydrological modelling in a changing world. Prog. Phys.
554 Geogr.: Earth Environ., 35, 249–261, <https://doi.org/10.1177/030913331140255>,
555 2011.

556 Renner, M., Seppelt, R., and Bernhofer, C.: Evaluation of water-energy balance
557 frameworks to predict the sensitivity of streamflow to climate change. Hydrol.
558 Earth Syst. Sci., 16, 1419–1433, <https://doi.org/10.5194/hess-16-1419-2012>,
559 2012.

560 Roderick, M.L., and Farquhar, G.D.: A simple framework for relating variations in runoff
561 to variations in climatic conditions and catchment properties, *Water Resour. Res.*,
562 47, <https://doi.org/10.1029/2010WR009826>, 2011.

563 Sankarasubramanian, A., Vogel, R.M., and Limbrunner, J.F.: Climate elasticity of
564 streamflow in the United States, *Water Resour. Res.*, 37, 1771–1781,
565 <https://doi.org/10.1029/2000wr900330>, 2001.

566 Schaake, J., and Liu, C.: Development and application of simple water balance models
567 to understand the relationship between climate and water resources, *New*
568 *Directions for Surface Water Modeling*, IAHS Red Book series n°181, 343–352,
569 <https://iahs.info/uploads/dms/7849.343-352-181-Schaake-Jr.pdf>, 1989.

570 Schreiber, P.: On the relationship between precipitation and river flow in central Europe
571 (Über die Beziehungen zwischen dem Niederschlag und der Wasserführung der
572 Flüsse in Mitteleuropa). *Zeitschrift für Meteorologie*, 21, 441–452, 1904.

573 Singh, R., Wagener, T., van Werkhoven, K., Mann, M. E., and Crane, R.: A trading-
574 space-for-time approach to probabilistic continuous streamflow predictions in a
575 changing climate – accounting for changing watershed behavior, *Hydrol. Earth*
576 *Syst. Sci.*, 15, 3591–3603, <https://doi.org/10.5194/hess-15-3591-2011>, 2011.

577 Thornthwaite, C.W.: An approach toward a rational classification of climate. *Geog.*
578 *Rev.*, 38 (1), 55–94. <https://doi.org/10.2307/210739>, 1948.

579 Tixeront, J.: Prediction of streamflow (in French: Prévion des apports des cours
580 d'eau). IAHS publication, 63, 118-126, 1964.

581 Turc, L.: The water balance of soils: relationship between precipitations, evaporation
582 and flow (In French: Le bilan d'eau des sols: relation entre les précipitations,
583 l'évaporation et l'écoulement), *Annales Agronomiques, Série A*, 5, 491-595,
584 1954.

585 Yang, H., Yang, D., Lei, Z., and Sun, F.: New analytical derivation of the mean annual
586 water-energy balance equation, *Water Resour. Res.*, 44, W03410,
587 <https://doi.org/10.1029/2007WR006135>, 2008.

588 Yokoo, Y., Sivapalan, M., and Oki, T.: Investigating the role of climate seasonality and
589 landscape characteristics on mean annual and monthly water balances, *J.*
590 *Hydrol.*, 357, 255–269, <https://doi.org/10.1016/j.jhydrol.2008.05.010>, 2008.

591 Zhang, L., and Brutsaert, W: Blending the evaporation precipitation ratio with the
592 complementary principle function for the prediction of evaporation. *Water Resour.*
593 *Res.*, 57, e2021WR029729. <https://doi.org/10.1029/2021WR029729>, 2021.

594 Zhang, Y., Viglione, A., and Blöschl, G.: Temporal scaling of streamflow elasticity to
595 precipitation: A global analysis. *Water Resour. Res.*, 58, e2021WR030601.
596 <https://doi.org/10.1029/2021WR030601>, 2022.
597