Response to Reviewer #1

We thank the reviewer for the careful reading and the suggestions to improve the quality and readability of the manuscript. We have followed your suggestions and revised the manuscript accordingly. Please find our responses below.

This is an interesting paper, examining a model of the vertical POC flux in the ocean below the euphotic zone. I found the paper hard to follow in places, and this was in part because of the choice of English usage: I would strongly urge the authors to seek out a native English speaker to clean this up.

Answer. The English usage was checked and improved with the help of a native English speaker.

There have been quite a few papers published recently that take a similar approach to the problem of modeling particle flux in the ocean, and the authors cite all of these (Kriest and Oschlies, 2008; Omand et al., 2020; DeVries et al., 2014). However, it is unclear what this manuscript presents that is new when compared with these other papers. Indeed, as far as I can see, there is no detailed comparison of results (except to show that one of their analytical solution is equivalent to that of DeVries et al.). I would like to see an analysis of what new things we learn from this model.

Answer. Thank you for your comment. Indeed, we have emphasised (L. 50) that we rely on the known parameterisations in the models (Kriest and Oshlies, 2008; DeVries et al., 2014; Cram et al., 2018). The novelty of our study is the development of the Euler-Lagrangian approach and the application of the corresponding numerical algorithm to solve the problem. We have added explanatory text.

L. 203 "Unlike the models (Kriest and Oshlies, 2008; Cael and Bisson, 2018) that use the same "Martin curve" power-law dependence (32) for the concentration and mass flux of POM with the exponent β, the exponent in the obtained solution (28) depends not only on β but also on the parameters that characterize the sinking velocity (η) and the particle mass fractal dimension (ζ)."

L. 345 "In this work, we considered a simple Eulerian–Lagrangian approach for solving equations that describe the gravitational sinking of organic particles under the effects of the sizes and ages of the particles, temperature and oxygen concentration on their dynamics and degradation processes. In contrast to other approaches, our approach does not use particle spectrum equations (e.g., DeVries et al., 2014) explicitly or power-law particle size distribution assumptions (e.g., Kriest and Evans , 1999; Maerz et al., 2020). Unlike (Omand et al., 2020), we do not assume *a priori* the constancy of the particle flux in depth in steady state problem. Instead, solutions are found for the Euler equation for the concentration of particles of a given size and the Lagrange equations for a sinking organic particle under the influence of microbiological degradation. In the stationary case, the problem is reduced to solving a system of ordinary differential equations of the first order, in contrast to (DeVries et al., 2014), where the solution of the hyperbolic equation of the first order for the particle distribution is found. In addition, the total concentration and flux of the POM are found by summation over the particle distribution at $z' = 0$, whereas in (DeVries et al., 2014) the summation is carried out over all depths. Our approach makes the particle transport model compatible with large-scale biogeochemical models and provides an opportunity to solve the non-stationary problem in the future using Eq. (1) complemented by the time derivative of $C_{p,d}$ and necessary parameterizations of the POM sinking processes."

L. 358 “Novel analytical solutions of the system of the one-dimensional Eulerian equation for the POM concentration and Lagrangian equations for the particle mass and depth were obtained for constant and age-dependent degradation rates…”

L. 374 “A new Eulerian–Lagrangian numerical approach for solving the problem in general cases was presented. The algorithm includes time steps for Lagrangian variables (sinking velocity and particle mass) and Eulerian depth steps for the concentration of particles of size *d*. This enables the inclusion of different parameterizations of interacting degradation and sinking processes (e.g., DeVries et al., 2014; Cram et al., 2018; Omand et al., 2020; Alcolombri et al., 2021). However, in this study, we limited ourselves to the case where the degradation rate depends on the age of the organic particle, the temperature of the sea water and the concentration of oxygen. Notably, the developed numerical algorithm is suitable for arbitrary dependencies of mass and sinking velocity on the particle diameter. The proposed numerical method was tested on the obtained analytical solutions.”

The model contains many assumption (as stated by the authors), but there is little to no analysis of the consequences of these assumptions. For example, everything is assumed to be a power-law (the mass-size relationship, the sinking velocity etc.) and while this makes things analytically tractable, it is unclear what observational evidence there is for them. For example, size distributions are often assumed to be power-law, but in reality this assumption often holds over a relatively small size range.

Answer. Thank you for your important comment.

Power dependencies are used for two reasons. The first reason is that the power law can be an effective method of parameterization, which, as the reviewer noted, allows one to obtain analytical solutions. Table 1 contains references to works that provide experimental data for the parameters of power dependencies. Note that the developed numerical algorithm suits arbitrary dependencies on the particle diameter. The second reason is that power dependences reflect fundamental properties of processes in nature, e.g. self-similarity of the formation of aggregates. The text and references to the papers with a critical analysis of these approximations were also added:

L. 92 “The measurements of (McDonnell and Buesseler, 2010) show that formulations of sinking velocity as a function of only equivalent particle size can be insufficient because the shapes of the particles (e.g., faecal pellets) can significantly affect the sinking velocity. Fig. 1 from (Cael et al. , 2021) also demonstrates the difficulties of describing the sinking velocities of particles of various sizes, shapes and 90 structures with a single universal dependence. Therefore, Eq. (4) should be considered only a first approximation when describing the complex dynamics of particles.”

L. 379 “Notably, the developed numerical algorithm is suitable for arbitrary dependencies of mass and sinking velocity on the particle diameter.”

The model is a steady state model, and it's unclear if such an assumption is a reasonable one. For example. export fluxes out of the euphotic zone can vary significantly over time periods of days. So whilst I'm not opposed to the use of the steady state assumption, I do wonder about its validity.

Answer. Thank you for pointing out this issue. Yes, time-dependent export fluxes in bloom periods can be important factors in the euphotic layer and upper twilight zone in Polar oceans. Our approach to solving the model equations needs extension for non-stationary Eq. (1), which is out of this paper's scope. We added the text:

L. 67 "We limit ourselves to large-scale climatological processes that cover the water column below the euphotic layer to the bottom.We assume that the effects of time variability on the POM flux are relatively small far from this layer, and we consider the steady states of these fluxes."

L. 355 "Our approach makes the particle transport model compatible with large-scale biogeochemical models and provides an opportunity to solve the non-stationary problem in the future using Eq. (1) complemented by the time derivative of $C_{p,d}$ and necessary parameterizations of the POM sinking processes."

Line 93, the mass loss is proportional to particle mass, not volume. The relationship in Equation (4) makes the correspondence between mass and volume unclear. For example, is the diameter the equivalent spherical diameter, is the volume the conserved volume or the encased volume?

Answer. We have made the following changes to the manuscript according to your comments:

L. 96 "Parameter $\theta = 1$ when the degradation rate is proportional to the particle mass, and $\theta = 2/3$ when the degradation rate is proportional to the surface area of the particle (Omand et al., 2020)."

L. 78 "The relationship between the organic matter mass *md* and diameter *d* of porous particles can be parameterized according to the particle fractal dimension."

L. 70 "The Euler equation for the POM concentration $C_{p,d}$ [kg m^{-3}] for particles of equivalent spherical diameter *d* [m] is written as…"

L. 105 "Furthermore, we suppose that the mass loss is proportional to the mass of the particle ($\gamma = \gamma_0$, $\theta = 1$) and does not depend on temperature or oxygen concentration (γ_0 = const)."

Line 109: I must be missing something here, because it's unclear to me that, practically, z-prime can never be larger than the inverse of psi. This follows from re-writing equation (10) and realizing that the constants eta, gamma0, and zeta are all positive. What am I missing?

Answer. For $d_0 = 20 \cdot 10^{-6}$ m and $d_0 = 200 \cdot 10^{-6}$ m, and for parameters $\eta, \varsigma, \gamma_0, c_w$ presented in Table 1 the values ψ^{-1} are 45.4 m and 672 m, respectively. Below these depths ($z' > \psi^{-1}$), only the trivial zero solutions for $d, W_{p,d}, C_{p,d}$ has physical meaning. We added estimates for the layer thickness:

L. 192 "The finite thickness of the layer of sinking particles with parameters given in Table 1 varies in the range from 45.4 m at $d_0 = 20$ μm to 9937 m at $d_0 = 2000$ μm. "

The authors also need to make their notation more consistent. For example, in equation (15) we get the definition for C_{p,d}. But in equation (16) this becomes C_{p,d,i}. Also, in equation (16), n_d becomes n. In equation (17) we are apparently integrating with respect to *a constant (d_0 having been defined as the initial particle diameter in equation (8)). So, the notation needs to be tidied up throughout the paper, not just in these places.*

Answer. We corrected the notations accordingly to your comments.

$z' = 0$. To obtain the size distribution of $C_{p,d}(0)$, we use a small increment of particle size Δd_0 under the assumption that the concentration is uniform within the interval Δd_0. Then, the distribution $C_{p,d}(0)$ is given by

$$C_{p,d}(0) = M_0 d_0^{-\epsilon} m_{0,d} \Delta d_0 = M_0 c_m d_0^{\zeta-\epsilon} \Delta d_0. \tag{15}$$

The total concentration C_p is calculated as the sum of concentrations $C_{p,k}$ in the k-th interval of size d over the total number of n_d intervals:

$$C_p(z') = \sum_{k=0}^{n_d} C_{p,k} = M_0 c_m \sum_{k=0}^{n_d} d_{0,k}^{\zeta-\epsilon} H(z')(1-\psi z')^{\frac{\zeta-\eta}{\eta}} \Delta d_0, \tag{16}$$

where $d_{0,k} = k\Delta d_0 + d_0^{min}$, $\Delta d_0 = (d_0^{max} - d_0^{min})/n_d$, and d_0^{min} and d_0^{max} are the minimal and maximal values, respectively, of d_0. At $\Delta d_0 \to 0$, the total concentration of sinking POM C_p [kg m^{-3}] in the range from d_0^{min} to d_0^{max} can be calculated as

$$C_p(z') = M_0 c_m \int_{d_0^{min}}^{d_0^{max}} \tilde{d}_0^{\zeta-\epsilon} H(z')(1-\psi z')^{\frac{\zeta-\eta}{\eta}} \, d\tilde{d}_0. \tag{17}$$