Response to Reviewer #3

This paper presents a combined Eulerian-Lagrangian formulation of the organic particles descent while taking into account their size and age among other things, resulting in a non-trivial relation among them, unlike suggested by previous studies. This is a well-written article with strong positioning regarding motivation, contribution and place among the previous literature. My detailed major and minor comments, mainly refraining to comment on the formulations, are organized below in the order of appearance in the manuscript.

Answer. Thank you very much for the very thoughtful comments on our paper. We have followed your suggestions and revised the manuscript accordingly. Please find our responses below.

While the contribution is clear around line 45, I would recommend highlighting the novelty by explaining the unique contribution in the context of limitations of previous attempts in the literature.

Answer. The novelty of our study is the development of the Euler-Lagrangian approach and the application of the corresponding numerical algorithm to solve the problem. We have added explanatory text.

L. 45 "We solved the Eulerian equation for the concentration of particles of a given size and the Lagrangian equations for a sinking organic particle under the influence of microbiological degradation. It allows incorporation of the proposed algorithm into biogeochemical global ocean models with relative ease."

L. 203 "Unlike the models (Kriest and Oshlies, 2008; Cael and Bisson, 2018) that use the same "Martin curve" power-law dependence (32) for the concentration and mass flux of POM with the exponent β, the exponent in the obtained solution (28) depends not only on β but also on the parameters that characterize the sinking velocity (η) and the particle mass fractal dimension (ζ)."

L. 345 "In this work, we considered a simple Eulerian–Lagrangian approach for solving equations that describe the gravitational sinking of organic particles under the effects of the sizes and ages of the particles, temperature and oxygen concentration on their dynamics and degradation processes. In contrast to other approaches, our approach does not use particle spectrum equations (e.g., DeVries et al., 2014) explicitly or power-law particle size distribution assumptions (e.g., Kriest and Evans , 1999; Maerz et al., 2020). Unlike (Omand et al., 2020), we do not assume *a priori* the constancy of the particle flux in depth in the steady state problem. Instead, solutions are found for the Euler equation for the concentration of particles of a given size and the Lagrange equations for a sinking organic particle under the influence of microbiological degradation. In the stationary case, the problem is reduced to solving a system of ordinary differential equations of the first order, in contrast to (DeVries et al., 2014), where the solution of the hyperbolic equation of the first order for the particle distribution is found. In addition, the total concentration and flux of the POM are found by summation over the particle distribution at $z' = 0$, whereas in (DeVries et al., 2014) the summation is carried out over all depths. Our approach makes the particle transport model compatible with large-scale biogeochemical models and provides an opportunity to solve the non-stationary problem in the future using equation (1) for different parameterizations of the POM sinking processes."

L. 358 “Novel analytical solutions of the system of the one-dimensional Eulerian equation for the POM concentration and Lagrangian equations for the particle mass and depth were obtained for constant and age-dependent degradation rates…”

L. 374 “A new Eulerian–Lagrangian numerical approach for solving the problem in general cases was presented. The algorithm includes time steps for Lagrangian variables (sinking velocity and particle mass) and Eulerian depth steps for the concentration of particles of size *d*. This enables the inclusion of different parameterizations of interacting degradation and sinking processes (e.g., DeVries et al., 2014; Cram et al., 2018; Omand et al., 2020; Alcolombri et al., 2021). However, in this study, we limited ourselves to the case where the degradation rate depends on the age of the organic particle, the temperature of the sea water and the concentration of oxygen. Notably, the developed numerical algorithm is suitable for arbitrary dependencies on the particle diameter. The proposed numerical method was tested on the obtained analytical solutions.”

One major comment I have is that the modeling results section needs to be more specific in terms of presenting the results. The authors are showing and referring to the results but not describing and discussing them.

Answer. Thank you for the comment. We extended the description of results and added a discussion in Sect. 6 (L. 318).

L. 307 “Notably, profiles C_p and F_p in Fig. 3c, 3f and 4c, 4f are quite close despite the differences between the temperature and oxygen concentration profiles in the 20-30 ◦N band of the Atlantic and Pacific Oceans (Fig. S1a-S1b). These profiles in the colder, oxygen-saturated waters of the Southern Ocean (Fig. S1c) attenuate more slowly with depth.”

Same goes for the validations -- I found it hard to follow which parts agree with the measurements more than others. Both of the aspects need to be addressed in the results section for the paper to be strong.

Answer. We added estimates of the model accuracy for different spectral resolutions of the particle sizes n_d :

L. 216 “The relative maximal absolute errors [%] of the calculated AIDR and ADDR solutions for C_p and F_p are presented in Table S1. We compare the solutions at spectral resolutions $n_d = 100$ and $n_d = 10$ with the baseline calculation at $n_d = 990$. These estimates demonstrate the necessity of fine resolution of the spectre of particles at the lower boundary of the euphotic zone for obtaining accurate profiles of the POM concentration and sinking flux. In this case, the particle concentration profile is more sensitive to the spectral resolution than the sinking flux profile is.”

When comparing simulation results and measurement data, it is important to keep in mind that (L. 389) “discrepancies between the model predictions and observations were caused by incomplete descriptions of processes and uncertainties in model parameters, as well as variability in the measured POM concentration and flux profiles owing to vertical and horizontal variability in the ocean fields.”

L. 283 “When the modelling results are compared with the measurement data, the significant scatter of the measurement data presented in Figs. 4--6 must be noted. This scatter is due both to the difficulties of measuring the concentration and flux of particles and to regional differences in the influx of particles and in the surrounding ocean.”

When saying Fig 4-6, as in line 270, I'd recommend being specific when referring to figures and panels to show results.

Answer. We referred figures accordingly to your suggestion.

L. 281 “These profiles are compared with normalized measurements in the subtropical zones of the Atlantic (Fig. 4) and Pacific (Fig. 5) Oceans and in the Atlantic and Pacific sectors of the Southern Ocean (Fig. 6) to consider the effects of temperature and oxygen concentration on POM.”

L. The solutions with η = 0.63 decay more slowly than those obtained for η = 1.17 as follows also from analytic solutions in Figs. 4a, 4d, 5a, 5d, 6a, 6d.

L. 296 “As shown in Figs. 4b, 4e, 5b, 5e, 6b, and 6e, the dependence of the degradation rate on temperature significantly affected the C_p and F_p profiles; namely, it enhanced the degradation of sinking particles in the upper layers of the ocean and suppressed it in the deep layers of the ocean.”

“As follow” is a typo in line 272

Answer. Thank you. We corrected this typo.

Another major comment is that the paper is missing the significance in terms of closing the loop and relating back the findings back to the implementation in the earth system/biogeochemical models as the authors had described in the introduction section.

Answer. We reworked and added text accordingly:

L. 355 “Our approach makes the particle transport model compatible with large-scale biogeochemical models and provides an opportunity to solve the non-stationary problem in the future using equation (1) for different parameterizations of the POM sinking processes.”

L. 398 “Notably, to obtain analytical solutions and demonstrate the numerical Eulerian--Lagrangian approach, significant simplifications were made in the description of the particle dynamics. In particular, the particle sinking velocity was described in the Stokes approximation. The aggregation and fragmentation of particles, mineral ballasting, ocean density stratification, and temporal changes in particle flows were not considered. While some simplifications can be eliminated by using a numerical approach, others require significant generalization. This applies particularly to the description of particle ballasting mechanisms. On the one hand, ballast affects the sinking of particles, but on the other hand, ballast minerals can protect organic matter from degradation (Cram et al., 2018). The processes of fragmentation and consumption of sinking particles, which are important in the upper mesopelagic layer, are poorly understood (Burd et al., 2024). Comparison of calculation results for different parameter values (e.g. η) did not reveal the advantage of one parameter value for both C_p and F_p, which may be due to the incompleteness of the description of the processes of the simplified model used. Therefore, for the effective application of the proposed approach in biogeochemical models, a parameterization of the main process controls of the biological pump mechanism based on data from natural and laboratory measurements is necessary.”

Please include significance and follow-up work recommendations before the conclusion. The authors have shown appreciations of the limitations of the work at various places in the text, I'd recommend synthesizing them at the end before conclusions.

Answer. Thank you for the suggestion. The reworked text is given in answer to the previous comment.

In light of my comments above, I'd recommend minor revisions to the manuscript before it is published. Good luck and congratulations on this useful work.

Answer. Thank you for the encouraging comments and suggestions.