Point-to-point response to the reviewers' comments

We sincerely thank the reviewers for the insightful comments and constructive suggestions, which have significantly helped us improve the quality of our manuscript. We have carefully considered all points and have revised the manuscript accordingly. Below, we provide a point-by-point response to the comments, the line numbers marked in red indicate the locations where revisions have been made in the revised manuscript.

Reviewer #1

The SVD-3/4DVar method, proposed by Qiu et al. (2007), is considered a pioneering achievement in the field of four-dimensional ensemble variational data assimilation (4DEnVar). This manuscript provides a valuable exploration of the practical application of the SVD-3DVar method and demonstrates certain innovative merits, meeting the publication criteria of this journal. The following suggestions are provided for further improvement:

Comment 1:

Regarding the choice of methodology, it is recommended that you explain why SVD-4DVar was not adopted in favor of SVD-3DVar, while briefly analyzing the core challenges of the latter.

Response:

We appreciate the reviewer's suggestion. The decision to adopt SVD-3DEnVar instead of SVD-4DEnVar was primarily motivated by two factors:

Computational Efficiency and Operational Feasibility: The main objective of this study is to improve computational efficiency for operational typhoon forecasting, so the relatively simple 3DVar scheme is chosen for experimentation. (line 127)

Observation Frequency: The sea surface wind observations used in this paper for real-data experiments are available only every 6 hours. This low temporal resolution does not fully leverage the temporal continuity advantages of 4DEnVar. (line 109)

We acknowledge that SVD-3DEnVar has its own challenges, which we now discuss in the revised manuscript (Section 5). The primary limitations include: The use of a single localization scale, which may not optimally handle multi-scale observations (e.g., surface, satellite, and radar); The limited ability of linear combinations of singular vectors to represent strongly nonlinear relationships, especially for unobserved variables. These challenges will guide our future work on multi-scale assimilation and machine-learning-enhanced methods. (line 532-564)

Comment 2:

It should be noted that the singular value decomposition (SVD) of matrix A is extremely

challenging in practice due to its large dimensions ($N_x + N_y$), posing significant difficulties in terms of both storage and computation. A discussion on this aspect is recommended.

Response:

We fully agree with the reviewer. To address the computational and storage challenges of performing SVD on the large matrix A, we implemented a local patch assimilation strategy. This approach significantly reduces the effective dimensions of A by horizontal and vertical localization. Only observations within a specified horizontal (l_h) and vertical (l_v) radius from the central grid point are included. As a result, both N_x (model variables in the local patch) and N_y (observations within the local patch) are drastically reduced. This makes the SVD computationally feasible without sacrificing the flow-dependent covariance information. We have added a clarification in Section 3.1 to explain this strategy. (line 158)

Comment 3:

While equation (11) provides the Gaussian weight function, the localization scheme used in SVD-3DVar should be presented in more detail to enhance the completeness of the paper.

Response:

Thank you for this suggestion. We have expanded the description of the localization scheme in Section 3.1. The revised text now reads:

"The Gaussian weight function defined in Equation (12) is applied to each observation within the local patch. The horizontal and vertical localization scales (σ_h and σ_v) control the rate at which observation influence decays with distance. This localization ensures that only observations within a specified radius significantly impact the analysis increment at the center of the local patch, thereby mitigating spurious long-range correlations and improving the stability and accuracy of the assimilation." (line 194)

Comment 4:

In recent years, 4DEnVar methods have advanced rapidly. To reflect an up-to-date understanding of the field, it is advisable to include references to relevant studies published between 2022 and 2025.

Response:

We thank the reviewer for this suggestion. We have now incorporated several recent references on 4DEnVar advancements in the Introduction and Conclusion sections, including:

Inverarity et al. (2023) on hybrid En-4DEnVar in the Met Office system;

Berre and Arbogast (2024) on hybrid covariances at Météo-France;

Lu and Wang (2024) on scale-dependent localization in hurricane forecasting;

Thiruvengadam and Wang (2025) on convective-scale 4DEnVar;

Wang et al. (2025) on CubeSat radiance assimilation.

Zhang et al. (2022) on NLS-4DVar method for PM2.5 forecasts.

These additions help contextualize our work within the evolving landscape of ensemble-variational methods. (line 120-124, 544-550)

Comment 5:

Regarding the generation of initial samples, several classical works (e.g. those by Evensen) have achieved high memory efficiency. It would be beneficial to reference these works and discuss their relevance to the present method. From a practical perspective, the main computational burden in parallelization typically lies in the ensemble forecast component, which should also be addressed.

Response:

We agree with the reviewer. The initial perturbation generation in SVD-3DEnVar is based on the Gaussian random field method introduced by Evensen (1994), which is known for its memory efficiency and statistical robustness. However, the original implementation was limited to 2D square grids with odd numbers of points, which motivated our multi-dimensional and parallel optimizations. We have mentioned Evensen's foundational work and clarify how our optimizations build upon it. (line 207, 243)

Regarding the computational burden of ensemble forecasts: yes, the ensemble integration is indeed the most computationally intensive part in parallel implementations. However, since ensemble members are independent, they can be run concurrently if sufficient computational resources are available. While this study focuses on optimizing the perturbation generation and assimilation steps, we acknowledge the resource demands of ensemble forecasting and will address this in future work. (line 82)

Comment 6:

As this is an ensemble-based method, it is recommended that the ensemble sample update strategy in SVD-3DVar is explained briefly to improve the completeness of the methodological description.

Response:

We have added the following explanation to Section 1:

"Unlike traditional ensemble assimilation schemes (such as EnKF), which directly update each ensemble member during cyclic assimilation, SVD-3DEnVar only assimilates and updates the control forecast. Therefore, after each assimilation cycle, the updated analysis field must be perturbed again to generate the initial conditions for the next cycle's ensemble forecast. This approach has the advantage of avoiding filter divergence and, in the presence of model errors, performs better than EnKF (Qiu et al., 2007)." (line 72)

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Reviewer #2 (Dr. Nima Zafarmomen)

The paper targets computational bottlenecks in the SVD-3DEnVar data assimilation scheme and proposes two main engineering improvements: (i) a redesigned perturbation-field generation algorithm that directly produces 3D, grid-conforming perturbations with multi-process parallelism and without intermediate I/O, and (ii) a parallel local-patch assimilation framework that partitions work, balances load, and reduces memory duplication via node-level pointer sharing. Using the TRAMS 3.0 model, the authors report large wall-clock reductions: 3D perturbation generation from ~22 minutes to 2.2 seconds and end-to-end assimilation from ~1700 minutes (serial) to <15 minutes with 150 nodes, alongside preliminary OSSE and real-data tests that show reasonable analysis increments and some improvement in typhoon forecasts.

Overall, the paper tackles a very practical problem: making SVD-3DEnVar fast and memory-efficient enough for realistic, large-domain applications. I recommed it for publication after considering these comments:

Comment 1:

How do you prescribe and verify the target covariance statistics of the perturbation fields (variance, horizontal and vertical correlation lengths, cross-variable correlations)?

Response:

The perturbation fields are generated following Evensen (2003). First, a set of smooth two-dimensional random perturbation fields with zero mean and unit variance is generated. Then, a fixed weighted blending scheme is applied (40% from the perturbation information of adjacent filled layers, 60% from newly generated two-dimensional random fields) to produce three-dimensional perturbation fields with a certain vertical correlation scale. After superimposing different three-dimensional perturbations for each variable, the fields are integrated forward for a period of time to allow the perturbations between different variables to develop reasonable multivariate correlations (e.g., physically consistent relationships between temperature and pressure).

The control of variance and horizontal correlation scale for the two-dimensional random perturbation fields is based on the following principles:

For a continuous two-dimensional field $q = q(x, y)$, its Fourier transform can be written as:

$$q(x, y) = \iint_{-\infty}^{\infty} \hat{q}(\mathbf{k}) e^{i\mathbf{k}\cdot\mathbf{x}} d\mathbf{k} \qquad (R1)$$

where $\hat{q}(\mathbf{k})$ are the Fourier coefficients, the wavenumber vector K is defined as $\mathbf{k} = (\kappa_l, \gamma_p)$, and κ_l and γ_p are the wavenumbers in the x and y directions, respectively.

Discretizing on an $N \times M$ horizontal grid:

$$q(x_n, y_m) = \sum_{l,p} \hat{q}(\kappa_l, \gamma_p) e^{i(\kappa_l x_n + \gamma_p y_m)} \Delta\mathbf{k} \qquad (R2)$$

where $x_n = n\Delta x$, $y_m = m\Delta y$, $\kappa_l = \frac{2\pi l}{x_N} = \frac{2\pi l}{N\Delta x}$, $\gamma_p = \frac{2\pi p}{y_M} = \frac{2\pi p}{M\Delta y}$, and $\Delta\mathbf{k} = \Delta\kappa\Delta\gamma = \frac{(2\pi)^2}{NM\Delta x\Delta y}$.

Assuming the Fourier coefficients take the form:

$$\hat{q}(\kappa_l, \gamma_p) = \frac{c}{\sqrt{\Delta\mathbf{k}}} e^{-\frac{\kappa_l^2 + \gamma_p^2}{\sigma^2}} e^{2\pi i \phi_{l,p}} \qquad (R3)$$

where c is a normalization constant controlling the variance, σ is a bandwidth parameter determining the correlation length, and $\phi_{l,p} \in [0,1]$ is a uniformly distributed random number.

Substituting (R3) into (R2) yields the spatial field expression:

$$q(x_n, y_m) = \sum_{l,p} \frac{c}{\sqrt{\Delta\mathbf{k}}} e^{-\frac{\kappa_l^2 + \gamma_p^2}{\sigma^2}} e^{2\pi i \phi_{l,p}} e^{i(\kappa_l x_n + \gamma_p y_m)} \Delta\mathbf{k} \qquad (R4)$$

This can be interpreted as multiplying a Gaussian-shaped filter (spectral window) and random phase in wavenumber space, followed by an inverse Fourier transform to obtain the spatial random field. Therefore, the horizontal correlation length and variance of the two-dimensional random perturbation field can be controlled by adjusting the parameters σ and c. (line 207, 255)

Reference:

Evensen G. 2003. The Ensemble Kalman Filter: Theoretical Formulation and Practical Implementation, Ocean Dynamics 53, 343–367.

Comment 2:

Are perturbations generated multivariately (i.e., with controlled cross-variable balance), or independently per variable with post hoc smoothing?

Response:

Currently, perturbations for each variable are generated independently. By integrating the independently generated perturbations added to the model initial conditions forward for a period of time, cross-variable covariance relationships develop among the perturbations. (line 258)

Comment 3:

What are the precise observation operator and error model used for the 10 m sea-surface winds (e.g., stability-dependent surface-layer mapping, bias correction, representativeness error)? How are coastal/land points handled, and are rainy scenes screened?

Response:

(a) Observation operator: The 10 m wind speed (u_{10}, v_{10}) is diagnosed from the wind speed (u_x, v_x) at the model's first layer height z_x, and then horizontally interpolated to observation locations. The diagnosis considers atmospheric stability through the dimensionless wind shear function Φ_x (the stability function in Monin–Obukhov similarity theory). The formulas are:

$$\begin{cases} u_{10} = u_x \dfrac{\Phi_{10}}{\Phi_x} \\ v_{10} = v_x \dfrac{\Phi_{10}}{\Phi_x} \end{cases} \tag{R5}$$

where Φ_{10} and Φ_x are the dimensionless wind gradient functions at 10 m height and the model's first layer height z_x, respectively.

Under different stability conditions, Φ_x is expressed as:

$$\Phi_x = \begin{cases} -10 \ln \frac{z_x}{z_0}, (Ri_b > Ri_c = 0.2) \\ -5\left(\frac{Ri_b}{1.1 - 5Ri_b}\right) \ln \frac{z_x}{z_0}, (Ri_c \geq Ri_b > 0) \\ 0, (Ri_b = 0) \\ 2 \ln\left(\frac{1+x}{x}\right) + 2 \ln\left(\frac{1+x^2}{x}\right) - 2\tan^{-1} x + \frac{\pi}{2}, (Ri_b < 0) \end{cases} \tag{R6}$$

$$x = \left(1 - \frac{16z}{L}\right)^{\frac{1}{4}} \tag{R7}$$

Here, Ri_b is the bulk Richardson number, Ri_c is a threshold for strongly stable conditions, and z_0 is the surface roughness. (line 440)

(b) **Bias correction:** The satellite-derived sea surface wind data have been bias-corrected prior to assimilation. (line 116)

(c) **Representativeness error:** In the SVD-3DEnVar scheme, only the leading N singular vectors are retained when fitting observation increments, which effectively truncates short-wave information in observations. Therefore, the observation representativeness error has minimal impact on the assimilation results. (line 452)

(d) **Coastal/land treatment:** The model uses static surface data to distinguish between ocean and land grid points. Over land, roughness length z_0 is specified based on land cover type. Over ocean, z_0 is diagnosed from sea surface wind speed using the Charnock (1955) relation:

$$z_0 = z_{ch}\frac{{u_*}^2}{g} + 0.00001 \qquad (R8)$$

where z_{ch} is the Charnock coefficient, u_* is the friction velocity, and g is gravitational acceleration. Differences in roughness calculation between land and sea surfaces further influence the diagnosis of 10 m winds through Eq. (R6). (line 445)

Reference:

Charnock, H. (1955), Wind stress on a water surface. Q.J.R. Meteorol. Soc., 81: 639-640. https://doi.org/10.1002/qj.49708135027

(e) **Rain screening:** The satellite-retrieved sea surface wind product used in this study is a blended product that incorporates longer-wavelength microwave radiometer data with better cloud-penetration capability, thereby mitigating the impact of rainfall on wind retrievals. (line 114)

Comment 4:

Please clarify the SVD/variational notation: define Λ_p explicitly, correct the SVD equation symbols, and detail the truncation criterion for K (energy, cross-validated error, or fixed)?

Response:

a) Λ_p has been corrected to Λ_K, where K is the truncation order. The definition of Λ_K has been added in the manuscript: "Λ_K is the diagonal matrix consisting of the first K largest singular values of the ensemble perturbation matrix **A**." (line 174)

(b) **Selection of truncation order K:**

K must be less than the rank of $\mathbf{A}$ (i.e., $K < r = \text{rank}(\mathbf{A})$) and cannot exceed the ensemble size M (since only the first r singular values are non-zero, and M limits the maximum effective dimension of ensemble perturbations). Additionally, truncation should retain the "dominant variance" of the ensemble perturbations, typically requiring that the sum of squares of the first K singular values accounts for ⩾95% of the total variance from all non-zero singular values. In this study, with an ensemble size of 30, the truncation order is fixed at $K = 27$. (line 167, 379)

Comment 5:

I storngly recommend to expand your introduction and cite below paper.

Assimilation of sentinel‐based leaf area index for modeling surface‐ground water interactions in irrigation districts

Response:

The suggested reference has been added at the end of the first paragraph in the Introduction:

"Beyond meteorological forecasting, data assimilation has also been effectively applied in hydrological and environmental modeling to integrate multi-source observations, such as combining satellite-derived vegetation indices with in-situ measurements to improve the analysis of land surface and subsurface processes (e.g., Zafarmomen et al., 2024)." (line 43)

Comment 6:

Abstract: Consider explicitly stating that the main novelty is the computational optimization and parallelization that makes SVD-3DEnVar suitable for operational use, plus a first real-data test with satellite-derived sea surface winds.

The abstract has been revised to emphasize the novelty:

"To bridge this gap towards operational readiness, this study introduces key computational optimizations: a new three-dimensional perturbation field generation scheme that supports multi-process parallelism and can directly generate any specified grid, and an efficient parallel implementation scheme tailored for the local patch assimilation in the SVD-3DEnVar scheme." (line 17)

Comment 7:
Notation consistency:

Ensure all symbols in the equations are defined once and consistently (e.g., Λ vs Λ_P in Equation (10); clarify if Λ_P is the same eigenvalue matrix as in Eq. (4) or modified).

Make sure the dimension of vectors and matrices in Eqs. (1)–(10) is always clear (model vs observation subspaces).

Response:

The notation regarding Λ_p has been corrected as explained in the response to Comment 4. All equation symbols have been reviewed for consistency, and vector/matrix dimensions (model vs. observation subspaces) are explicitly stated in the revised manuscript. (line 159, 174)

Comment 8:

Equation (11) Gaussian localization:

Check the condition in the piecewise definition:

You currently use (r_h ≤ l_h and r_v ≤ l_v) vs (r_h > l_h or r_v ≥ l_v). It might be clearer and more symmetric to use strict/≤ consistently.

Consider giving an example of actual physical localization scales (in km) corresponding to l_h, l_v, σ_h, σ_v.

Response:

Equation (11) in manuscript has been revised to:

$$\mathrm{w}(\sigma_h, \sigma_v) = \begin{cases} exp\left(\left(-\frac{{r_h}^2}{{\sigma_h}^2}\right) + \left(-\frac{{r_v}^2}{{\sigma_v}^2}\right)\right), & (r_h \le l_h \; and \; r_v \le l_v) \\ 0, \quad (r_h > l_h \; or \; r_v > l_v) \end{cases}$$

In this study, l_h is set to 10 grid points (approximately 90 km), and l_v equals the number of model layers. Both σ_h and σ_v are set to 3 grid points, corresponding to about 27 km and 1.5 km, respectively. These parameters are detailed in Section 4.1. Note that grid counts are used as units for convenience in implementation. These are preliminary settings; future work will refine them and develop more effective optimization methods for data assimilation. (line 192)

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Reviewer #3

This manuscript presents a technical advancement in the SVD-3DEnVar data assimilation scheme, focusing primarily on improving computational efficiency through a newly proposed three-dimensional ensemble perturbation generation method and a parallelization strategy. The authors evaluate this optimized framework using the TRAMS 3.0 model in both OSSE and real-data experiments.

The practical contribution of this study is evident, particularly the reported reduction in wall-clock time (from approximately 1,700 minutes to less than 15 minutes), which is highly relevant for operational numerical weather prediction. However, the manuscript requires substantial improvement in its scientific presentation. Several critical issues were identified regarding the mathematical rigor of the methodology, the theoretical consistency of the OSSE results, and the clarity of the experimental design. Specifically, the explanation of the load-balancing mechanism within the parallel strategy is vague, and multiple inconsistencies between the text and figures undermine the credibility of the findings.

Therefore, I recommend Major Revisions before this manuscript can be considered for publication.

Comment 1：Clarification on Parallelization and Load Balancing (Line 255)

The authors state: "The approach adopted in this study allows for synchronous invocation of all processes... effectively avoiding the problem of idle waiting... thus significantly improving resource utilization."

This explanation is scientifically insufficient. "Synchronous invocation" implies starting tasks simultaneously, but it does not inherently solve the computational load balancing problem. In a local patch-based domain decomposition, observations are rarely uniformly distributed. Consequently, using a static domain decomposition inevitably results in some processes handling significantly more observations than others. How does "synchronous invocation" prevent processes with fewer observations from finishing early and idling?

The authors seem to imply that a task redistribution mechanism is in place, but it is not clearly described. A detailed explanation of how the workload is balanced across processes is required (e.g., are grid points dynamically redistributed based on observation density or computational cost?).

Response:

Thank you for highlighting this crucial point. Our parallel strategy involves a two-step process to achieve load balancing.

Initial Parallel Screening: The model grid is statically partitioned. Within each partition, all processes work in parallel to screen each grid point. The criterion for marking a point as "to be assimilated" is whether the number of valid observations within its local patch meets or exceeds the preset truncation order (K). (line 288)

Dynamic Task Redistribution: After this global parallel screening, all marked grid points are collected. These points (now a scattered set, not a regular grid) are then evenly redistributed among all available processes. This ensures that each process receives a nearly equal number of assimilation tasks, effectively balancing the computational load. (line 292)

Through the above two steps, the assimilation computational tasks of SVD-3DEnVar can be essentially evenly distributed across various nodes.

We have added this detailed explanation to Section 3.3 of the manuscript.

Comment 2: Theoretical Validity of OSSE Results (Figure 7)

In Figure 7, the analysis increment for the u-wind component (7b) appears almost identical to the "true error" (7a) in both spatial pattern and magnitude. According to data assimilation theory, the analysis increment is the product of the Kalman gain and the innovation (observation minus background). Unless the observation error (R) was set to zero or the background error covariance (B) was inflated to an unrealistic magnitude, the analysis increment should not match the true error so perfectly. This result raises serious questions about the experimental setup or the plotting (e.g., potential confusion between innovation and increment). The authors must verify this result and provide a physical or theoretical justification.

Response:

We appreciate the reviewer's careful scrutiny. The close match is expected and validates the experimental design and algorithm behavior under the specific OSSE conditions:

Dense, Perfect Observations: In the OSSE, direct observations of the u-wind component are provided at every model grid point within layers 8-32. This creates an exceptionally dense and spatially complete observing network for this variable. (line 373)

Methodology: For computational convenience, in the implementation described in this paper, we did not directly minimize the objective function (10) through iterative calculations. Instead, following the approach of Qiu and Chou (2005), we employed the method of least squares to solve for α:

$$\Delta y = \sum_{r=1}^{K} \alpha_r b_r^d \qquad (11)$$

By solving the algebraic Equation (11), the coefficients α are obtained, and the required analysis increment Δu is computed using Equation (7):

$$\Delta u = \sum_{r=1}^{K} \alpha_r b_r^u$$

In this SVD-3DEnVar implementation, the coefficients α are determined solely from observational information Δy. When sufficient observations are available (i.e., the number of observations exceeds or equals the truncation order K), the system is well-posed, a stable minimum-norm solution can be obtained because the solution is constrained to the low-dimensional subspace spanned by the singular vectors. This eliminates the dependence on background error statistics required in traditional methods and simplifies the assimilation process. (line 183)

On the other hand, since equation (11) only retains the first K singular vectors to fit the observation increments, it effectively filters out small-scale observational increment information (corresponding to singular vectors with relatively small singular values). Therefore, even when random noise is added to the observations in the OSSE experiment, it does not significantly impact the assimilation results. This is one of the differences between the SVD method and traditional assimilation methods, namely its lower sensitivity to random observational errors. (line 181, 407)

Reference: Qiu, C. and Chou, J.: Four-dimensional data assimilation method based on SVD: Theoretical aspect, Theor. Appl. Climatol., 83, 51 – 57, https://doi.org/10.1007/s00704-005-0162-z, 2005.

Related content has been added to Sections 3.1 and 4.2: specifically, the formulas in Section 3.1 have been supplemented and refined, while a further elaboration on the reasons why the analysis increments are close to the true errors has been provided in Section 4.2.

Comment 3: Ambiguity in Observation Processing (Line 108)

The manuscript mentions: "valid data are selected according to program input requirements after quality control."

This description is too vague for a research article. How exactly are "valid data" selected? Does this process involve spatial thinning, super-obbing, or specific domain checks? A concrete description of the selection criteria is necessary to ensure reproducibility.

Response:

We agree that the description was too brief. For the specific experiments presented in this paper:

OSSE Experiment: Simulated u-wind observations were used directly at their native model grid points without additional thinning or super-obbing. (line 373)

Real-Data Experiment (Sea Surface Winds): The multi-source satellite wind product (described in Section 2) underwent its quality control and bias correction during the retrieval process. For ingestion into our SVD-3DEnVar system, we applied a simple background check: observations were rejected if the absolute difference between the observation and the background (mapped to observation space) exceeded a threshold (set to 20 m/s for wind speed). Due to the lower resolution of observations compared to the model, no additional spatial thinning or super-obbing was applied for these preliminary tests. (line 449)

Comment 4: Mathematical Rigor and Definitions (Section 3.1)
The mathematical description of the SVD-3DEnVar scheme lacks precision:

- Line 135: The matrix AA is rectangular; therefore, it has singular values, not "eigenvalues."

- Undefined Symbols: In Eq. (4) (A=BΛV^T), the term V^T is not defined. Similarly, in Eq. (6) (x=bα), the variable αα appears without a proper definition. The authors should explicitly define these variables to aid reader understanding.

Response:

Thank you for catching these issues. We have corrected them in the manuscript.

"Eigenvalues" has been replaced with "singular values" throughout. (line 158)

In the text following Equation (4), we have added: "...where Λ is a diagonal matrix of singular values..., and B and V are orthogonal matrices containing the left and right singular vectors of A, respectively." (line 158)

Following Equation (6), we have added: "...where $\alpha = (\alpha_1, \alpha_2, ..., \alpha\_K)$^T is the vector of coefficients to be determined by minimizing the cost function." (line 168)

Comment 5: Overgeneralization of Results (Figure 8)

Based on a single OSSE case shown in Figure 8, the authors claim that "the SVD-3DEnVar scheme can significantly improve typhoon track and intensity forecasts." This statement is too strong for a single idealized experiment. It would be more appropriate to state that the scheme demonstrates potential for improvement in this specific case.

Response:

We agree with the reviewer. The statement has been toned down as suggested. The relevant sentence in Section 4.2 now reads:

"The DA experiment effectively reduces the forecast bias in both the typhoon track and intensity for this specific case. These results indicate that after assimilating the wind field, the SVD-3DEnVar scheme demonstrates the potential to improve typhoon track and intensity forecasts in this idealized framework." (line 427)

Comment 6: Interpretation of "Operational Feasibility"

The manuscript frequently emphasizes the operational applicability of the proposed scheme based on reduced wall-clock time. However, operational feasibility depends not only on speed but also on resource availability and system stability. Given that some results rely on a large number of computing nodes (up to 300 nodes), it would be beneficial for the authors to discuss whether similar performance gains can be achieved under more constrained computational resources, which is a common reality for many operational centers.

Response:

This is a valuable point. Our scaling test up to 300 nodes was to explore the saturation point of parallelism. As seen in Fig. 5a, the performance gain (reduction in wall-clock time) slows significantly beyond ~150 nodes for our test configuration. More importantly, substantial gains are achieved with far fewer resources.

Using approximately 30 nodes (with 64 cores each), the assimilation time is reduced to about 1 hour, which is already a dramatic improvement from the original serial execution.

The current experiments assimilate very dense data (effectively all grid points in the OSSE). In an operational setting, realistic observations would be much sparser after standard quality control and thinning procedures. This would further reduce the computational cost, potentially allowing runtime targets (e.g., under 30 minutes) to be met with fewer than 60 nodes.

Since operational ensemble prediction systems already require tens to hundreds of nodes to run the ensemble forecasts, dedicating a comparable subset for an efficient assimilation step is feasible and represents a significant step toward operational readiness. We will add a brief discussion on this in Section 5 (Conclusion). (line 509)

Comment 7: Technical Corrections and Visualization Issues

1. Figure 4: The red boxes representing parallel domains are difficult to distinguish because the model grid points are also represented by lines. I suggest representing the model grid points as dots **and using** lines only for the red boxes **to improve visual clarity.**

Response:

Figure 4: We have changed the representation of the model horizontal grid points from a grid of lines to black crosses as stated in the response, which significantly improves the distinction

from the red partition boxes. (line 312)

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2. Figure 10 Issues:

- Visibility: The black line indicating the cross-section in Fig. 10a is obscured by the wind vectors. Please use a contrasting color (e.g., gray or magenta) or increase the line thickness.

Response:

We have changed the wind vector arrows from black to a high-contrast purple to make the overlaid black cross-section line more visible without altering the line itself, maintaining consistency across subplots. (line 478)

- Physical Interpretation: The cross-section passes through the typhoon center. It is physically puzzling why the u-wind analysis increments (Fig. 10b) appear overwhelmingly positive across the center.

Response:

We apologize for the confusion. Figure 10b plots the increment in total wind speed, not the u-component. The background field underestimated the wind speed in the typhoon core (Fig. 9c), so a positive increment across the center is physically consistent for wind speed magnitude. (line 460, 478)

- Mismatches: The caption for Fig. 10d states it shows v-wind, but the unit label in the image is ππ (dimensionless pressure). Additionally, the caption describes panels up to (j), but the layout and labels need to be checked for consistency.

Response:

We have corrected the caption and verified all labels. Panel (d) correctly shows the π (Exner pressure) increment. The reference to non-existent panels (j) has been removed. (line 480)

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3. Section Titles: The titles for Section 4.2 and Section 4.3 are identical ("Results of OSSE Experiment"). Section 4.3 discusses real-data assimilation and should be titled accordingly (e.g., "Results of Real-Data Assimilation Experiment").

Response:

Section Titles: The title of Section 4.3 has been changed to "4.3 Assimilation of Sea Surface Wind Observations". (line 438)

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4. Figure 11 Caption: The caption is too brief. It needs to be expanded to clearly describe what panels (a) and (b) represent (e.g., "Comparison of typhoon tracks (a) and maximum wind speed (b) between CTL and DA experiments...").

Response:

Figure 11 Caption: The caption has been expanded as suggested: " Figure 11Comparison of (a) typhoon tracks and (b) 10-m maximum wind speed (units: m s^{-1}) among the control experiment (CTL), the data assimilation experiments (DA1, DA2, DA3), and observations (OBS) for Typhoon Yagi (2024). Forecasts start at 00:00 UTC on 6 September 2024." (line 498)

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5. Typos and Grammar:

- Line 16: "This paper constructed..." → "**constructs**" (or presents/proposes).

- Line 83: "microphysical schem" → "**scheme**".

- Line 261: "Figure 5a showed..." → "**shows**".

- Line 286: "The results shows that..." → "The results **show** that...".

- Line 445: "assimilate atmospheric variable" → "atmospheric **variables**".

Response:

All noted corrections have been made in the manuscript ("constructs", "scheme", "shows", "show", "variables"). We have also performed a thorough check for tense and grammar throughout the paper.