Response to Reviewer 1

We sincerely thank the reviewer for the insightful comments and constructive suggestions, which have significantly helped us improve the quality of our manuscript. We have carefully considered all points and have revised the manuscript accordingly. Below, we provide a point-by-point response to the comments.

Comment 1:

Regarding the choice of methodology, it is recommended that you explain why SVD-4DVar was not adopted in favor of SVD-3DVar, while briefly analyzing the core challenges of the latter.

Response:

We appreciate the reviewer's suggestion. The decision to adopt SVD-3DEnVar instead of SVD-4DEnVar was primarily motivated by two factors:

Computational Efficiency and Operational Feasibility: The main objective of this study is to improve computational efficiency for operational typhoon forecasting, so the relatively simple 3DVar scheme is chosen for experimentation...

Observation Frequency: The sea surface wind observations used in our real-data experiments are available only every 6 hours. This low temporal resolution does not fully leverage the temporal continuity advantages of 4DEnVar.

We acknowledge that SVD-3DEnVar has its own challenges, which we now discuss in the revised manuscript (Section 5). The primary limitations include: The use of a single localization scale, which may not optimally handle multi-scale observations (e.g., surface, satellite, and radar); The limited ability of linear combinations of singular vectors to represent strongly nonlinear relationships, especially for unobserved variables. These challenges will guide our future work on multi-scale assimilation and machine-learning-enhanced methods.

Comment 2:

It should be noted that the singular value decomposition (SVD) of matrix A is extremely challenging in practice due to its large dimensions ($N_x + N_y$), posing significant difficulties in terms of both storage and computation. A discussion on this aspect is recommended.

Response:

We fully agree with the reviewer. To address the computational and storage challenges of performing SVD on the large matrix A, we implemented a local patch assimilation strategy. This approach significantly reduces the effective dimensions of A by horizontal and vertical localization. Only observations within a specified horizontal (l_h) and vertical (l_v) radius from the central grid point are included. As a result, both N_x (model variables in the local patch) and N_y (observations within the local patch) are drastically reduced. This makes the SVD computationally feasible without sacrificing the flow-dependent covariance information. We have added a clarification in Section 3.1 to explain this strategy.

Comment 3:

While equation (11) provides the Gaussian weight function, the localization scheme used in SVD-3DVar should be presented in more detail to enhance the completeness of the paper.

Response:

Thank you for this suggestion. We have expanded the description of the localization scheme in Section 3.1. The revised text now reads:

"The Gaussian weight function defined in Equation (11) is applied to each observation within the local patch. The horizontal and vertical localization scales (σ_h and σ_v) control the rate at which observation influence decays with distance. This localization ensures that only observations within a specified radius significantly impact the analysis increment at the center of the local patch, thereby mitigating spurious long-range correlations and improving the stability and accuracy of the assimilation."

Comment 4:

In recent years, 4DEnVar methods have advanced rapidly. To reflect an up-to-date understanding of the field, it is advisable to include references to relevant studies published between 2022 and 2025.

Response:

We thank the reviewer for this suggestion. We have now incorporated several recent references on 4DEnVar advancements in the Introduction and Conclusion sections, including:

Inverarity et al. (2023) on hybrid En-4DEnVar in the Met Office system;

Berre and Arbogast (2024) on hybrid covariances at Météo-France;

Lu and Wang (2024) on scale-dependent localization in hurricane forecasting;

Thiruvengadam and Wang (2025) on convective-scale 4DEnVar;

Wang et al. (2025) on CubeSat radiance assimilation.

These additions help contextualize our work within the evolving landscape of ensemble-variational methods.

Comment 5:

Regarding the generation of initial samples, several classical works (e.g. those by Evensen) have achieved high memory efficiency. It would be beneficial to reference these works and discuss their relevance to the present method. From a practical perspective, the main computational burden in parallelization typically lies in the ensemble forecast component, which should also be addressed.

Response:

We agree with the reviewer. The initial perturbation generation in SVD-3DEnVar is based on the Gaussian random field method introduced by Evensen (1994), which is known for its memory efficiency and statistical robustness. However, the original implementation was limited to 2D square grids with odd numbers of points, which motivated our multi-dimensional and parallel optimizations. We have mentioned Evensen's foundational work and clarify how our optimizations build upon it.

Regarding the computational burden of ensemble forecasts: yes, the ensemble integration is indeed the most computationally intensive part in parallel implementations. However, since ensemble members are independent, they can be run concurrently if sufficient computational resources are available. While this study focuses on optimizing the perturbation generation and assimilation steps, we acknowledge the resource demands of ensemble forecasting and will address this in future work.

Comment 6:

As this is an ensemble-based method, it is recommended that the ensemble sample update strategy in SVD-3DVar is explained briefly to improve the completeness of the methodological description.

Response:

We have added the following explanation to Section 1:

"Unlike traditional ensemble assimilation schemes (such as EnKF), which directly update each ensemble member during cyclic assimilation, SVD-3DEnVar only assimilates and updates the control forecast. Therefore, after each assimilation cycle, the updated analysis field must be perturbed again to generate the initial conditions for the next cycle's ensemble forecast. This approach has the advantage of avoiding filter divergence and, in the presence of model errors, performs better than EnKF (Qiu et al., 2007)."