

Dear Editor,

These are our responses to the Referees and your reports.

Referee 1

R: What is, however, missing, is the physical meaning, the insight gained by formally applying such an analysis. The paper is lacking that completely. The authors should absolutely add what the physical meaning of their method is even applied to the event they are using it. Without such an effort, result, information I cannot accept the paper.

The manuscript presents a method but does not apply it nor say what will be expected when applied to observations (which is intended to be applied!).

A: We newly explain our goal and outcome on page 5, ll. 114-124 and ll. 132-137. The main goal was a search for an indication of increased geomagnetic activity in terms of harmonic content changes of the Ap index. In February 13-14, 2001, the substorm demonstrated decrease of higher frequency magnitudes and increase of lower frequency ones.

R: My personal procedure reading a paper is after the title and a glance at the Abstract (which in most cases does not tell much) to look for the final Section on the Conclusions/Outlook/Application to become informed where and why the paper has been written. This is missing here.

A: We modified and expanded Sec. 4.

Referee 2

R: The abstract claims that the study analyzes the Dst index, but the manuscript contains no such analysis. It also states that the spectral content prior to the substorm growth phase is examined, yet no analysis relating to the substorm growth phase appears in the paper.

A: The abstract was modified. A substorm is analyzed, see p. 5, ll. 120-124 and Fig. 1.

R: A substantial portion of Section 2 simply reproduces standard FFT formulations. Since these are well-established and widely known, it is unnecessary to present them in such detail. Moreover, the expressions for a_{nl} and b_{nl} are mathematically equivalent to the formulation in terms of C_{nl} and ϕ_{nl} . Because only C_{nl} and ϕ_{nl} are used in the subsequent analysis, the extensive derivations of unused quantities are redundant.

A: It is not FFT but an analysis using a Fourier series for a piecewise linearly interpolated A_p (which becomes a continuous function). The coefficients a_{nl} and b_{nl} are derived first for such a case, and then via them, c_{nl} and ϕ_{nl} are obtained.

R: Line 101. The statement about "linear interpolation" is unclear. It is not clear what type interpolation has been performed, how the interpolation modifies the sampling rate, or how the Nyquist frequency is affected. This requires explicit clarification.

A: In addition to above explanation, we added in the manuscript an explicit formula how the linear interpolation is made, p. 4, Eq. (14). Because the reconstructed A_p agree well with original data, we conclude that our approach works.

R: Line 114-115. In other words, all coefficients and phases are calculated in (3)-(5) for every three-hour interval? It is unclear whether a moving time-window is used, what the total window length is (e.g., 27 days?), and how the three-hour stepping is implemented. This should be explained precisely.

A: Moving time window length T is 27 days and it is centered at the point of interest. The step is 3 hours. Details are explained in p. 4, ll. 83-87.

R: Figure 1, The inverse FFT reconstruction of A_p is a routine check applicable to any FFT analysis. It does not provide new insight into either the method or the dataset. This figure should be removed or moved to supplementary material.

A: It is not an FFT method but a new approach using a Fourier series, so Fig. 1 is important.

R: Standard practice in space-physics FFT analysis is to express coefficients as functions of frequency (or period), not as a function of index n . The physical meaning of n in Figures 2-6 is not immediately obvious, and readers cannot interpret the results without knowing the corresponding frequencies or periods.

A: It is not an FFT method but an analysis using the Fourier sum. We added the relationship between index n and frequency on p. 4, ll. 93-94.

R: It is very hard to interpret Fig.2, Fig.3. If the goal is to relate coefficient variations to substorms or magnetic storms, the relevant geomagnetic indices (e.g., AL/AE for substorms or Dst for storms) should be plotted alongside the spectral coefficients on the same timescale. Without this, interpretation is very limited.

A: We supplemented Figs. 1 and 2 by the Dst index.

R: Fig.5 and Fig.6. The manuscript does not justify why phase is normalized by n , nor does it explain the near-linear trend seen in the figures. This raises concerns that the behavior might be an artifact of the analysis method rather than a physical signal.

A: We added explanation in p. 4, ll. 97-102. The linear trend is not always present for all harmonics, but only during weak geomagnetic activity.

R: A major issue is the absence of contextual background on the Ap/Kp indices. Ap is the linearized form of Kp and provides a widely used measure of magnetospheric convection strength (e.g., Thomsen 2004; Rowland & Wygant 1998; Wang et al. 2025). Large-scale magnetospheric convection is driven by dayside and nightside magnetic reconnection (e.g., Siscoe & Huang 1985; Cowley & Lockwood 1992; Dai et al. 2024; Zhu et al. 2024, 2025) and penetrates deep into the inner magnetosphere (e.g., Wygant et al. 1998; Dai et al. 2015; Thaller et al. 2015).

A: We modified Introduction accordingly and included the suggested references.

Editor

E: The abstract gives the impression that the method is applied to Dst, whereas this is not actually done in the manuscript.

A: The abstract was modified.

E: The introduction should offer a coherent overview of the field, rather than a disconnected listing of prior work that does not contribute meaningfully to the formulation of the paper's aim.

A: Introduction was largely modified.

E: The introduction should at the end announce the aim of the paper and how it is organized.

A: We did it.

E: The referees reported a strong technical tone in the paper with insufficient physical interpretation.

E: The referees reported insufficiently fulfilled claims in the paper e.g. the analysis of the growth phase of a sub-storm.

E: The paper could gain major relevance if an analysis of an actual sub-storm event is presented.

A: We explicitly stated our aim and analyzed a substorm.

E: Section 2 is more related to the FFT method rather than describing a model. Its title should be reconsidered accordingly.

A: We made change to Method. But it is not FFT.

E: Some paragraphs contain very long sentences that are very difficult to digest by the reader e.g. page 6 line 149 to line 154.

A: Many parts were rewritten to make them clearer.

E: Page 7, line 174, the second sentence misses a full stop.

A: Fixed.

Thank you for important comments and suggestions.

The Authors.