

Reply to Andy Hogg

I thank the referee for his thoughtful comments and for his substantial contributions to APE theory over many years, particularly in the global APE framework context. I agree that several issues merit clarification and revision. I address each point in turn below and will implement the indicated changes in the revised manuscript.

Long-range interactions *This paper examines the definition of Available Potential Energy as per the Lorenz Reference State (LRS). One significant issue with the LRS is that it is defined relative to the global density field. This means that, in principle, altering the density in the Arctic Ocean may alter the energetics in the Southern Ocean, even though it is unclear how that information could be communicated across the planet.*

Response I appreciate the concern. The key point in the manuscript is that the LRS is dynamically passive: it is introduced as a reference against which dynamical anomalies are defined, not as an active agent that transmits influence. As such, whether one regards the LRS as global or local does not introduce new interactions or time/space scales into the dynamics; it is a bookkeeping device that helps uncover dynamical information in the momentum and energy balances that would otherwise remain hidden. Buoyancy oscillations is an example of dynamics that can only be uncovered by the introduction of a dynamically passive reference state.

To make this explicit, I will clarify in Section 3 that the momentum equations are most transparently written in terms of anomalous forces. Writing the pressure as $p = p_0(z) + \delta p$, with $p_0(z)$ dynamically passive, the non-hydrostatic primitive equations can be expressed as

$$\frac{D\mathbf{v}}{Dt} + f \mathbf{k} \times \mathbf{u} + \frac{1}{\rho_\star} \nabla \delta p = b \mathbf{k}, \quad (1)$$

where $b = -\frac{g}{\rho_\star} (\rho(S, \theta, p_0(z)) - \rho_0(z))$ and $\rho_0(z) = -p'_0(z)/g$. The introduction of $p_0(z)$ does not alter the fundamentally local character of the forces entering the momentum balance, nor does it imply any long-range coupling beyond that already present in the governing equations. I will revise Section 3 to make this point more prominent.

Nature of the Lorenz reference state *Since the LRS is often regarded as the zero-APE state, some have interpreted it as “real”, yet there is no practical way to attain it in a complex fluid. In most cases it is hypothetical; what is gained by measuring potential energy relative to an unattainable state?*

Response This is an important conceptual issue. In the manuscript I propose addressing it in terms of observability. If the LRS is tied to observables (directly or indirectly), then it is not merely a formal or hypothetical construct. This way, its ‘reality’ can be more objectively assessed and discussed. Two distinct notions are helpful:

- Direct observability: near equilibrium, the LRS relates to locally measurable buoyancy frequencies. Predictions for N_0^2 can be tested against direct measurements of buoyancy oscillation frequencies. Establishing this in a controlled limit is a necessary first step; if the proposition failed near rest, it would be unlikely to hold more generally.

- Indirect observability: different definitions of the “non-available” component (i.e., Σ_{heat}) lead to distinct, testable dynamical consequences. For example, the surface-forced production/destruction of the dynamical part Σ_{dyn} depends on the choice of Σ_{heat} . The APE-consistent choice (based on the LRS) yields surface production forms that align with empirical evidence for when surface fluxes destabilize the water column, whereas other choices (e.g., based on potential enthalpy) do not. I will make these distinctions explicit in Sections 3–4 and provide additional examples.

Local character of the LRS *The paper aims to show that the LRS can be regarded as a local quantity and linked to the governing equations. I would be delighted if this were established, but I remain unconvinced and believe major improvements are needed for publication.*

Response Thank you for the candid assessment. The revision will strengthen the argument along two lines: (i) clarifying the controlled, near-rest result as a baseline—a necessary foundation for broader claims—and (ii) setting out a more general anomalous-force formulation that demonstrates how observable quantities (notably N_0^2) constrain the reference fields. Together these revisions should render the intent and limitations of the argument clearer.

Limitations of Section 3 *The gravity analogy is imperfect; g varies, but can be measured locally. Buoyancy frequency is also measurable locally, which gives a local approximation to APE that suffices for linear internal waves. However, as shown by Hughes, Hogg and Griffiths (2009), the local linear approximation is insufficient for the large-scale overturning and associated mixing. It has not been shown that Section 3’s arguments extend beyond the linear range; more work is needed.*

Response I agree that the local linear approximation has limitations for large-scale energetics and mixing, as highlighted by [1]. Section 3 is not intended to claim sufficiency of the linear approximation for all purposes. Rather, its purpose is to establish a baseline: in the near-rest limit, the LRS couples to local observables such as N_0^2 . This provides a clear “existence proof” for observability in a controlled setting.

To bridge toward more general regimes, I will add the following formulation. Define the parcel’s neutral level $z_r = z_r(S, \theta)$ by $b(S, \theta, z_r) = 0$ and the displacement $\zeta = z - z_r$. For adiabatic, isohaline motion $w = Dz/Dt = D\zeta/Dt$, and the vertical balance becomes

$$\frac{D^2\zeta}{Dt^2} + \frac{1}{\rho_\star} \frac{\partial \delta p}{\partial z} + \int_0^\zeta N_0^2(S, \theta, z_r + \zeta') d\zeta' = 0, \quad (2)$$

with

$$b(S, \theta, z) = \int_0^\zeta b_z(S, \theta, z_r + \zeta') d\zeta', \quad (3)$$

$$b_z(S, \theta, z) = -N_0^2(S, \theta, z) = \frac{g}{\rho_\star} \left(\frac{d\rho_0}{dz}(z) + \frac{g \rho_0(z)}{c_s^2(S, \theta, p_0(z))} \right). \quad (4)$$

Equation (2) shows that, away from the linear limit, vertical motions comprise forced and free nonlinear buoyancy oscillations across turbulence, internal waves, and balanced motions—each depending (sometimes intricately) on N_0^2 . Inferring $\rho_0(z)$ and $p_0(z)$ in these regimes is a nontrivial inverse problem, but not an impossibility. I will clarify these points and add an explicit citation to [1] to acknowledge this limitation and context.

Static energy asymptotics as a basis for APE theory (Section 4) *I accept the static-energy framework and that APE can be written from the dynamical component. However, Equation (8) relies on the Lorenz definition of the reference state, which remains global. Thus the separation presumes global knowledge and does not demonstrate an “external constraint.” Moreover, there is a genuinely non-local APE effect: e.g., if the northern-hemisphere water were magically made 10 kg m^{-3} denser, it would sink below the southern-hemisphere water, altering energetics remotely.*

Response I agree that, formally, different choices of Σ_{heat} are conceivable. The question is whether they are physically acceptable. The decomposition is meaningful only if the “non-available” part is chosen so that the remaining “available” part exhibits the observed dynamical signatures. This imposes strong constraints.

A particularly diagnostic test concerns the surface-forced production or destruction of Σ_{dyn} by heat and freshwater fluxes. Denoting the net surface heat flux by Q_{net} , the surface freshwater density by $\rho_f = \rho(0, T, p)$, and net evaporation minus precipitation by $E - P$ (m s^{-1}), we obtain

$$F_{\text{dyn}} = \left(\frac{T - T_r}{T} \right) Q_{\text{net}} + \left[\mu - \mu_r - (T - T_r) \frac{\partial \mu}{\partial T} \right] \rho_f S (E - P), \quad (5)$$

with

$$T_r = \frac{\partial \Sigma_{\text{heat}}}{\partial \eta}, \quad \mu_r = \frac{\partial \Sigma_{\text{heat}}}{\partial S}. \quad (6)$$

For the APE-consistent, LRS-based Σ_{heat} , F_{dyn} reduces to the exact APE production form [e.g., 1, 2, 3] and is positive when surface fluxes destabilize the water column, consistent with empirical understanding. By contrast, if Σ_{heat} were defined via potential enthalpy [4], for which $T_r = \theta$ and $\mu_r = \mu$, one obtains $F_{\text{dyn}} = 0$, implying that surface fluxes do not contribute to APE production—contrary to observations and established energetics. I will expand Section 4 with this comparison and additional examples, thereby reducing the perception of arbitrariness and clarifying the “external constraint” language (which I will rephrase to avoid any implication of causal forcing by the LRS itself).

Regarding the thought experiment: an abrupt, global-density modification would take the system far from equilibrium and trigger a global adjustment via acoustic and internal gravity waves and balanced motions. The subsequent reorganisation of APE and BPE reflects the system’s dynamical response to the imposed perturbation, not an intrinsic non-local action of APE. I will clarify this distinction in the text.

Summary *I remain unconvinced that the LRS can be considered a local quantity. The nonlocal generation of APE is less problematic if one avoids treating the LRS as real or attainable. A hierarchy of approximations to the exact LRS (from local-linear to semi-local) is likely the most useful path.*

Response I appreciate these suggestions. The revised manuscript will:

- Emphasize observability as the organizing principle (direct in near-rest limits; indirect via dynamical constraints such as surface-forced production).
- Clarify that the LRS’s role is as a passive reference constraining the decomposition, not as a driver of dynamics.
- Expand Section 4 to demonstrate how physically consistent constraints single out the LRS-based Σ_{heat} .

With these revisions, I believe the manuscript will address the concerns raised and meet the standards for publication.

References

- [1] G. O. Hughes, A. M. Hogg, and R. W. Griffiths. Available potential energy and irreversible mixing in the meridional overturning circulation. *J. Phys. Oceanogr.*, 39:3130–3146, 2009.
- [2] J. A. Saenz, R. Tailleux, E. D. Butler, G. O. Hughes, and K. I. C. Oliver. Estimating lorenz’s reference state in an ocean with a nonlinear equation of state for seawater. *J. Phys. Oceanogr.*, 45:1242–1257, 2015.
- [3] V. E. Zemskova, B. L. White, and A. Scotti. Available potential energy and the general circulation: partitioning wind, buoyancy forcing, and diapycnal mixing. *J. Phys. Oceanogr.*, 45:1510–1531, 2015.
- [4] T. J. McDougall. Potential enthalpy: a conservative oceanic variable for evaluating heat content and heat fluxes. *J. Phys. Oceanogr.*, 33:945–963, 2003.