

We are indebted to the Reviewers for their valuable feedback. The resulting revisions to the manuscript are described below.

REVIEWER 1:

Reviewer comment:

The work is **technically deep, well structured, and fills a meaningful gap** between traditional DO methods (e.g., DISORT) and the needs of strongly anisotropic scattering applications such as pigment modeling and atmospheric backscattering. The mathematical exposition is extensive and complements the numerical demonstrations.

However, for readers in atmospheric physics or hydrology, the connection to physical interpretation (e.g., energy conservation, reciprocity, flux closure) needs more clarity.

Response to comment:

Thank you for this evaluation and valuable feedback. One of the benefits of using Galerkin's Method is that it is an orthogonal projection from the Hilbert space of differentiable functions to the Hilbert space of piece-wise continuous functions, so it preserves conservation properties under the correct weights. This includes energy conservation, reciprocity, and flux closure. Additional language to that effect has been added to the paper. Please see the subsection 'Conservation Properties' in the revised manuscript.

Reviewer comment:

Show explicitly that the method conserves energy for typical phase functions

Response to comment:

The tool uses a variation of the Sinkhorn-Knopp normalization on the phase kernel to ensure a conservative scattering matrix. This is presented in the new subsection, "Conservation Properties".

Reviewer comment:

Explain how the natural reflectance assumption relates to physical boundary conditions

Response to comment:

The general natural reflectance solution for the top boundary and the general natural reflectance solution for the bottom boundary are classes of solutions to the general radiative

transfer equation and together provide enough degrees of freedom to solve for the specific solution to any two-boundary physical boundary condition. Clarifying language has been added to the subsection “Satisfying the General Two-Boundary Problem”.

Reviewer comment:

Provide discussion on limitations of the method for non-homogeneous or non-plane-parallel media

Response to comment:

Like many current tools (excepting those that utilize Monte Carlo methods), in Eigenflux, non-homogeneous media must be organized into distinct layers of homogeneous media. The tool provides routines for combining the solutions to the homogeneous layers into a single solution for the non-homogeneous media. The tool does not currently handle non-plane-parallel media, although the addition of pseudo-spherical media is contemplated for future work. Clarifying language has been added to the Introduction.

Reviewer comment:

Numerical Experiments Are Extensive but Lack Error Metrics

Figures show intensity distributions, eigenvalues, and transparency depths, but there is no table or section that:

- reports numerical error norms,
- quantifies convergence with mesh refinement,
- shows sensitivity to the choice of Chebyshev points or mesh density.

To strengthen the numerical section, consider adding:

- L2 or L ∞ error convergence plots
- Benchmark comparisons using known analytic solutions (e.g., Rayleigh scattering)
- Mesh resolution sensitivity analysis

Response to comment:

A new subsection, “Accuracy and Timing Studies” has been added that includes L2 error metrics. Comparisons are between the solutions and the analytic equation that is being solved, and

between EigenFlux and the most popular current tool, DISORT. For example, the mesh refinement graph (one of the nine figures in the new subsection) is:

