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Title: Physical characterization of the boundary separating safe and unsafe AMOC overshoot behaviour

Authors: Aurora Faure Ragani and Henk A. Dijkstra

Point-by-point reply to reviewer #2

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We thank the reviewer for their careful reading and for the useful comments on the manuscript. The comments of the reviewer are in italic font below and our response is in normal font.

Overview

The paper studies the recovery or its failure around an AMOC breakdown for an intermediate complexity model depending on a freshwater forcing parameter. It has been found before that for certain values of the parameter the model possesses a branch of steady states whose stable large amplitude region is identified with the AMOC. The branch undergoes a fold bifurcation that is identified as an AMOC tipping point. In this paper the parameter is chosen piecewise linear in time such that it overshoots the fold value for an intermediate time interval. Numerical computations are presented for several choices of the time parameterisation showing "safe" scenarios that give recovery of the AMOC, as well as "unsafe" ones, where the recovery fails. The results are interpreted physically, suggesting that the lateral salinity advection can distinguish these. Finally, a comparison with analytical results for a scalar fold point are presented. I think the topic is timely and of broad interest, fitting to Earth System Dynamics. However, I have a few comments that should be addressed in a revision.

Author's reply:
Thank you for your positive evaluation of the paper.

Main comments:

1. *In order to shorten the manuscript, I recommend to revise section 4. On the one hand, (13) and the discussion on how to achieve the normal*

form is not needed, and in fact misleading as noted in the next point. On the other hand, I do not think section 4.3 is needed as the formulae appear to be directly taken from Li et al 2019.

Author's reply:
The analysis is indeed based on the results of Li et al. (2019), but the new element is the piecewise continuous forcing, so the formulae are not directly taken from that paper.

Changes in manuscript:
To improve readability, we will move many of the details of the analytical analysis in section 4.3 to an appendix.

2. *In section 4.4 it seems the authors use Ψ_A as an ad hoc variable X and suggest this relates to a closed ODE (13). However, such a relation requires a center manifold reduction and the resulting coefficients can strongly differ from those obtained here. Hence, I would strongly recommend to do this, and I do not see much added difficulty: one just needs a kernel eigenvector of the linearisation at the fold point, and the corresponding adjoint kernel eigenvector. I do not understand what the authors later mean by Lyapunov-Schmidt reduction.*

   Author's reply:
   We addressed this issue already in section 4.4 of the original paper as a reason why the results of the analytic model and the numerical model differ. It is outside the scope of this paper to compute the center manifold reduction for such a complicated ($\sim$ 500,000 degrees of freedom) model as one also needs also to determine the nonlinear terms. The Lyapunov-Schmidt method is very similar to the center manifold reduction method (but the reduction is algebraic instead of dynamic). In fact, in many finite-dimensional cases, they produce equivalent reduced equations (especially for stationary bifurcations).

   Changes in manuscript:
   We will explain in more detail the reason for the mismatch of the numerical and analytical results and also mention the center manifold reduction method in section 4.3.

Minor/typo:

We thank the reviewer for pointing out the corrections; we will follow all suggestions in the revised paper.

1. *Abstract: I wonder about the definite formulation "[The AMOC] is an important tipping element". In line 16 this is rather differently only "potential".*

   Author's reply:
   We will remove 'potential' in line 16.

2. *l5: What is meant by the term "fully-implicit (global ocean model)"? I think it would deserve a brief explanation.*

   Author's reply:
   Agreed. We will include a short explanation.

3. *In section 3.1: Is there some significance of the times in years mentioned here?*

   Author's reply:
   No, the time scale just serves as illustration.

4. *l65: Explain why the cos(theta) part is removed from F_S (also elsewhere, e.g. in (8), (9)).*

   Author's reply:
   The cos(theta) part comes from integrating over a spherical surface.

5. *Fig 1b: The upper part of the dashed line branch looks a bit edgy. Is this getting smoother with smaller continuation stepsize?*

   Author's reply:
   Indeed, but this is not crucial here so not recomputed.

6. *l95: I think there is a typo: the parameter in the third time interval in (4) should be time-independent.*

   Author's reply:
   No, there is no typo. After t_2, the forcing is constant.

7. *l384: course should be coarse*

   Author's reply:
   Well spotted and this will be corrected.