

Review #2 of “Discrete differential geometry of fluvial landscapes”

Ben Kargère

First, I appreciate the major reorganization of the paper. I realize this wasn’t easy, and it makes the manuscript much more pleasant to read. Unfortunately, many of my initial comments have not been addressed, and the material has instead simply been rearranged. Some of the analysis underlying Section 5 is not accurate, which I did not notice during the first round of reviews.

The overall writing quality is still insufficient. The manuscript is unnecessarily repetitive and should be more concise in both content and presentation. Consider whether each sentence is necessary or merely repeats a point made elsewhere, and whether it can be stated more clearly in fewer words. The differential geometry section remains scattered and contains numerous errors, making it difficult to trust even where it is correct. I’ve tried to flag these sections in the line comments (though near the end of the paper I stopped). I still see a path to publication, but the authors need to revise substantially, cutting the large portions of the manuscript that read as unfounded, overly speculative, or merely promotional.

My second major comment is that the authors have not adequately addressed one of my original comments, perhaps due to misinterpretation. The authors dismissed my comment by stating:

“While it is true that curvature is only equal to the erosion rate at shallow slopes, it is still the leading-order term in the expansion. Thus, the dynamics of the hilltop to hillslope transition are defined by the rate at which the magnitude of curvature decays with increasing slope.”

This interpretation is not accurate. There was also a typo in my review, and the correct expansion is

$$\mathbf{q}_s \approx -D\nabla z - \frac{D}{S_c^2} |\nabla z|^2 \nabla z + \dots, \quad -\nabla \cdot \mathbf{q}_s = D\nabla^2 z + \nabla \cdot \left(D \frac{|\nabla z|^2}{S_c^2} \nabla z \right) + \dots.$$

The linear diffusive/Laplacian term is leading order only when $|\nabla z|/S_c \ll 1$. In the OCR this condition holds only in a small neighborhood near hilltops, and not throughout the hilltop to hillslope transition. For slopes comparable to the critical slope, the nonlinear term is of the same order as the linear diffusive term, so the expansion is **not** asymptotically ordered. Therefore the Laplacian/linear diffusive flux does not control this steep transition region (and presumably nor does the DL-SP erosion). Instead, this region is governed in large part by the nonlinear (but more realistically nonlocal) slope-dependent fluxes.

More generally, steep regions can’t be well described by erosion laws whose physical basis is rooted in a shallow-slope approximation. As noted in my prior review, this fact is intimately connected to the results presented here. As written, placing Eq. (1) immediately before describing the OCR study site suggests that the equation adequately represents the OCR, but (at the minimum) its limitations should be clearly noted.

Line Comments

1, 3: “to understand” is used in both of the first two sentences.

8: “at a point” is unnecessary.

11: “systematic errors” is still misleading.

20: “with relief generally increasing with the horizontal scale of measurement ” is not clear.

36: Use active language. Start with “Several parallel” and end with “DEM processing.”

Paragraph starting on line 36: The details of how curvature is used in geomorphology are discussed in the next section, and the final sentence here is vague. I think the last two sentences of this paragraph could be cut and the paragraph merged with the following one.

36: Either use “models” for all of the items or only the first: “models of stresses, sheet joint development, and bedrock fold structure.”

45: Ending the sentence with “with rich potential” is awkward. “Extracts valuable surface curvature information(?)”

47: Is “continuous” the right word? It seems you may mean “consistent,” but “continuous” suggests something different.

48: Unnecessary and awkward. Cut after dynamics?

54: “has”

62: If Cayley and Maxwell reached essentially the same conclusion, their discussions should be merged and the emphasis placed on the mathematical results rather than attribution.

62: Remove “prominent physicist” and everything but their last name. This applies throughout the paper (Gauss and Euler), since the 19th century academics being cited are well known for obvious reasons and mentioning it doesn’t add substance.

71: Comma after models.

73-74: “models model”

75: “Channels themselves are defined on the basis of curvature” was already noted on line 65. This distracts from the goal of arriving at Eq. (1).

81: Use $z(x, y, t)$ to show that it is a PDE. You could also write $A(x, y, z(x, y, t))$ and remove “at a given point,” though that’s a matter of preference.

83: Note that this is the detachment-limited stream-power model.

84: $\partial z/\partial t$

85: Eq. (1)

84-92: Eq. (2) is unnecessary. I suggest making a new paragraph starting with steady state.

87: It’s obvious that the parameters refer to those in the equation since they were just defined.

93: Awkward wording. “directly from the curvature tensor, providing” (?)

94: Awkward wording. “At a point,” “assumed to,” and “alike” are unnecessary.

107: “Identified” and “documented” are unnecessary. Remove the comma after topography.

110: with little.

Section 3: This section still contains many inaccuracies. I have noted some of them, but the list is far from exhaustive.

116: “The term” is unnecessary. Comma after or.

117: “The tools of” is unnecessary. Switch “developed” and “in part.”

118: Awkward and indirect wording. “Curvature is either ‘intrinsic’ or ‘extrinsic.’” (?)

121: I don’t think ‘reference frame’ is technically correct. “How the surface sits in space?”

122: “Discretely sampled” is beside the point here. The geometric argument in this section would hold even at infinite resolution.

Paragraph starting on line 122. The mathematical notation remains inconsistent. The hats could reasonably be confused with unit vectors, so primes would be better here. Points should be scalars (as on line 127). \mathbf{dx} , \mathbf{dy} , \mathbf{du} , and \mathbf{dv} should not be described as displacement vectors and should not be bolded. The displacement vectors should be ds and ds' , not \mathbf{ds} .

134: Not a reference frame.

135, 607: Refer to Euler and Gauss by last name only.

141: As who? Colloquial.

141: Continuity of the surface is not enough for the curvature to vary smoothly. Curvature depends on second derivatives, so it should be twice continuously differentiable.

145: As noted in my last review, cut the parenthetical note. The equation is already provided, and it is clear that it is unrelated to number theory or complex analysis.

147: Direction, not path (as on line 151). Not a reference frame either.

147: Coordinate system, not reference frame. This definition would fit better near line 120, alongside the definition of intrinsic versus extrinsic.

150: an extrinsic quality

156: cut “both,” the comma after transformations, and the second “that it.”

Figure 2: Use 3-D and 2-D. b. is repeated.

Paragraph starting on line 165: This disrupts the flow. It should be removed or, at a minimum, substantially shortened and not presented as a separate paragraph.

Paragraph starting on line 170. I do not think this is accurate. Consider flipping the landscape upside down. Does this still hold?

175: Misleading, since $K_M \neq \frac{1}{2} \nabla \cdot (\nabla z)$ for non-shallow slopes, as noted in my prior review.

176: Add a comma after “opposite” or revise the sentence structure.

177: Put the comma inside the quote or simply remove the ’ around minimal surfaces. Also, “which have arisen” is awkward.

179: The comment about perfect saddles doesn’t add much.

180-182: It’s clear that perfect saddles are not discussed here, so trim and merge the sentences to resolve the awkward wording in the second. The connection to process regimes and drainage area is already clear.

184-192: Move this section to join the paragraph starting on line 122 to rigorously (and correctly) explain Figure 2. This section should also define the tangent vectors $\frac{\partial \mathbf{r}}{\partial u}$ and $\frac{\partial \mathbf{r}}{\partial v}$, using them to get $d\mathbf{s}$.

187: “via as”

190: r_1 , r_2 , and r_3 serve no clear purpose. You already use x , y , and z in a Cartesian coordinate system, so you may as well keep that notation. As noted in my previous review, you may also want to use something other than z here for the multivalued case (cliffs), which can’t occur for a single-valued $z(x, y, t)$ as in Eq. (1) (Stark and Stark, 2022).

192: Note $ds = |d\mathbf{s}|$

207: “Resultant curvature values reference” is an awkward wording.

216: Use $\kappa(\lambda)$ to distinguish it from Eq. (3).

217: Check all equation reference notation, as noted in my prior review.

218: “The result is given by” is unnecessary. Equation 12 and 13 should be combined.

225: Use present tense: “are found” or “are obtained.” This applies throughout this section.

245: It appears that this convention was also used earlier in lines 170-175.

Section 3.4/Line 249: The distinction drawn here is misleading. It’s the same approach, just expressed in different notation.

260: I am skeptical of this claim. Fundamentally it’s the same math, so this appears to reflect the computational implementation rather than the method itself. Given this, Section 3.4 seems unnecessary and could be moved to an appendix or removed.

268-270: Combining these sentences would be cleaner.

277: “have been extensively applied in geomorphology, with applications including” is clunky.

278: “such that the resulting topography is equal to its actual value only in center of the grid, and is elsewhere damped towards the margins” reads clunky.

287: Citation style.

290: This sentence is unnecessary.

316: “Curves are pulled” is unclear. Add a comma after wavelengths.

325: Drainage area on hillslopes is especially dependent on grid resolution, as noted in my previous review. This should be made clear here.

341: This assumption is not well supported and the reasoning is incorrect. K_M is not generally proportional to $\nabla \cdot (\nabla z)$. Stability and instability are concepts from dynamical systems theory (phase plane), and the suggested interpretation of K_G as indicating of the dynamical stability of the surface is not appropriate. The cited references do not support this interpretation.

344: comma after geometry.

347: This sentence doesn’t add information and should be removed.

360: This sentence should be removed.

376: remove after transition.

381: The convergence/divergence interpretation is not clear.

383: This sentence is unclear.

385: This sentence is broken and unclear.

387: The end of this sentence is redundant, since it is obvious that $25\% > 18\%$.

388: It’s not clear what this sentence adds.

392: This sentence is obvious and adds nothing of substance.

399: Remove the note after the dash.

401: The tone here feels overly promotional.

Section 5.2: At this point in the paper, it feels unnecessary to go through all of this again, even if the results are new. It may be more concise to combine the Gaussian and mean curvature interpretations and treat the landscape together. This would need to be written more concisely to avoid becoming convoluted.

424: This note is not useful and should be removed.

Section 6, Fig. 15: These sections come very late in the paper. These ideas have not appeared earlier in the draft, so it is hard to treat them seriously. Remove them or move them to an appendix. If kept and moved earlier (as they were in the earlier manuscript), they will need a much clearer and more concise explanation.

461: “And yet” is an awkward construction.

506: It's not clear what's meant by this comment.

556-560: This is not accurate, as noted earlier.

The paper should conclude sooner, and the discussion needs to be severely shortened. Section 6 should not appear where it is. Remove it or move it to an appendix (provided it doesn't make inaccurate claims).

References

Stark, C. P. and Stark, G. J.: The direction of landscape erosion, *Earth Surface Dynamics*, 10, 383–419, doi: 10.5194/esurf-10-383-2022, 2022.