

1 **Improving the computation efficiency of a source-oriented chemical mechanism for the
2 simultaneous source apportionment of ozone and secondary particulate pollutants**

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13 **Abstract:**

14 Source-oriented chemical mechanisms enable direct source apportionment of air
15 pollutants by explicitly representing precursor emissions and their reaction products in
16 atmospheric models. These mechanisms use source-tagged species to track emissions and
17 their evolution. However, scalability was previously limited by the large number of reactions
18 required for interactions between two tagged species, such as the NOx-NOx or VOC-NOx
19 reactions. This study improves computational efficiency and scalability with a new method
20 that tracks the total concentration of tagged species, reducing the n^2 second-order reactions
21 for n sources into $2n$ pseudo first-order reactions. The overall production and removal rate of
22 individual species remain unchanged in the new approach. The number of reactions and
23 number of model species increase linearly with the number of source types, thus greatly
24 improving the computation efficiency. In addition, a source-oriented Euler Backward
25 Iterative (EBI) solver was implemented to replace the Gear solver used in previous
26 applications of the source-oriented mechanism. The source-oriented EBI solver has been
27 assessed by comparing predicted results with the Gear solver. Good agreement between
28 those two methods has been achieved, as the results from the EBI scheme are linearly
29 correlated to Gear and average of absolute relative error is below 5%. In the timing
30 assessment, the proposed EBI scheme can effectively reduce the total chemistry time by 73%

31 to 90% for grids with different resolutions, which leads to the reduction of total simulation
32 time by 46% to 74%. The proposed source-oriented scheme is efficient enough for practical
33 long-term source apportionment applications on nested domains.

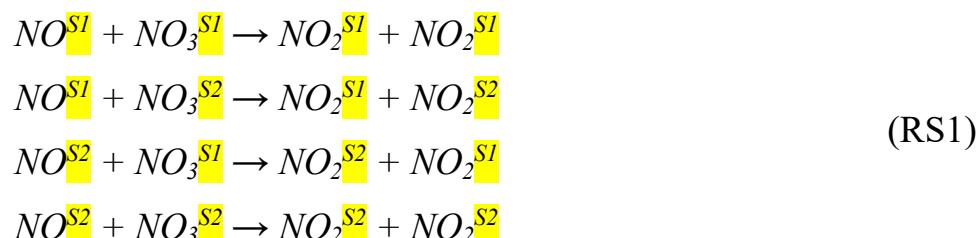
34 **Keywords:** Atmospheric chemical solver, Euler Backward Iterative, Source apportionment
35

36 1 Introduction

37 1.1 Source-oriented chemical mechanisms

38 Source-oriented air quality models have been used extensively in source apportionment
39 modeling studies to determine source (or source region) contributions to NO_x (Zhang & Ying,
40 2011), VOCs (Ying & Krishnan, 2010), secondary inorganic (Ying & Kleeman, 2006) and
41 organic aerosols (Wang et al., 2018), and ozone (Wang et al., 2019a; Wang et al., 2020). In
42 these models, source-tagged species and their reactions are introduced in the gas phase
43 chemical mechanisms to track primary emissions and their reaction products from different
44 sources. For the source apportionment of secondary aerosol products from gas-to-particle
45 partitioning, aerosol and cloud processes are also modified to include additional model
46 species to represent the semi-volatile products from different sources.

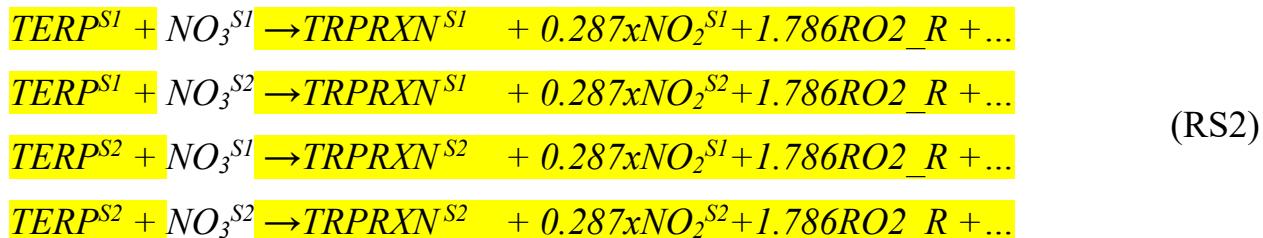
47 While this is conceptually simple, the source-oriented mechanisms are computationally
48 expensive because the number of reactions increases almost quadratically with the number of
49 source types due to reactions that involve two source-tagged species. For example, consider
50 the simple reaction of $\text{NO} + \text{NO}_3 \rightarrow 2\text{NO}_2$, if the source-oriented mechanism is designed to
51 track two explicit sources, four reactions in reaction set 1 (RS1) are needed:



52 where the superscript^{Sn} are tags attached to the name of the species to differentiate their
53 source-origin. For a total of N_s sources of NO_x to be tracked explicitly, N_s^2 reactions are
54 needed instead of one reaction. As there are quite a number of such $\text{NO}_x + \text{NO}_x$ reactions in
55 the gas phase inorganic chemistry, the number of reactions needed for the chemical
56 mechanism grows quickly. The necessity to deal with N_2O_5 , which can be generated from
57 NO_2 and NO_3 from different sources, is handled with double-source-tagged species $\text{N}_2\text{O}_5,\text{Sij}$.

58 In addition to a potential quadratic scaling of the number of reactions, the number of N₂O₅
59 species also increases quadratically with the number of explicit sources, leading to
60 near-quadratic growth of the overall number of species when the number of types to track
61 gets higher.

62 In ozone source apportionment calculations, it is also necessary to track the sources of
63 primary emitted VOCs as well as some of their reaction products in addition to the sources of
64 NO_x (Wang et al., 2020). Some of the unsaturated VOCs such as olefins can react with the
65 NO₃ radical. In the source-oriented mechanism, the number of reactions needed for these
66 VOC+NO₃ reactions also increases quadratically, as shown in RS2 below, using the ethene
67 (ETHE) + NO₃ reaction from the SAPRC-07 mechanism (Carter, 2010) as an example for
68 two sources. For accurate VOC source apportionment calculations that involve reactions
69 between two source-oriented species, such quadratic dependence of source types and
70 reaction numbers also arises (Ying & Krishnan, 2010).



71 Due to the necessity of explicitly handling some or all of these reactions in
72 source-oriented mechanisms, the source-oriented modeling approach is computationally
73 intensive so that previous applications were limited to up to 9 explicit sources for secondary
74 nitrate in a single run (Kleeman & Cass, 2001; Ying et al., 2004; Ying et al., 2014). In some
75 previous work for VOC and secondary organic aerosol source apportionment, only one
76 explicit source was tracked at a time to simplify the reactions and to reduce the computation
77 burden (Ying & Krishnan, 2010; Wang et al., 2018). However, multiple model runs are
78 needed to determine the contributions from all sources. To make the source-oriented
79 approach practical for a larger number of source types, it is necessary to improve the
80 computation efficiency of the source-oriented approach.

81 1.2 Numerical solution of stiff ODEs for gas phase reaction kinetics

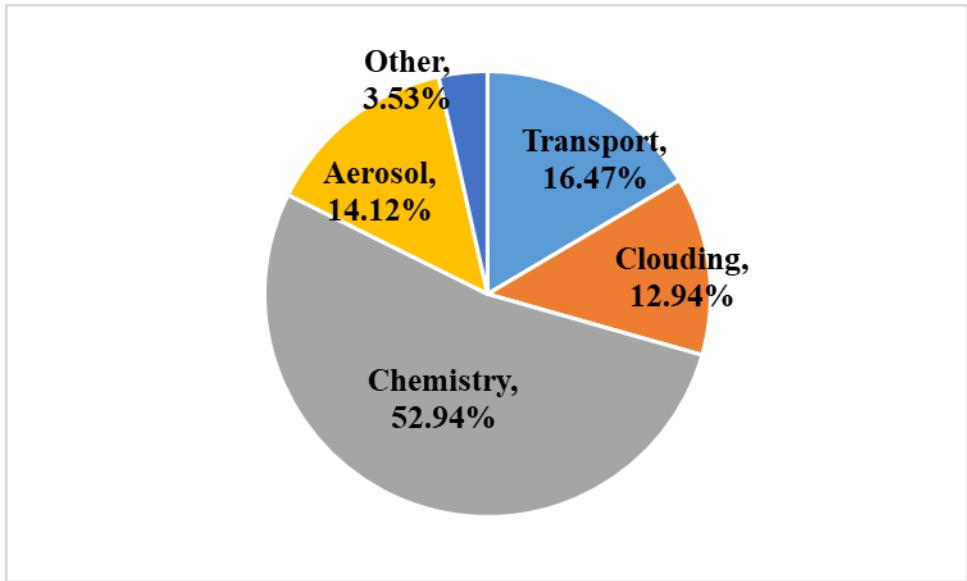
82 The gas phase chemical reaction kinetics are described by of a non-linear system of stiff
83 ordinary differential equations (ODEs) which must be solved to predict the transient
84 evolution of the concentrations of gas species. One of the most widely used schemes is the

85 Gear method (Ralph, 1973)

$$(I - h\beta_s J) \Delta \vec{C}_t^{m+1} = -\vec{C}_t^m + \sum_{j=1}^s \alpha_j \vec{C}_{t-jh} + h\beta_s \frac{d\vec{C}_t^m}{dt} \quad (1)$$

86 where h is the integration time step; \vec{C}_t^m is the vector of species concentrations to be
 87 solved for time t during the m^{th} iteration; $\Delta \vec{C}_t^{m+1}$ is the correction vector to estimate \vec{C}_t^{m+1} ; J
 88 is the Jacobian matrix of partial derivatives for all species that $J_{i,k} = \frac{\partial^2 C_{i,t}}{\partial C_{k,t} \partial t}$. It is calculated
 89 either analytically or numerically initially based on \vec{C}_{t-h} and updated when necessary using
 90 the most recent values of \vec{C}_t ; I is the identity matrix; s is the order of the accuracy; β_s and
 91 α_j are scalar multipliers that depend on the order of the method. For each iteration, the new
 92 concentrations for the next time step t is evaluated as $\vec{C}_t^{m+1} = \vec{C}_t^m + \Delta \vec{C}_t^{m+1}$. The iteration stops
 93 when $\Delta \vec{C}_t^{m+1}$ becomes less than a provided error. A practical solver also needs to
 94 automatically adjust the time step size h , the order of accuracy s and recalculate the **Jacobian**
 95 matrix when necessary to ensure that the estimated error in one time step is less than a
 96 prescribed criteria (Jacobson, 2006).

97 The advantage of using the Gear solver is that it is a general stiff solver so that no special
 98 modifications are needed for a specific chemical mechanism. However, it is computationally
 99 intensive as it involves evaluating the Jacobian matrix and performing LU factorization for
 100 the left-hand side matrix. A Sparse-Matrix Vectorized Gear (SMVGEAR) solver was
 101 developed by Jacobson and Turco (1994) and has been included in a number of atmospheric
 102 chemical transport models (Zhang et al., 2011; Hu et al., 2012; Hu et al., 2014). The
 103 SMVGEAR solver was also used previously to solve the gas phase reaction kinetics in the
 104 source oriented CTM (Shi et al., 2017; Li et al., 2021). **Test based on the reported Texas Air**
 105 **Quality Study 2006 ozone episode showed that a source-oriented SAPRC-07 mechanism that**
 106 **simultaneously performs the source apportionment of NO_x, SO₂, primary VOCs, HCHO, and**
 107 **ozone for 16 sources needs 11 times of the computation time of the original**
 108 **non-source-oriented mechanism (Parrish et al., 2009).** The gas chemistry is the most
 109 time-consuming step that normally takes more than half of total simulation time, as shown in
 110 Figure 1.



111
 112 **Fig. 1.** Typical fraction of time spent in the scientific processes in the source-oriented
 113 CMAQ model. This is based on a 36-km resolution domain (160 rows*130 columns*44
 114 layers), 6 source types and solved using the SMVGEAR solver.

115 The Euler Backward Iterative (EBI) (Hertel et al., 1993) is a faster method to solve the
 116 stiff ODE systems arising from a gas-phase photochemical mechanism. The basis of this
 117 method is the backward Euler method as shown in equation (2),

$$C_{i,t} = \frac{C_{i,t-h} + hP_{i,t}}{1 + hL_{i,t}} \quad (2)$$

118 where $C_{i,t}$ and $C_{i,t-h}$ are the concentrations of species i at time t and $t-h$, respectively; h
 119 is the integration time step; $P_{i,t}$ and $L_{i,t}$ are the chemical production and loss terms,
 120 respectively, evaluated using the concentrations of the species at time t . Equation (2)
 121 represents a set of coupled non-linear equations and a solution can be obtained by first
 122 evaluating the production and loss terms using the concentrations from the previous time
 123 step $t-h$ to calculate an initial estimation of the species concentrations for the time step t
 124 based on equation (2). These concentrations are applied to update the P and L terms so that
 125 and updated estimation of the species concentrations for time step t are obtained. This
 126 procedure is repeated until the changes in $C_{i,t}$ for a new iteration are less than a prescribed
 127 value.

128 For atmospheric photochemical reactions, there are several families of species whose
 129 concentrations are strongly coupled in reversible reactions. The general backward Euler
 130 method described above has a slow rate of convergence or even fails to converge. In the EBI
 131 solver, four family groups of strongly coupled species, are excluded from the general
 132 equation (2): (1) NO, NO₂, O₃ and O(³P), (2) OH, HO₂, HONO and HNO₄, (3) peroxyacetyl

radical ($\text{CH}_3\text{C(O)OO}$, or C_2O_3) and peroxyacetyl nitrate (PAN), and (4) NO_3 and N_2O_5 . For these species, analytical solutions of four sets of non-linear algebraic equations (Eqs. 9-12 in Hertel et al., 1993) are applied to determine their concentrations at time t instead of using the P and L terms with equation (2). The detailed mathematical procedures involved in obtaining these analytical solutions are listed in Appendices A.1-A.4 of Hertel et al. (1993).

The accuracy of the EBI solver has been evaluated against more accurate solvers (Hertel et al., 1993) and is the default choice in the CMAQ model for a number of built-in chemical mechanisms. However, it cannot be directly used to solve the source-oriented chemical mechanisms since the solution procedures of aforementioned four strongly coupled groups only calculate the total concentrations of species, and source-tagged species are not included in the explicit solutions. Thus, for source-oriented mechanisms, groups of tagged reactive nitrogen species require additional solution steps. Direct replication of the procedures for total concentrations to treat the source tagged species are infeasible, as this would dramatically increase the computational cost and the difficulty of source code implementation. An EBI solver capable of handling the chemical mechanisms with source tagged species and their reactions while maintaining brevity and ease of implementation, is highly desirable. Ideally, it should be able to predict the concentrations of source-tagged reactive nitrogen species based on their pre-determined total concentrations, which would greatly improve the computational efficiency and consequently enhance the applicability of the source-oriented air quality model.

The objective of this study is to develop a computationally efficient source-oriented gas phase chemical mechanism for the simultaneous source apportionment of O_3 and other gaseous pollutants such as CO, primary VOCs, NO, NO_2 , SO_2 and NH_3 . The mechanism, when linked with a proper source-oriented aerosol mechanism, can be used to determine the sources contributions to nitrate, sulfate and ammonium ion. The method for improving the efficiency of the source-oriented mechanism through simplification of reaction representation and modification of the EBI ODE solver for source-oriented nitrogen species is described in Section 2. Section 3 details the testing of the improved mechanism and the source-oriented EBI solver.

2 Method

163 **2.1 Reduce the number of reactions and source-tagged species**

164 In the original source-oriented model, a general reaction set that involves two
 165 source-tagged species as reactants for N_s source types can be written in the following form:

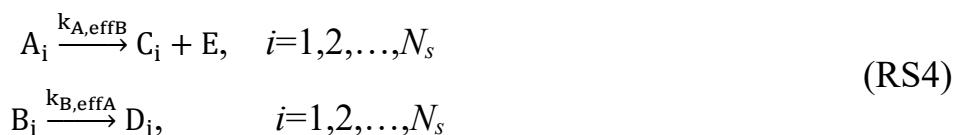


166 where A, B, C and D are source-tagged model species and the subscripts denote source
 167 origin index of these species. For simplicity, assuming that C and D are the reaction products
 168 from A and B , respectively. E represents a general product whose source-origin is not tracked
 169 in the model simulation. k_{AB} is the second order reaction rate coefficient, which is the same
 170 for all the reactions in this reaction set. Reaction sets RS1 and RS2, as shown in the
 171 examples in Section 1, can both be expressed in this form. The loss rate of A_i is calculated
 172 using equations (3a) and (3b):

$$\frac{d[A_i]}{dt} = -k_{AB}[B_1][A_i] \dots - k_{AB}[B_{N_s}][A_i] = -k_{A,\text{eff}B}[A_i] \quad (3a)$$

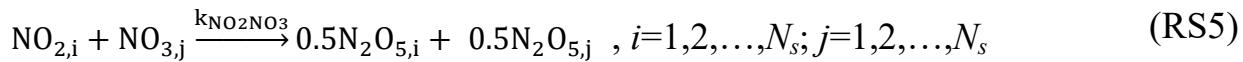
$$k_{A,\text{eff}B} = k_{AB}[B_{\text{tot}}] = k_{AB} \sum_{j=1}^{N_s} [B_j] \quad (3b)$$

173 where $k_{A,\text{eff}B}$ is the pseudo-first order reaction rate coefficient for $[A_i]$ based on the total
 174 concentration of B , $[B_{\text{tot}}]$, as defined in equation (3b). **This method is valid for reactions with**
 175 **rate laws of the form rate**= $k[A]^m[B]^n\dots$, where the rate is proportional to the product of
 176 reactant concentrations raised to their orders. A similar set of equations can be derived for the
 177 loss rate of individual tagged species B_i . Thus, the N_s^2 second-order reactions represented by
 178 RS3 can be equivalently described by the following $2N_s$ pseudo first-order reactions,



179 For the non-typed product E , it can appear in either the A_i reactions or the B_i reactions
 180 and it is easy to show that the formation rate of E based on RS4 is the same as these from
 181 RS3.

182
 183 The double-tagged N_2O_5 species and their reactions can be simplified as well. For $N_2O_{5,ij}$
 184 which represents N_2O_5 based on NO_2 from source i and NO_3 from source j , it can be
 185 equivalently written as $0.5N_2O_{5,i} + 0.5N_2O_{5,j}$ as shown in reaction set RS5, in terms of
 186 preserving the source contributions to NO_2 and NO_3 ,



With this simplification, as well as the pseudo-first order reaction technique described above, the reactions of N_2O_5 formation from N_s types of NO_x can be expanded into $2N_s$ reactions with N_s tagged N_2O_5 species as shown in the general reaction set RS4, where A is NO_2 , B is NO_3 and C and D are N_2O_5 species that have the same source tag as A and B , respectively. This **dual-tagged reaction reduction method** significantly decreases the number of species as well as the number of reactions for the source-oriented mechanism.

The total concentration of tagged species needed in equation (3b) for the pseudo first order reaction rate coefficient need not be tracked separately **in the dual-tagged reaction reduction method**. Instead, they are calculated on-the-fly and then used to calculate the pseudo first-order rate coefficients for the reactions of the tagged species shown above. The function that calculates the reaction rates to be used in the stiff ODE solvers needs to be modified to recognize these special pseudo first-order reactions. The CMAQ model is capable of dealing with these special pseudo-first order reaction natively with its included mechanism preprocessor (CHEMMECH). An example input to the CHEMMECH on how the reactions are constructed for $\text{NO} + \text{NO}_3 \rightarrow 2\text{NO}_2$ for 10 source types is illustrated in List 1 of Appendix.

2.2 Source-oriented Euler Backward Iteration (EBI) scheme

2.2.1 Solution for family of source-tagged species in coupled reversible reactions

For source apportionment of ozone and secondary inorganic aerosols, the reactive nitrogen species NO , NO_2 , HONO , HNO_4 , PAN , NO_3 and N_2O_5 are source-tagged. For PAN , its source is determined by the source of NO_2 while the peroxyacetyl group is not source-tagged.

The standard solution procedure of the source-tagged species in the source-oriented EBI method includes two major steps: (1) evaluating of total concentrations of these tagged species using equation sets (9), (10) and (11) and (12) in Hertel et al. (1993); and (2) predicting the concentrations for each tagged species based on the total concentrations. In the following, equations for step (1) are summarized first, followed by equations for step (2). Equations are separately listed for each family.

The first set of equations (4a-4d) are used to solve the total concentrations of NO , NO_2 , O_3 and $\text{O}({}^3\text{P})$. These equations are based on the corresponding ones in Hertel et al. (1993).

$$[NO_{tot}]_t = [NO_{tot}]_{t-h} + hP'_1 + h(r_{1,2} + J_1)[NO_{2,tot}]_t - hk_{1,3}[O_3]_t[NO_{tot}]_t - hr_{2,1}[NO_{tot}]_t - hL'_1[NO_{tot}]_t \quad (4a)$$

$$[NO_{2,tot}]_t = [NO_{2,tot}]_{t-h} + hP'_2 + hr_{2,1}[NO_{tot}]_t + hk_{1,3}[O_3]_t[NO_{tot}]_t - h(r_{1,2} + J_1)[NO_{2,tot}]_t - hL'_2[NO_{2,tot}]_t \quad (4b)$$

$$[O_3]_t = [O_3]_{t-h} + hJ_2[O(^3P)]_t - hk_{1,3}[NO_{tot}]_t[O_3]_t - hL'_3[O_3]_t \quad (4c)$$

$$[O(^3P)]_t = [O(^3P)]_{t-h} + hP'_{12} + hJ_1[NO_{2,tot}]_t - hJ_2[O(^3P)]_t - hL'_{12}[O(^3P)]_t \quad (4d)$$

In the above equations, species with a subscript *tot* represent the total concentration of a set of tagged species, which is calculated by adding the concentrations of the individual tagged species; *h* is the size of the current time step; $r_{1,2}$ is the production rate coefficient of NO from NO_2 , excluding photo-dissociation and J_1 is the photolysis rate of NO_2 to form NO; $k_{1,3}$ is the second-order rate coefficient for $NO+O_3$ reaction to form NO_2 ; $r_{2,1}$ is the pseudo-fist order rate constant for the production of NO_2 from NO from all other pathways, excluding the $NO+O_3$ reaction. J_2 is the first-order reaction rate constant for $O(^3P)+O_2$ to form O_3 . The terms P'_1 , P'_2 , and P'_{12} account for all the remaining production for the total concentrations of NO, NO_2 and $O(^3P)$ in the mechanism, and the terms L'_1 , L'_2 , L'_3 , and L'_{12} are the losses of the total concentrations of NO, NO_2 , O_3 and $O(^3P)$, respectively. Analytical solutions for the total concentrations based on 4a-4d were derived and described in detail in the Appendix A of Hertel et al. (1993) and are not repeated here.

Once the total concentrations of NO, NO_2 , O_3 and $O(^3P)$ are solved, concentrations of the source-tagged NO and NO_2 are solved from the following two equations,

$$[NO_i]_t = [NO_i]_{t-h} + hP'_{1,i} + h(r_{1,2} + J_1)[NO_{2,i}]_t - hk_{1,3}[O_3]_t[NO_i]_t - hr_{2,1}[NO_i]_t - hL'_1[NO_i]_t \quad (5a)$$

$$[NO_{2,i}]_t = [NO_{2,i}]_{t-h} + hP'_{2,i} + hr_{2,1}[NO_i]_t + hk_{1,3}[O_3]_t[NO_i]_t - h(r_{1,2} + J_1)[NO_{2,i}]_t - hL'_2[NO_{2,i}]_t \quad (5b)$$

where *i* is the source index for the tagged species. For each *i*, the two unknowns $[NO_i]_t$ and $[NO_{2,i}]_t$ are solved analytically using the following equations,

$$[NO_i]_t = \frac{1}{\det(\mathbf{A})} (A_{22}b_1 - A_{12}b_2) \quad (6a)$$

$$[NO_{2,i}]_t = \frac{1}{\det(\mathbf{A})} (-A_{21}b_1 + A_{11}b_2) \quad (6b)$$

$$\mathbf{A} = \begin{bmatrix} 1 + hk_{1,3}[O_3]_t + hr_{1,2} + hL'_1 & -h(r_{2,1} + J_1) \\ -h(r_{2,1} + k_{1,3}[O_3]_t) & 1 + hr_{1,2} + hJ_1 + hL'_2 \end{bmatrix} \quad (6c)$$

$$\mathbf{b} = \begin{bmatrix} [NO_i]_{t-h} + hP'_{1,i} \\ [NO_{2,i}]_{t-h} + hP'_{2,i} \end{bmatrix} \quad (6d)$$

233 In equations 6a and 6b, $\det(\mathbf{A})$ is the determinant of the 2 x 2 matrix \mathbf{A} , as defined in
 234 equation 6c.

235 The second set of equations are for the total concentrations of OH, HO₂, HONO and
 236 HNO₄, as shown in equations (7a) - (7d),

$$[OH]_t = [OH]_{t-h} + hP'_4 + hr_{4,5}[HO_2]_t + hr_{4,19}[HONO_{tot}]_t - hL_4[OH]_t \quad (7a)$$

$$\begin{aligned} [HO_2]_t = [HO_2]_{t-h} + hP'_5 + hr_{5,4}[OH]_t + hr_{5,21}[HONO_{4,tot}]_t - 2hk_{5,5}[HO_2]_t^2 \\ - hL'_5[HO_2]_t \end{aligned} \quad (7b)$$

$$[HONO_{tot}]_t = [HONO_{tot}]_{t-h} + hr_{19,4}[OH]_t - hL_{19}[HONO_{tot}]_t \quad (7c)$$

$$[HONO_{4,tot}]_t = [HONO_{4,tot}]_{t-h} + hr_{21,5}[HO_2]_t - hL_{21}[HONO_{4,tot}]_t \quad (7d)$$

237 The notations and symbols used in the above equations are similar to those used in
 238 equations (5a-5d). $r_{4,5}$ and $r_{4,19}$ are pseudo first order production rate coefficients of OH
 239 from HO₂ and HONO, respectively. $r_{5,4}$ and $r_{5,21}$ are pseudo first order production rate
 240 coefficients of HO₂ from OH and HNO₄, respectively. $k_{5,5}$ is the HO₂+HO₂ self-reaction
 241 rate coefficient. $r_{19,4}$ and $r_{21,5}$ are pseudo first order rate coefficients for the production of
 242 HONO and HNO₄ from OH+NO and HO₂+NO₂, respectively. The terms P'_4 and P'_5 account
 243 for all the remaining production of OH and HO₂ and the terms L_4 , L'_5 , L_{19} and L_{21} account
 244 for all the other losses of OH, HO₂, HONO and HNO₄, respectively. Analytical solutions for
 245 (7a)-(7d) were also derived and described in detail in Appendix A of Hertel et al. (1993). As
 246 the OH and HO₂ concentrations are determined, concentrations of individual HONO and
 247 HNO₄ from different sources are solved using equations 8a and 8b, respectively.

$$[HONO_i]_t = \frac{[HONO_i]_{t-h} + hr_{19,4}^i[OH]_t}{1 + hL_{19}} \quad (8a)$$

$$[HNO_{4,i}]_t = \frac{[HNO_{4,i}]_{t-h} + hr_{21,5}^i[HO_2]_t}{1 + hL_{21}} \quad (8b)$$

248 where $r_{19,4}^i$ and $r_{21,5}^i$ are pseudo first order production rate coefficients of HONO and
 249 HNO₄ from source i due to NO and NO₂ from the same source with OH and HO₂,
 250 respectively. Concentrations of NO_i and NO_{2,i} for the current timestep has already been

251 determined using (6a) and (6b) and these concentrations will be applied to calculate $r_{19,4}^i$
 252 and $r_{21,5}^i$ used in the above two equations.

253 The third set of equations are for C_2O_3 and PAN:

$$[C_2\text{O}_3]_t = [C_2\text{O}_3]_{t-h} + hP'_8 + hr_{8,9}[\text{PAN}_{tot}]_t - 2hk_{8,8}[C_2\text{O}_3]_t^2 - hL'_8[C_2\text{O}_3]_t \quad (9a)$$

$$[\text{PAN}_{tot}]_t = [\text{PAN}_{tot}]_{t-h} + hr_{9,8}[C_2\text{O}_3]_t - hL_9[\text{PAN}_{tot}]_t \quad (9b)$$

254 In the equation, it is assumed that in the source-oriented mechanism, PAN is a
 255 source-tagged species and its source is based on the source of NO_2 but C_2O_3 is not
 256 source-tagged. This is sufficient for the source apportionment of ozone, as described in the
 257 Section 2.1. A quadratic equation for C_2O_3 can be obtained (see Appendix A of Hertel et al.
 258 1993). In $r_{9,8}$, total NO_2 concentration at the current timestep t has already been determined
 259 by solving equations 5a-5d. Once the concentrations of C_2O_3 at timestep t is solved, the
 260 concentrations of each of the tagged PAN species can be used by:

$$[\text{PAN}_i]_t = \frac{[\text{PAN}_i]_{t-h} + hr_{9,8}^i[C_2\text{O}_3]_t}{1 + hL_{21}} \quad (9c)$$

261 where the $r_{9,8}^i$ includes the concentration of $\text{NO}_{2,i}$ (NO_2 attributed to source i) at the
 262 current timestep t .

263 The last set of equations treated specially in the source-oriented EBI solver is for NO_3
 264 and N_2O_5 , as shown in equations (10),

$$[\text{NO}_{3,tot}]_t = [\text{NO}_{3,tot}]_{t-h} + hP'_{15} + hr_{15,16}[\text{N}_2\text{O}_{5,tot}]_t - hL_{15}[\text{NO}_{3,tot}]_t \quad (10a)$$

$$[\text{N}_2\text{O}_{5,tot}]_t = [\text{N}_2\text{O}_{5,tot}]_{t-h} + hr_{16,15}[\text{NO}_{3,tot}]_t - hL_{16}[\text{N}_2\text{O}_{5,tot}]_t \quad (10b)$$

265 The two equations are linear equations and can be solved easily for $\text{NO}_{3,tot}$ and $\text{N}_2\text{O}_{5,tot}$.
 266 The $\text{NO}_{3,i}$ and $\text{N}_2\text{O}_{5,i}$ (as discussed in section 2.1, sources of N_2O_5 can be tracked with a
 267 single type instead of double typed) can be solved explicitly based on the two equations as
 268 well, as shown in (10c) and (10d),

$$[\text{NO}_{3,i}]_t = \frac{(1 + hL_{16})([\text{NO}_{3,i}]_{t-h} + hP_{15}^{i'}) + hr_{15,16}[\text{N}_2\text{O}_{5,tot}]_{t-h}}{(1 + hL_{15})(1 + hL_{16}) - h^2r_{15,16}r_{16,15}} \quad (10c)$$

$$[\text{N}_2\text{O}_{5,i}]_t = \frac{(1 + hL_{15})[\text{N}_2\text{O}_{5,tot}]_{t-h} + hr_{16,15}([\text{NO}_{3,i}]_{t-h} + hP_{15}^{i'})}{(1 + hL_{15})(1 + hL_{16}) - h^2r_{15,16}r_{16,15}} \quad (10d)$$

269 Note that $r_{16,15}$ includes the total concentration of NO_2 thus is the same as that used in
 270 equation (10b). However, the production of NO_3 from other reactions do have to be source
 271 specific, thus the $P15$'s used in equation (10c) and (10a) are different.

272 To avoid round-off errors introduced in the calculation for the source-tagged species
273 involved in these four groups so that the sum of the source-tagged species always exactly
274 matches the total concentrations, their concentrations are readjusted by the pre-determined
275 total concentrations of photochemical species by solving the algebraic equations for the
276 special groups in original EBI scheme.

277 2.2.2 Successive under-relaxation

278 During the testing of the above algorithm, non-converging oscillations were sometimes
279 observed, mostly due to the low concentration source-tagged species. The iterative process
280 used in the EBI solver can be modified to include a relaxation factor α so that the
281 concentration array at end of each iteration is updated by a weighted average of the results
282 from the previous iteration and the present estimated values in the current iteration, as shown
283 in equation (11),

$$C_{i,iter}^{update} = (1 - \alpha)C_{i,iter-1} + \alpha C_{i,iter} \quad (11)$$

284 The selection of α influences only the number of iterations required for convergence, not
285 the final converged solutions. Generally, larger α values lead to faster convergence, but have
286 a higher chance of fall into oscillation. Based on the testing, $\alpha=0.8$ appears to be a
287 conservative choice that always lead to convergence. A dynamic under-relaxation scheme
288 using a set of varying α values between 0.79 and 1.0, based on the number of iterations in the
289 EBI scheme, is shown to lead to faster convergence. This is further discussed in the Results
290 section.

291 2.3 Test mechanism and model set up

292 To evaluate how much improvement in computation efficiency can be achieved by using
293 the simplified reaction representation and source-oriented EBI solver, a series of
294 source-oriented mechanisms for simultaneous attribution of ozone and secondary inorganic
295 aerosol were constructed based on the SAPRC-07 photochemical mechanism and
296 implemented in CMAQv5.2. The SAPRC-07 mechanism is chosen instead of the more
297 recent versions of SAPRC because it is faster with fewer species and reactions, and thus is
298 more suitable for simulations requiring rapid response, such as operational air quality
299 forecasting and for source apportionment of ozone and secondary inorganic aerosols. The
300 source-oriented mechanism based on this will be applied in a future air quality forecasting
301 model that also forecasts source-tagged species concentrations and source-region

302 contributions to air pollution. As the primary goal of this paper is to evaluate the efficiency
303 of the gas phase algorithm, aerosol results despite being enabled in the simulations along
304 with cloud processes, are not included in the analyses described below.

305 The tested SAPRC-07 mechanism used in this study contains a total of 134 species and
306 341 reactions. Among these species, 15 species are reactive nitrogen species. For each of
307 these species, tagged species are used to track their source origins. Reactions involving these
308 species are expanded in the source-oriented mechanism. In addition, CO, SO₂ and sulfuric
309 acid (SULF) were also expanded in the source-oriented mechanism. To evaluate source
310 contributions to ozone, 14 primary VOC species were also treated as source-oriented species
311 in addition to source-tagged non-reactive O₃ species to track contributions from different
312 sources of NO_x and VOCs to O₃ formation. As HCHO is an important oxidation product
313 from several parent VOCs, sources of secondary HCHO from the first-generation oxidation
314 of parent VOCs are also tracked. Details for the source apportionment of O₃ has been
315 described by Wang et al. (Wang et al., 2019a; Wang et al., 2019b) and are not repeated here.
316 While the dual-tagged reaction reduction method and EBI solver for source apportionment
317 can be employed separately, their integration could lead to enhanced performance and more
318 significant benefits. Two versions of the source-oriented SAPRC-07 mechanisms are
319 prepared. The first version (V1) uses double-tagged N₂O₅ and fully expanded reactions
320 without the pseudo first order reactions described in Section 2. The second version (V2) is
321 based on single-tagged N₂O₅ and a dual-tagged reaction reduction treatment applied to fully
322 expanded V1 mechanism, as described in section 2.1. Both mechanisms are constructed to
323 track emissions from ten source types. The number of reactions and species in each
324 mechanism is listed in Table 1, while the accuracy of this method is presented in the Fig A1
325 which shows the EBI solver's predictions scattered on the 1:1 line when compared to the
326 SMVGEAR solver's results for of tagged species concentrations and the total.
327

328 **Table 1.** Computation time needed for a one-day simulation using two different versions of
329 the source-oriented chemical mechanism and two versions of the ODE solvers. Both versions
330 are capable of tracking 10 different source types in a single simulation.

	V0 SMV GEAR	V1 SMV GEAR	V2 SMVGEAR				V2 EBI*
Domain resolution	36km	36km	36km	12km	4km	36km	12km
# of total species	134	512				422	

# of total reactions	341	2845	1376					
Total Chem. Time [^] (min)	15.3	133	49.9	172	325	13.4	15.1	32.4
Total Sim. Time [^] (hr)	0.419	2.63	1.32	3.59	6.27	0.707	1.03	1.63
Chem %	61%	84%	63%	80%	86%	32%	24%	33%
Chem time reduction wrt. V1	12%	-	62%	-	-	90%	-	-
Chem time reduction wrt. V2-S [#]	31%	-	-	-	-	73%	91%	90%
Total time reduction wrt. V1	16%	-	50%	-	-	73%	-	-
Total time reduction wrt. V2-S [#]	32%	-	-	-	-	46%	71%	74%

* Simulation time based on dynamic under-relaxation coefficient.

[^] Wall-clock time

V2-S: V2 reaction mechanism with SMVGEAR solver

The testing is performed for **totally threedays** (July 1-3, 2020) simulation using three-level nested domains with 36, 12 and 4 km resolutions that cover eastern Asia, central and eastern China and Henan province in central China, respectively. The meteorology inputs were based on the Weather Research and Forecasting (WRF) model v4.1.4. The anthropogenic emissions are based on the Multi-scale Emission Inventory for China (MEIC, for 36- and 12-km domains, available from <http://www.meicmodel.org/>) and a local emission inventory (for the 4-km domain) (Lu et al., 2023). Emissions are grouped into six source categories, including five anthropogenic source sectors (power, industrial, residential, transportation and agriculture) and one biogenic sector, whose emission is based on MEGAN (Model for Emissions of Gases and Aerosol from Nature) v2.1(Guenther et al., 2006). The initial concentrations of the species **are** based on a 7-day non-source-oriented simulation. The boundary conditions for the 36-km domain are based on the clean continental vertical profiles included in the CMAQ model. The boundary conditions of the 12- and 4-km domains are based on results from the parent 36- and 12-km domains, respectively.

Three sets of simulations using the source-oriented mechanisms were conducted: (1) V1, solved using the SMVGEAR solver (V1-GEAR), (2) V2, solved with SMVGEAR (V2-GEAR), and (3) V2, solved with source-oriented EBI (V2-EBI). For the SMVGEAR a relative tolerance (RTOL) of 1×10^{-3} and an absolute tolerance (ATOL) of 1×10^{-9} were used. For the EBI solver, only a relative tolerance RTOL was used in the convergence check. For most of the species, a relative tolerance of 1×10^{-3} is used. **Exceptions include** pseudo-steady-state species like O_3P and O_1D , for which a tolerance of 1.0 is applied, indicating that a convergence check is not applicable. Integrate reaction rate analysis (IRR) was used in all three simulations as it is needed for the ozone source apportionment algorithm (Wang et al., 2019b). In addition to the source-oriented model simulations, two

358 base case simulations (V0) were conducted using the unmodified SAPRC-07 solved with the
359 SMVGEAR (V0-SMVGEAR) and the EBI (V0-EBI) solvers, respectively.

360 All the simulations were conducted on a Dell Precision-Tower 7810 working station
361 (2XE-2660-v4, 28/56 cores/threads and 256G of DDR4 RAM), and the run was in parallel in
362 a configuration of 8x6 domain decomposition.

363 3 Results

364 3.1 Timing results

365 For the first day simulation, the wall-clock time for the gas-phase mechanism as well as
366 the total run time were recorded using the time function (MPI_WTIME) in the Message
367 Passing Interface. An MPI_BARRIER call was issued before each MPI_TIME call to make
368 sure that the measured wall-clock time represents the actual time for a chemistry time step in
369 a static domain decomposition setting with imbalanced loads.

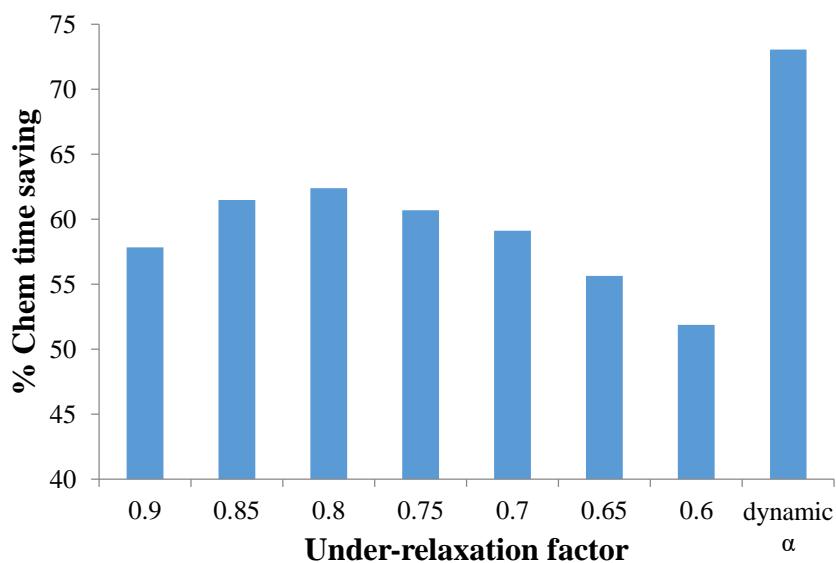
370 The wall-clock time for the 1-day simulation using the source-oriented EBI solver was
371 compared with the SMVGEAR solver and presented in Table 1. The fully expanded
372 source-oriented mechanism with SMVGEAR (V1-SMVGEAR) is the slowest and only the
373 36-km resolution simulation was conducted. The simplified reaction representation alone
374 (V2-SMVGEAR, see section 2.1) leads to a reduction of chemistry time by ~62% when
375 compared with V1-SMVGEAR (133 min to 49.9 min) and total computation time by ~50%
376 (2.63 hr to 1.32 hr). Using the source-oriented EBI on V2 (V2-EBI) further reduces the total
377 chemistry time to ~13.4 min. Compared to the V1-SMVGEAR, V2-EBI reduced the
378 chemistry time by ~90% and the total time by ~75% for a one-day simulation in the 36 km
379 domain.

380 The EBI solver represents a significant reduction in both chemistry time and total
381 computation time comparing to the SMVGEAR solver. When V2-SMVGEAR and V2-EBI
382 are compared, the total chemistry time saving of EBI scheme increases with grid resolution,
383 from 70% for 36 km to 90% for 4 km grid. As the result, the total simulation time saving
384 increases from 46% for coarse grid to 74% for fine grid. The increase in time saving of EBI
385 solver with grid resolution can be attributed to the smaller time step size which is determined
386 from flow Courant stability criterion. For fixed simulation duration (1 day in this study), the
387 required total number of chemistry steps increase dramatically with grid resolution, this
388 results in significant reduction in total chemistry time with faster EBI scheme for finer grid.

389 Therefore, for time consuming applications such as long-term source apportionment
390 simulation of nested domains, the time efficiency can be improved by a factor of 3 or more.

391 The V2-EBI results shown in Table 1 are based on the dynamic under-relaxation using an
392 iteration count dependent under-relaxation factor (α) as shown in Table 2. In this scheme, the
393 α is initially set to 1.0 and gradually decreases to smaller values. If the solution does not
394 converge in 15 iterations, a constant α of 0.79 is used. Using this dynamic α scheme is
395 demonstrated to be more efficient than using a constant α , as shown in Figure 2. The optimal
396 value for fixed α is 0.8, at which the total chemistry time could be reduced by ~62%, for the
397 36-km domain simulations, which is approximately 10% less than the dynamic α scheme.

398 The values of α in Table 2 have been fine-tuned through a series of numerical
399 experiments to optimize the convergence rate for the source-oriented chemical mechanisms.
400 The dynamic α strategy demonstrates superior performance due to its adaptive nature. Larger
401 α values in the early iterations aggressively propel a rapid movement of the solution vector
402 towards the region of the true solution. Subsequently, in later iterations, a more conservative
403 approach with smaller α values is adopted to gradually refine the solution and effectively
404 damp out residual errors and potential overshoots by assigning more weight to the solution
405 of the previous iteration. For the majority of test cases, convergence is achieved within 10
406 iterations. However, in instances where convergence is slower, a slightly larger adjustment
407 step (an increase in α between iterations 11 and 15) can be beneficial for a faster approach to
408 the true solution. The ultimate value of $\alpha=0.79$ is aimed at effectively damping oscillations in
409 the final convergence stages.



411 **Fig. 2.** Percentage chemistry time saving with respect to the SMVGEAR solver for the

412 identical source-oriented chemical mechanism for the 36-km domain. See text and Table 2
413 for the details of the dynamic under-relaxation scheme.
414

Table 2. Dynamic section of the under-relaxation factor (α) based on iteration count

Iteration #	α	Iteration #	α
1	1.00	6-7	0.79
2	0.90	8-10	0.78
3	0.85	11-15	0.80
4-5	0.81	≥ 16	0.79

3.2 Accuracy assessment of the source oriented EBI solver

The general accuracy of the EBI solver has been tested by Hertel et al. (1993). The accuracy of proposed source apportionment EBI scheme was evaluated by comparing predicted results with those from the SMVGEAR. For this comparison, results from the first day of a three-day simulation are primarily presented to highlight the maximum potential discrepancy between EBI and SMVGEAR.

Hourly average concentrations of the three days NO, NO₂, PAN, HONO, OH and HO₂ at each grid cell in the surface layer of the 36-km domain were selected as sample species for accuracy evaluation because these are the important species treated specially by the source oriented EBI solver. The maximum and mean values of the normalized error of these species are listed in the Table 3. For all these species, the maximum normalized error among all grid cells is less than 15% and the mean normalized error does not exceed 4% on the first day. Subsequently, the error gradually decays over the following two days, reaching an order of magnitude of 0.1% to 1% by the third day. This indicates that the accuracy of the source-oriented EBI scheme is acceptable, as the errors are anticipated to diminish further with increasing flow time.

Table 3. Max and mean values of the normalized error* for selected species in the 36-km domain.

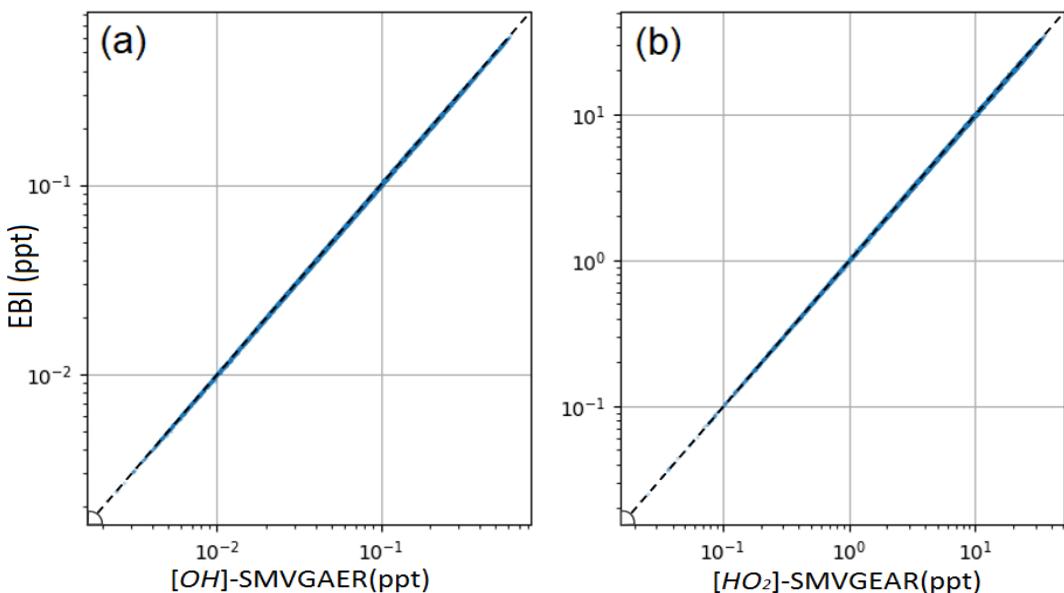
Species	Max Normalized error (%)			Mean Normalized Error (%)		
	Day1	Day2	Day3	Day1	Day2	Day3
NO_X0	14.45	8.83	6.49	3.01	2.41	1.14
NO_X1	5.92	3.49	1.85	2.50	1.41	1.05
NO_X2	2.30	1.48	1.05	0.0059	0.0034	0.0025
NO_X3	6.29	3.62	1.70	1.35	0.72	0.45
NO_X4	5.64	2.00	1.33	0.59	0.36	0.19
NO_X5	0.99	0.40	0.24	0.57	0.31	0.17

NO_X6	1.35	0.65	0.32	0.41	0.24	0.15
NO ₂ _X0	12.69	9.16	6.49	3.99	2.34	1.26
NO ₂ _X1	11.89	7.06	5.61	3.37	1.19	0.86
NO ₂ _X2	9.59	5.99	3.84	3.14	1.99	1.24
NO ₂ _X3	8.26	5.89	3.51	2.19	1.27	0.72
NO ₂ _X4	11.34	6.25	3.04	1.17	0.68	0.33
NO ₂ _X5	5.47	3.77	2.56	1.46	0.71	0.34
NO ₂ _X6	4.70	2.78	2.19	0.83	0.48	0.24
PAN_X0	9.52	6.41	4.29	2.98	1.73	1.04
PAN_X1	6.42	3.26	2.59	1.09	0.66	0.31
PAN_X2	8.12	5.76	3.64	1.23	0.75	0.48
PAN_X3	6.22	4.43	2.96	0.96	0.60	0.29
PAN_X4	5.99	2.96	1.60	0.75	0.58	0.37
PAN_X5	2.08	1.56	1.01	0.014	0.0074	0.0023
PAN_X6	4.97	2.25	1.33	0.672	0.41	0.24
HONO_X0	4.55	2.21	1.34	1.52	1.13	0.93
HONO_X1	5.49	2.42	1.87	1.13	0.79	0.42
HONO_X2	4.36	2.78	1.39	0.65	0.38	0.21
HONO_X3	13.89	9.64	6.05	1.56	0.81	0.32
HONO_X4	8.56	5.14	3.30	1.61	0.87	0.48
HONO_X5	7.41	4.78	2.39	1.32	0.95	0.64
HONO_X6	5.73	3.67	1.95	0.41	0.27	0.071
OH	3.01	2.17	1.61	0.19	0.129	0.064
HO ₂	5.15	3.73	2.36	0.35	0.17	0.053

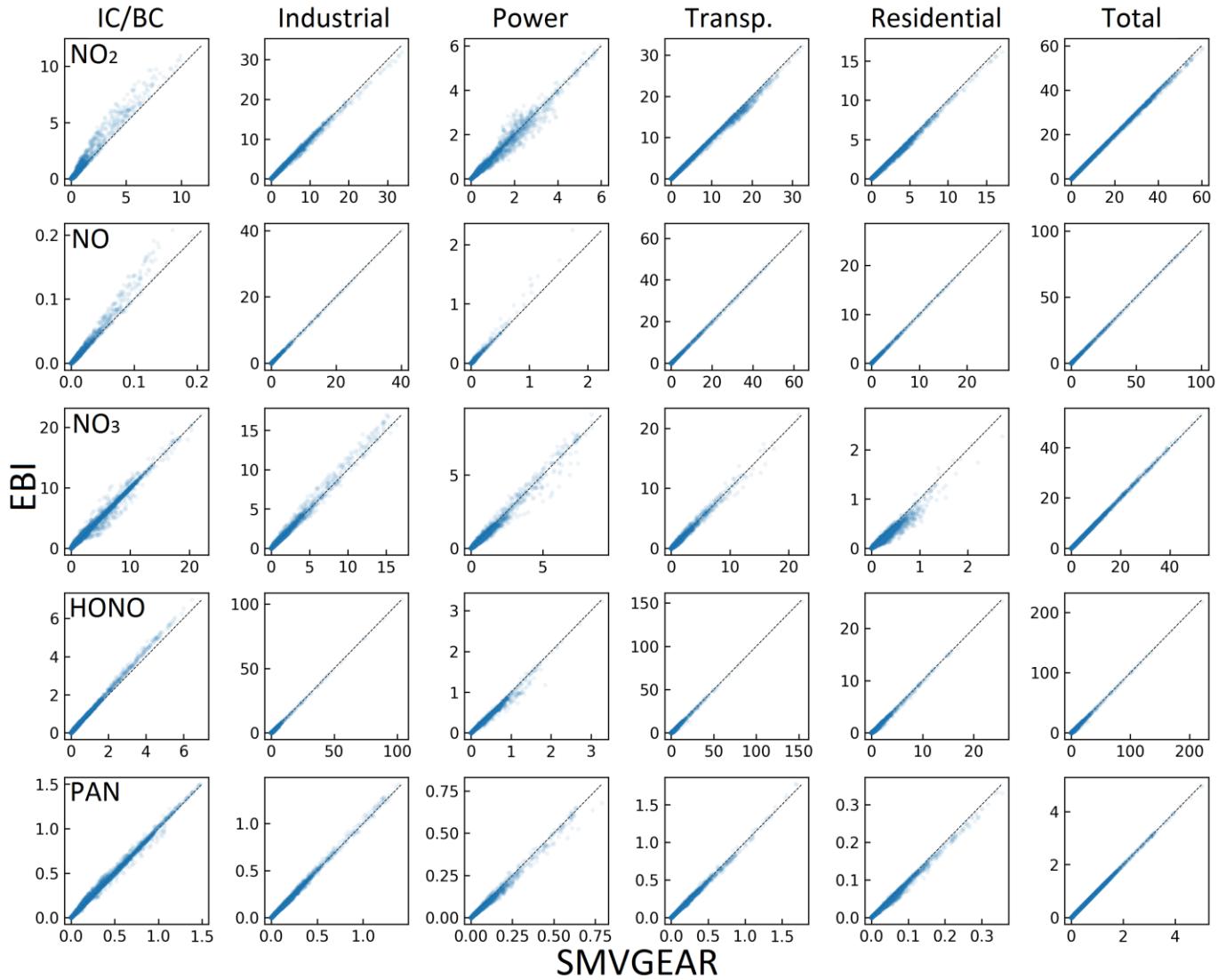
* Normalized error is calculated as $|C_{V2-EBI} - C_{V2-SMVGEAR}| / C_{V2_SMVGEAR}$. This is calculated for hourly concentrations for all the grid cells in the entire day. The mean normalized error is calculated by averaging the normalized error for all the grid cells.

For OH and HO₂, the EBI solver agrees with the SMVGEAR solver very well, with mean differences of $\sim 0.19\%$ and 0.35% , respectively. This indicates that the overall gas-phase chemistry is not significantly influenced by replacing the SMVGEAR solver with the EBI scheme. Figure 3 shows the comparison of all the hourly concentrations of OH and HO₂ at hour 6, which represent average concentrations between 1400-1500 local time. The EBI results agree with SMVGEAR solver results across all concentration ranges that span more than three orders of magnitude. Figure 4 shows the comparison of hourly concentrations of NO₂, NO, NO₃, HONO and PAN for hour 24 of day-1, with day-2 and day-3 results presented in Figures A2 and A3 respectively. For the total concentrations these species (i.e. sum of the concentrations of the source-tagged species), the source-oriented EBI predictions agree very well with the observations. For the individual source-tagged species, differences between the EBI and SMVGEAR results are highest among the NO₂, NO and HONO species that are used to track the initial and boundary contributions (IC/BC type, first

450 column of Figure 4). The results shown in the plots are essentially initial concentrations
 451 because boundary conditions for NO and NO₂ are quite low, and won't contribute to such
 452 high concentrations (see Figure 6 – the regional NO₂ and NO plots). The source-oriented EBI
 453 predictions for IC/BC species are biased high, while those for other tagged species are biased
 454 low, compared to those predicted using the Gear solver. The observed discrepancies arise
 455 because the dynamic stage solution predicted by the Eulerian backward scheme is generally
 456 slower, exhibiting a significant time lag. In this test, the IC/BC species inherited a
 457 concentration field from a 7-day non-source-oriented simulation, while the other tagged
 458 species started from near-zero concentrations. Consequently, the decay of IC/BC species
 459 (with no emission associated) was overestimated, while the accumulation of other tagged
 460 species (associated with emissions) was underestimated. These over-predictions are balanced
 461 by the general under-predictions of EBI for species associated with emissions, resulting in
 462 close agreement in total concentrations. Furthermore, the errors associated with the EBI
 463 prediction exhibit a decreasing trend with advancing flow time and diminish to within a
 464 tolerable range (on the order of 0.1%-1%) by the end of day 3. This phenomenon suggests
 465 that species concentrations asymptotically approach a new steady state dictated by external
 466 inputs, with emission intensity being the primary factor.



467
 468 **Fig. 3.** Comparison of predicted OH (a) and HO₂ (b) radicals in the surface layer of the
 469 36-km resolution domain using the source-oriented EBI(new) and the SMVGEAR(baseline)
 470 solvers. Concentrations at hour 6 (local time 1400-1500) are shown.



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Fig. 4. Predicted hourly-averaged NO_2 , NO , NO_3 , HONO and PAN concentrations for different source types in the last hour of the **day-1** simulation using source-oriented EBI(**new**) and the SMVGEAR(**baseline**) solver. Concentrations of all grid cells in the surface layer are included in the plot. Concentrations are in units of ppb for NO , NO_2 and PAN and in units of ppt for NO_3 and HONO .

Figure 5 shows the predicted hourly time series of OH , HONO , NO_2 and PAN at five grid cells that represent no emission, and low, medium, high and intense emission conditions, respectively. For HONO , NO_2 and PAN , predicted concentrations are shown for IC/BC type and the sum of the tagged species for other emission-related types. This again demonstrates that predicted OH from EBI and SMVGEAR agree with each other at all times under all emission conditions. The fraction of initial concentrations to the total concentration decreases as the emission intensity increases and the difference between EBI and SMVGEAR remains very small.

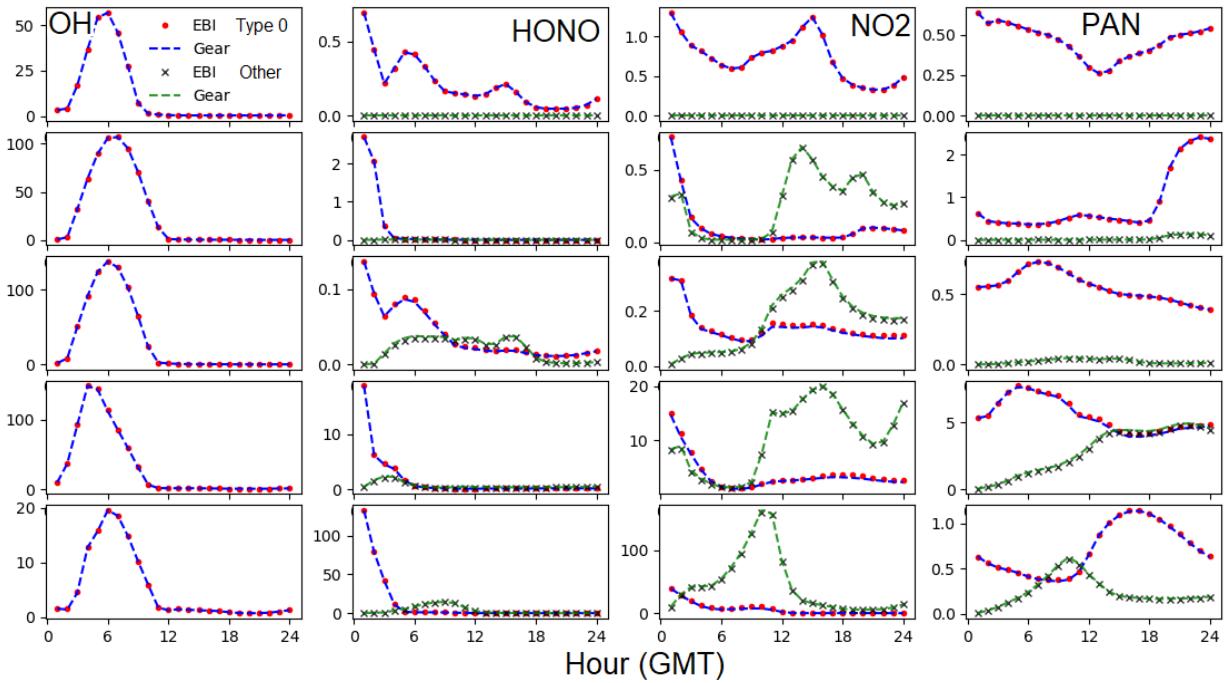


Fig. 5. Timeseries of OH, HONO, NO₂ and PAN at grid cells that represent different emission conditions (no emissions, low emission, medium emission, high emission and intense emission) for the one-day simulation. Units are ppq (parts per quadrillion) for OH, ppt for HONO and 0.1 ppb for NO₂ and PAN. Type 0 is the concentration for the IC/BC source type and “other” represents the sum of the concentrations of all other tagged species.

Figures 6 and 7 illustrate the spatial distribution of daily average concentrations predicted from two methods. HO, HO₂ and two tagged NO, NO₂ and HONO of 36 km grid were selected as sample species, results from proposed EBI scheme are very close to the SMVGEAR results. The concentration of X0 species predicted by EBI scheme at high concentration are higher than the Gear, the reason of such slight deviation is mainly caused by the aforementioned time lag in results of Euler backward scheme.

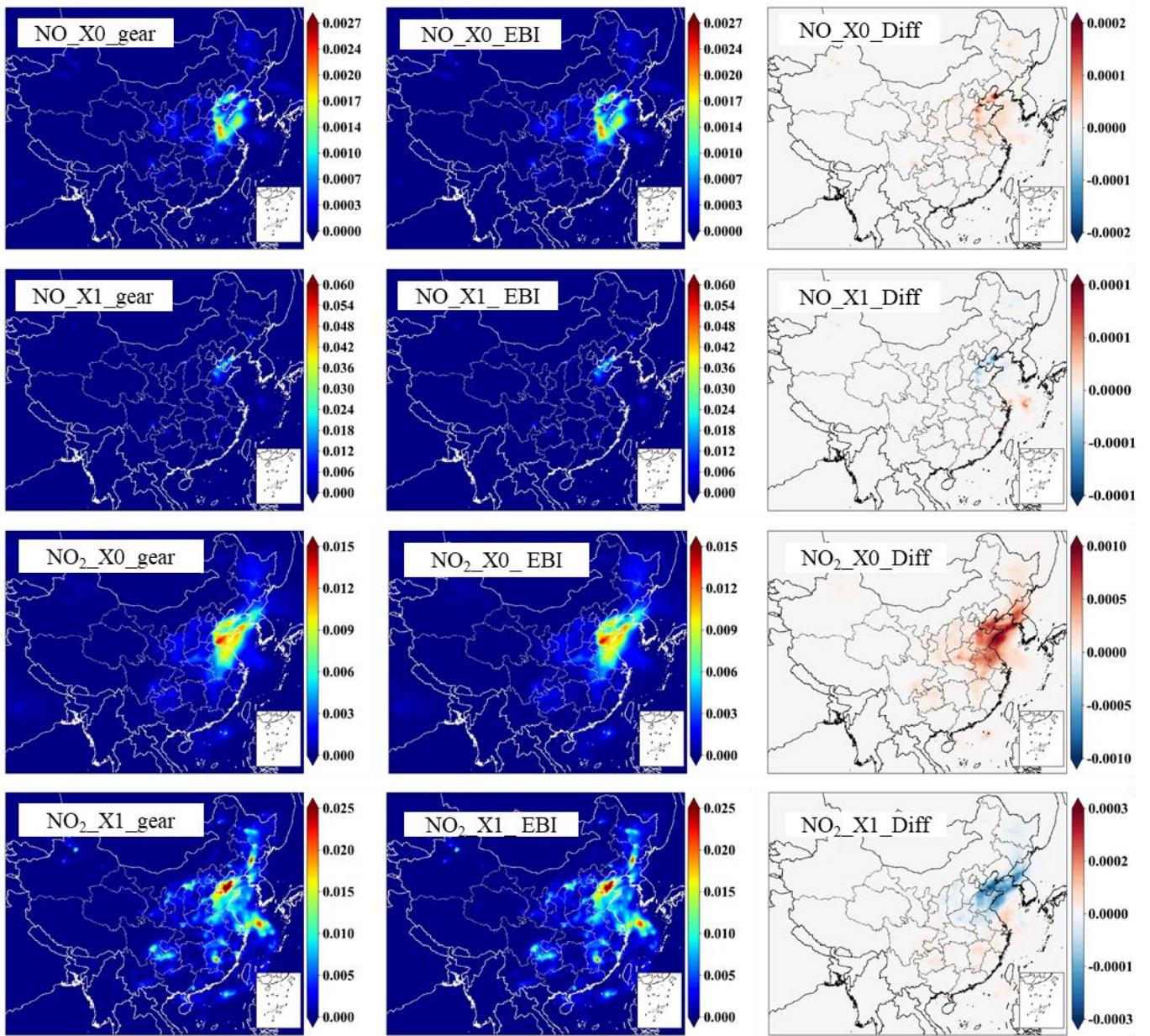
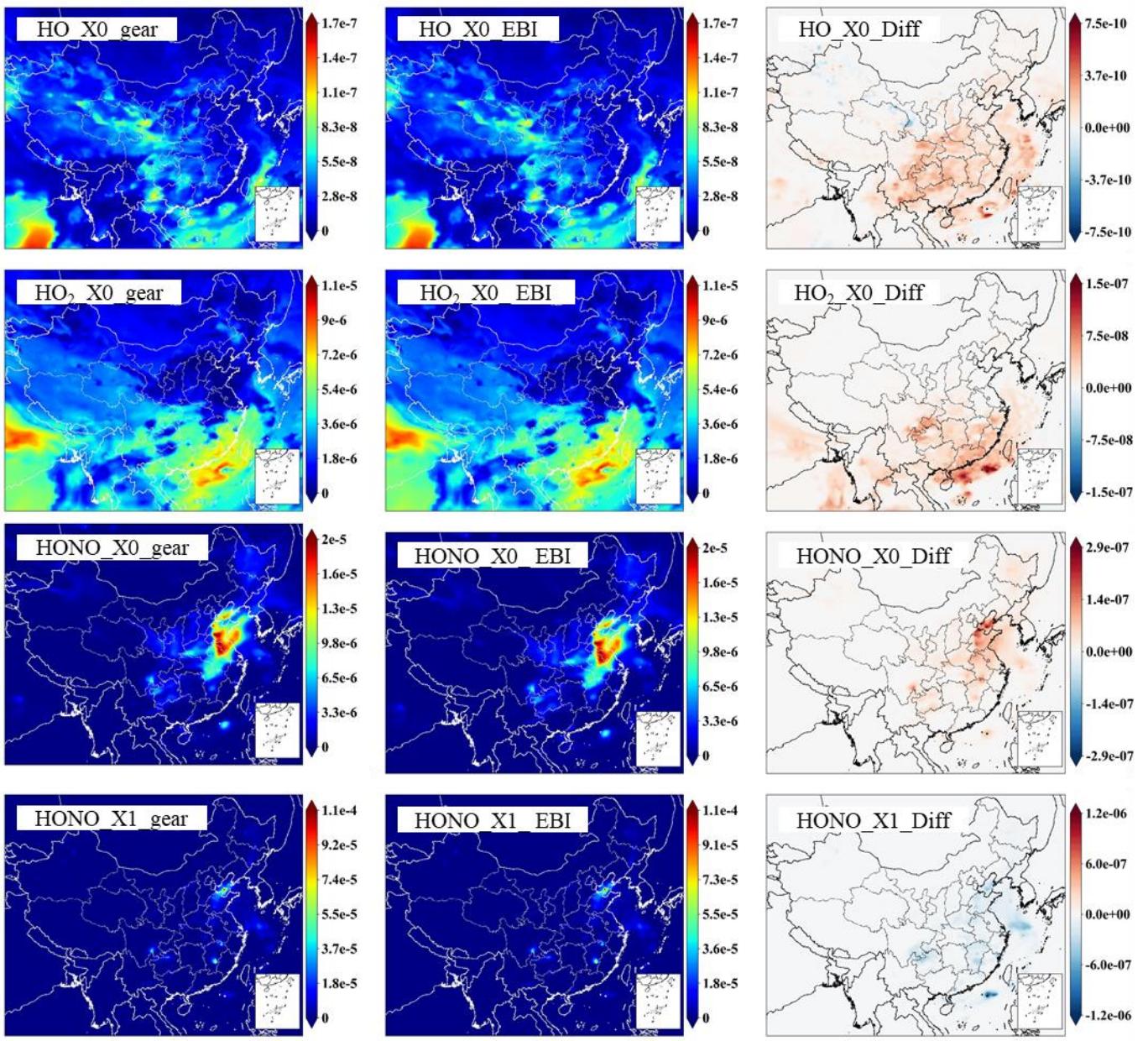


Fig. 6. Regional distribution of NO and NO_2 from a day-1 CMAQ simulation with 36km resolution and meteorological fields from WRFv4.1.4, units are ppm.



501
502 **Fig. 7.** Regional distribution of HO HO₂ and HONO from a day-1 CMAQ simulation with
503 160X130 36km resolution domain and meteorological fields from WRFv4.1.4, units are
504 ppm.
505

4 Conclusions

506 In this study, the computation efficiency and thus scalability of the source-oriented
507 approach is greatly improved with a new approach of dealing with these two-tagged-species
508 reactions. The new approach is based on tracking the total concentration of the source-tagged
509 species and reduce the n^2 number of second-order reactions into $2n$ pseudo first-order
510 reactions for chemical system with n sources, this method preserves individual species'
511 production and loss rates, thus leading to improved computational efficiency because the
512 total number of reactions increases linearly with the source number. Additionally, Euler
513 Backward Iterative (EBI) solver has been successfully implemented to the source-oriented
514 mechanism, with average of absolute relative error is below 5% and up to 90% chemistry

515 time reduction in comparison to SMVGEAR. While efficient source-oriented approach for
516 primary particles are already available to track a large number of sources simultaneously, the
517 efficient approach developed in this study has the potential to track a large number of sources
518 to evaluate their impact on secondary pollutant formation, and has the potential to be applied
519 in air quality forecasting models that provide source or source-region contribution
520 information for policy makers for better emission regulations under meteorological
521 conditions that exacerbate pollution.

522 **Data availability.** Data can be obtained upon request from the authors.

523 **Authorship contributions.** QX: Investigation, Methodology, Software, Visualization,
524 Writing & editing. ZJ: Visualization, Writing - review & editing. QY: Software, Data
525 curation, Validation, Visualization, Writing - review & editing. KW: Supervision, Writing -
526 review & editing, Visualization. FS: Writing, Investigation, Conceptualization, Methodology,
527 Software, Validation. RZ: Resources, Supervision, Visualization. M. K: Software,
528 Methodology.

529 **Declaration of competing interest.** The contact author has declared that neither they nor
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Appendix

List A1. The special reaction rate section (between the keywords SPECIAL and END SPECIAL) and reactions used to implement the source-oriented $\text{NO}_2 + \text{NO}_3$ reactions in dual-tagged reaction reduction, with 10 source types using the chemical mechanism preprocessor CHEMMECH for the CMAQ model. Due to the limitation of the current mechanism preprocessor, a dummy reaction (<10_dum>) is needed so that the original reaction rate can be included in calculation of the special rate constants. Using the special rate expression is signaled by including the symbol ‘?’ in the reaction rate coefficient expression.

```
SPECIAL =
RNO_NO3 = K<10_dum>*C<NO3> + K<10_dum>*C<NO3_X1> +
<10_dum>*C<NO3_X2>
    + K<10_dum>*C<NO3_X3> + K<10_dum>*C<NO3_X4> +
K<10_dum>*C<NO3_X5>
    + K<10_dum>*C<NO3_X6> + K<10_dum>*C<NO3_X7> +
K<10_dum>*C<NO3_X8>
    + K<10_dum>*C<NO3_X9>;
RNO3_NO = K<10_dum>*C<NO> + K<10_dum>*C<NO_X1> +
K<10_dum>*C<NO_X2> +
    K<10_dum>*C<NO_X3> + K<10_dum>*C<NO_X4> +
K<10_dum>*C<NO_X5> +
    K<10_dum>*C<NO_X6> + K<10_dum>*C<NO_X7> +
K<10_dum>*C<NO_X8> +
    K<10_dum>*C<NO_X9>;
...
END SPECIAL
...
<10_dum> dummy1 + dummy1 = dummy1 + dummy1 #1.80e-11@-110;
<10_ax0> NO = NO2 #1.0?RNO_NO3;
<10_ax1> NO_X1 = NO2_X1 #1.0?RNO_NO3;
<10_ax2> NO_X2 = NO2_X2 #1.0?RNO_NO3;
...
<10_ax9> NO_X9 = NO2_X9 #1.0?RNO_NO3;
<10_bx0> NO3 = NO2 #1.0?RNO3_NO;
<10_bx1> NO3_X1 = NO2_X1 #1.0?RNO3_NO;
<10_bx2> NO3_X2 = NO2_X2 #1.0?RNO3_NO;
...
<10_bx9> NO3_X9 = NO2_X9 #1.0?RNO3_NO;
```

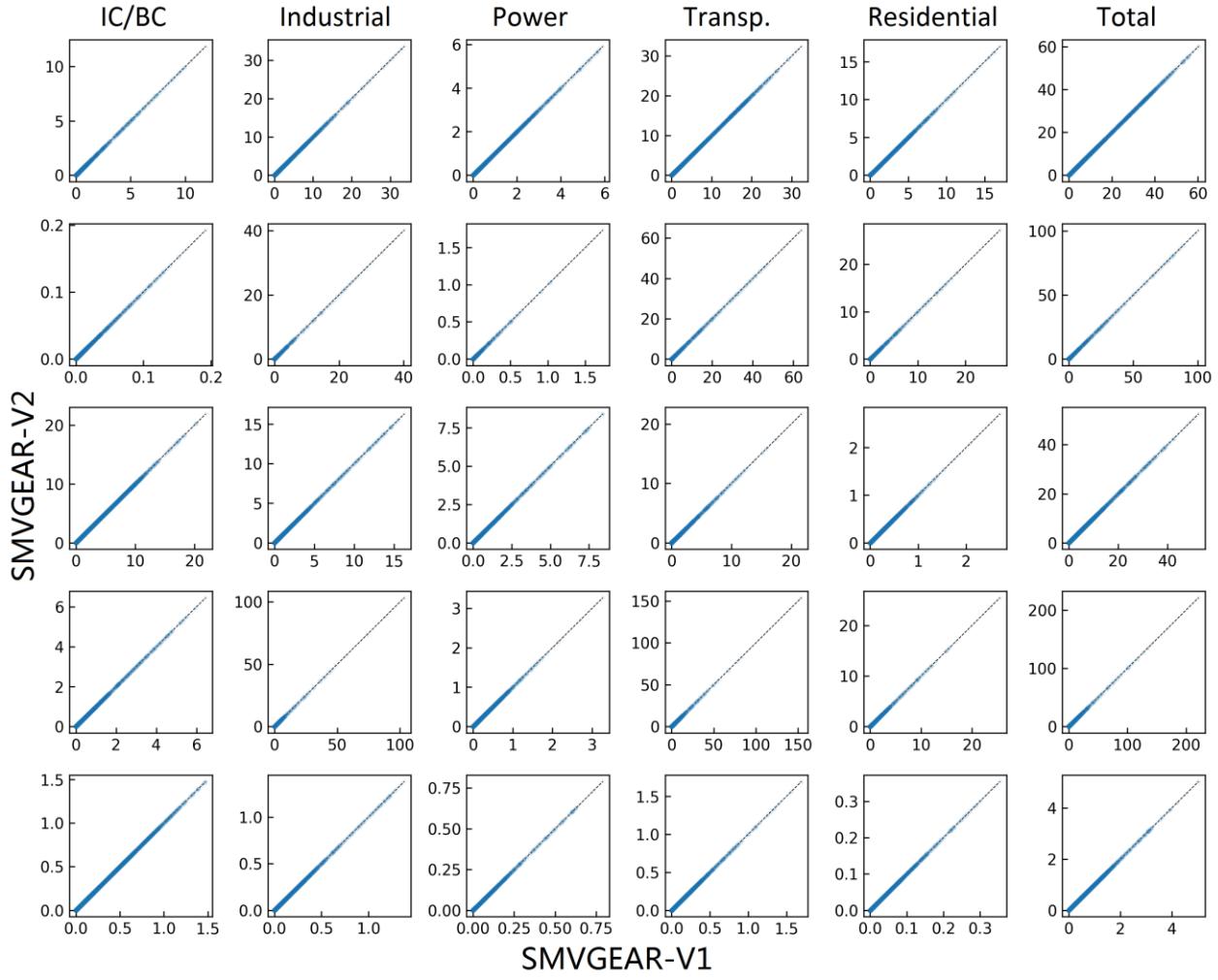
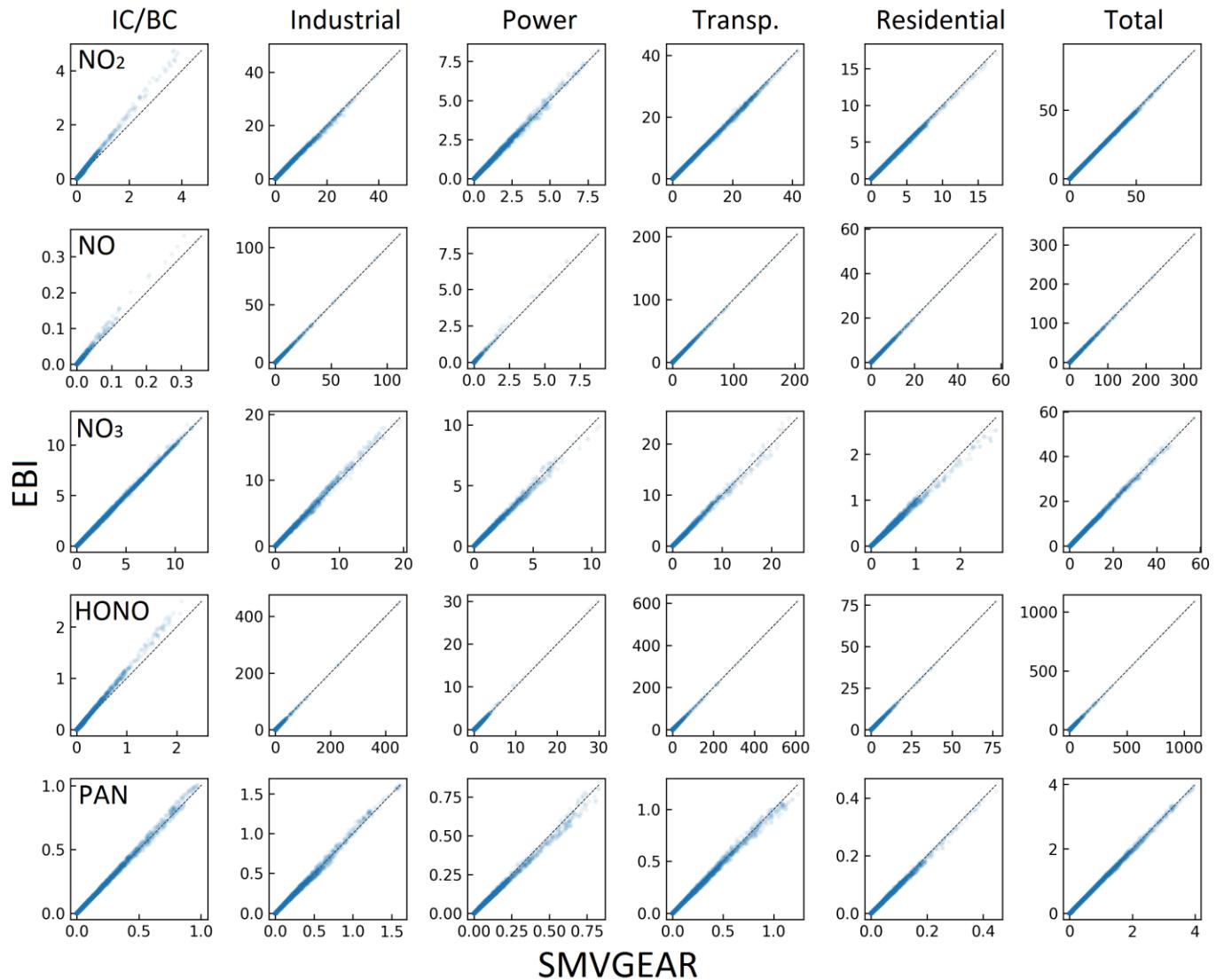
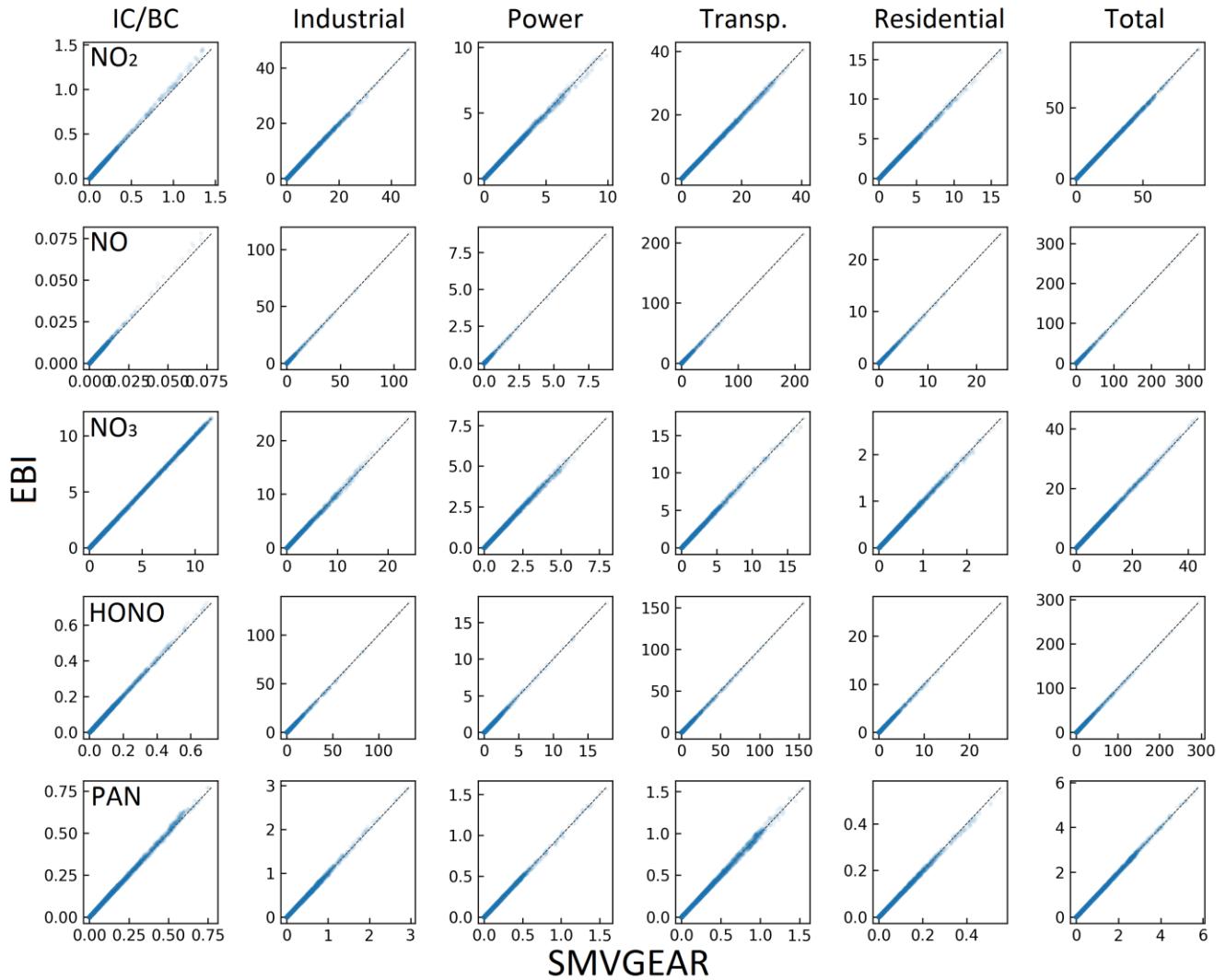


Fig. A1. Predicted hourly-averaged NO_2 , NO , NO_3 , HONO and PAN concentrations for different source types in the last hour of the day-1 simulation using SMVGEAR solver with fully expanded source-oriented (V1) and the dual-tagged reaction reduction method (V2) mechanism. Concentrations of all grid cells in the surface layer of 160X130 36km resolution domain are included in the plot. Concentrations are in units of ppb for NO , NO_2 and PAN and in units of ppt for NO_3 and HONO .



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Fig. A2. Predicted hourly-averaged NO₂, NO, NO₃, HONO and PAN concentrations for different source types in the last hour of the day-2 simulation using source-oriented EBI(new) and the SMVGEAR(baseline) solver. Concentrations of all grid cells in the surface layer of 160X130 36km resolution domain are included in the plot. Concentrations are in units of ppb for NO, NO₂ and PAN and in units of ppt for NO₃ and HONO.



620
621 **Fig. A3.** Predicted hourly-averaged NO₂, NO, NO₃, HONO and PAN concentrations for
622 different source types in the last hour of the day-3 simulation using source-oriented EBI(new)
623 and the SMVGEAR(baseline) solver. Concentrations of all grid cells in the surface layer of
624 160X130 36km resolution domain are included in the plot. Concentrations are in units of ppb
625 for NO, NO₂ and PAN and in units of ppt for NO₃ and HONO.