# Referee Report — Minor Revision

Title: Retrieving Atmospheric Thermodynamic and Hydrometeor Profiles Using a Thermodynamic-Constrained Kalman Filter 1D-Var Framework Based on Ground-based Microwave Radiometer

## Summary statement

This manuscript presents a novel Thermodynamic-Constrained Kalman Filter 1D-Var (TCKF1D-Var) framework to retrieve atmospheric profiles from ground-based microwave radiometer data. The method introduces a physically consistent approach using virtual potential temperature as the control variable, a ratio-based cost function, and a diagnostic microphysics closure.

The paper is clearly written and technically sound, addressing an important gap in cloudy-condition retrievals. Validation with radiosonde and EarthCARE data convincingly shows improved accuracy over conventional 1D-Var and ERA5. I find the study well-suited for Geoscientific Model Development and recommend minor revision before acceptance.

## General comments

The manuscript meets GMD's standards for methodological rigor and relevance. The proposed framework is innovative and the results are promising. However, several clarifications and small improvements are necessary to enhance transparency, reproducibility, and interpretability.

The authors should particularly clarify the theoretical justification of the ratio-based cost function, provide more quantitative evidence in the discussion of mid-tropospheric humidity degradation, and improve data/code availability in line with GMD policy.

## Specific comments

1. Clarification of the ratio-based cost function (Eq. 4)
   The use of a dimensionless, ratio-based cost function is an interesting innovation. Please elaborate on:
   a) How subtracting unity affects numerical stability and convergence behavior.
   b) Whether normalization issues arise when observed or simulated brightness temperatures approach zero.
   c) A brief comparison with the conventional covariance-weighted formulation.

2. Uncertainty quantification and statistical significance
   The performance metrics (bias, RMSE) are presented without uncertainty ranges. Please include standard deviations or confidence intervals, or indicate whether improvements are statistically significant.

3. Microphysics parameterization and coupling
   The coupling between the WSM3 single-moment microphysics scheme and the thermodynamic constraint is not entirely clear. Please expand on how liquid/ice water contents influence the state vector and cost function. A schematic or equation would be helpful.

4. Limited EarthCARE validation sample
   Validation is based only on July 2025 data (around 60 collocated profiles). Please explicitly acknowledge this limitation and discuss whether the conclusions may vary with season or location.

5. Figure readability and accessibility
   The font size in Figures 4–6 is rather small. Please adjust the figure layout so that all symbols, units, and legends are clearly readable and distinguishable.

**Technical corrections**

1. Correct minor grammatical errors (e.g., "profiles shows" → "profiles show"; "biase" → "biases").

2. Maintain consistent notation for virtual potential temperature (θv) in equations and figure captions.

3. Define all acronyms (ERA5, RTTOV-gb, CLWC, WSM3) upon first mention in both abstract and text.

4. Add final acknowledgements before publication.