Review Comment 1

Clarification of the ratio-based cost function (Eq. 4)

The use of a dimensionless, ratio-based cost function is an interesting innovation. Please elaborate on:

a) How subtracting unity affects numerical stability and convergence behavior.

b) Whether normalization issues arise when observed or simulated bightness temperatures approach zero.

c) A brief comparison with the conventional covariance-weighted formulation.

Author Response:

We thank the reviewer for the insightful comments and interest in our ratio-based cost function. Our detailed responses are as follows:

a) Effect of subtracting unity on numerical stability and convergence.

We sincerely thank the reviewer for the valuable comment and for pointing out this important numerical consideration. As described in the manuscript, subtracting unity in the cost function can "enhance the contribution of the initial analysis increment to the cost function and reduce the number of iterations" (lines 155–156). We fully acknowledge that, due to the finite precision of floating-point arithmetic, a potential loss of significant digits may occur, which could affect the numerical stability of the computation. To address this issue, both $\frac{H_{(x)}}{y}$ and $\frac{\theta_{v(x)}}{\theta_{v(x_o)}}$ are scaled to the range of -1 to 1 by multiplying them with an amplification factor calculated from $max(abs(1 \Big/ \frac{H_{(x)}}{y} - 1), abs(1 \Big/ \frac{\theta_{v(x)}}{\theta_{v(x_o)}} - 1))$, where $max()$ denotes the maximum operator and $abs()$ represents the absolute value operator. Moreover, all input values are converted to double precision prior to initializing the minimization algorithm, in order to further ensure numerical stability and robustness.

For clairity, we have added the discussion below to the manuscript: "Due to the finite precision of floating-point arithmetic, a loss of significant digits may occur, potentially compromising the numerical stability of the computation. To mitigate this issue, both the observed ($H_{(x)}/y$) and simulated ($\theta_{v(x)}/\theta_{v(x_o)}$) variables are normalized to the range of −1 to 1 using an amplification factor derived from $max(abs(1/\frac{H_{(x)}}{y}-1), abs(1/\frac{\theta_{v(x)}}{\theta_{v(x_o)}}-1))$, where $max()$ and $abs()$ denote the maximum and absolute value operators, respectively. Furthermore, all input values are converted to double precision before the initialization of the minimization algorithm to enhance numerical stability and robustness" (Lines 161 – 166).

b) Normalization issues near zero brightness temperature.

We sincerely thank the reviewer for raising this thoughtful question. Such cases may occasionally occur under clear-sky or non-weather conditions. However, this issue can be effectively mitigated by increasing the number of iterations during the minimization process, which ensures stable convergence. In addition, the normalization treatment, introduced in the revised manuscript (lines 161–166), can further alleviates the potential instability caused by near-zero brightness temperatures. Moreover, the quality control module also alleviates the potential instability by removing the observed brightness temperature whose departure against the simulation is smaller than the noise-equivalent temperature difference.

For clairity, we have added the the discussion below to the manuscript "To avoid normalization issues when observed and simulated brightness temperatures are very close to each other, the quality control module automatically discards GMWR channel observations whose brightness temperature departures from the simulated values are smaller than the noise-equivalent temperature difference" (Lines 166 – 169).

c) Comparison with the conventional covariance-weighted formulation.

We appreciate the reviewer's insightful comment regarding this issue. Specifically, the ratio-based cost function minimizes the relative deviation between observed and simulated brightness temperatures rather than their absolute difference. This approach effectively normalizes the residuals, rendering them dimensionless and ensuring more balanced contributions among different frequency channels. Moreover, the ratio-based formulation is less sensitive to multiplicative calibration or gain errors and better reflects the logarithmic response characteristics of microwave radiative transfer. These advantages have been demonstrated in previous studies and are now explicitly discussed in the revised manuscript.

In the revised manuscript, we have clarified the rationale and advantages of using the ratio-based cost function compared with the conventional difference-based formulation as follows: “It ensures balanced channel weighting, as each channel is normalized by its own magnitude and channels with smaller brightness temperatures are no longer underrepresented during optimization. It also achieves better physical consistency, since the ratio-based form is closer to the logarithmic radiative response of microwave observations, making the inversion more physically meaningful. Finally, it offers enhanced robustness to calibration biases, being less sensitive to multiplicative gain or calibration errors and therefore improving retrieval performance under low signal-to-noise conditions.” (Lines 169 – 175).

Review Comment 2

Uncertainty quantification and statistical significance

The performance metrics (bias, RMSE) are presented without uncertainty ranges. Please include standard deviations or confidence intervals, or indicate whether improvements arestatistically significant.

Author Response:

We sincerely appreciate the reviewer's valuable comment regarding the quantification of uncertainty and statistical significance. Following this suggestion, confidence intervals have been added to Figures 5, 6, 7, and 9 to better illustrate the variability and robustness of the results. Since the root-mean-square error (RMSE) values already provide a comprehensive measure of the overall deviations, no additional modifications were made to those metrics.

Review Comment 3

Microphysics parameterization and coupling:

The coupling between the WSM3 single-moment microphysics scheme and thethermodynamic constraint is not entirely clear. Please expand on how liquid/ice watercontents influence the state vector and cost function. A schematic or equation would behelpful.

Author Response:

We thank the reviewer for this constructive comment. In the revised manuscript, we have clarified the coupling between the thermodynamic constraint and the WSM3 single-moment microphysics scheme, and we have added a schematic (now Figure 4) to illustrate the iterative process.

As described in Section 3.2.2, the retrieval begins with the calculation of virtual potential temperature from the priori (background) profiles of pressure, temperature, and water vapor mixing ratio. The cost function minimization then adjusts these thermodynamic variables using the observed GMWR brightness temperatures to generate intermediate profiles. The WSM3 microphysics scheme dynamically updates cloud water and cloud ice mixing ratios based on the intermediate thermodynamic fields and the priori hydrometeor profiles, ensuring thermodynamic and microphysical consistency at each iteration. The updated hydrometeor contents (liquid and ice) subsequently influence the forward-simulated brightness temperatures through the radiative transfer operator, thereby affecting the cost function and its gradient. The iteration continues until the convergence criterion is met, yielding the final analysis fields of pressure, temperature, water vapor, and hydrometeors. The discussions above have been added to the manuscipt (Lines 185 – 194) read as follows: "The coupling between the thermodynamic constraint and the WSM3 single-moment microphysics scheme is illustrated in Figure 4. The procedure begins with the calculation of the virtual potential temperature from the priori thermodynamic and hydrometeor profile. These fields serve as the initial state for the cost function minimization, where the cost function iteratively adjusts the pressure, temperature, and water vapor mixing ratio using the GMWR brightness temperature observations to produce intermedium profiles. The WSM3 microphysics scheme then dynamically updates the cloud water and cloud ice mixing ratios based on the intermediate pressure, temperature, and water vapor profiles, together with the priori hydrometeor fields. This coupling ensures physical consistency between the thermodynamic state and the microphysical processes during each iteration. If the convergence criterion is satisfied, the resulting profiles of temperature, humidity, and hydrometeors are designated as the final analysis. Otherwise, the updated fields are fed back into the next iteration as new initial conditions until convergence is achieved.”

Review Comment 4

Limited EarthCARE validation sampleValidation is based only on July 2025 data (around 60 collocated profiles). Please explicitly acknowledge this limitation and discuss whether the conclusions may vary with season or location.

Author Response:

We appreciate the reviewer's valuable comment. We acknowledge that the validation dataset, consisting of approximately 60 collocated EarthCARE profiles, is limited in sample size. This constraint indeed restricts the statistical representativeness of the hydrometeor validation results.

However, July was deliberately selected as the test period because the prevailing large-scale circulation over North China during this month frequently produces various types of intense convective systems, including mesoscale convective complexes, squall lines, and stratiform precipitation events. These conditions make July particularly representative of the summer cloud and precipitation regimes in this region, allowing for the evaluation of retrieval performance under diverse hydrometeor conditions.

We also fully recognize that using only July data cannot capture the potential seasonal variability of cloud water and ice characteristics, and thus the current validation results may not fully reflect performance differences across different seasons or locations. We have explicitly discussed this limitation in the revised manuscript and plan to extend the validation to additional months and regions in future work to further assess seasonal dependence.

The discussion from (Lines 425 – 433) read as follows: "This study demonstrates that the TCKF1D-Var framework efficiently integrates thermodynamic constraints and microphysical closure into a unified variational retrieval system, substantially reducing biases, improving hydrometeor profile realism, and enhancing heavy-rain precursor detection. These results highlight its potential for continuous GMWR profiling and short-range nowcasting applications. However, current validation relies on about 60 collocated EarthCARE profiles from July 2025, which limits the statistical robustness of hydrometeor evaluation and the representativeness of seasonal variability. July was chosen because the prevailing synoptic patterns over North China frequently produce diverse convective systems—making it a suitable test period. Future work will extend evaluations across seasons and regions, employ more advanced microphysics and Bayesian uncertainty quantification, and incorporate multi-sensor fusion and scattering-aware radiative operators to further improve retrieval robustness and operational applicability."

Review Comment 5

Figure readability and accessibilityThe font size in Figures 4–6 is rather small. Please adjust the figure layout so that all symbols,units, and legends are clearly readable and distinguishable.

Author Response:

We appreciate the reviewer's helpful suggestion. The layouts of Figures 4–6 (now as Figures 5 – 7) have been revised to improve readability. Font sizes for all labels, units, and legends have been enlarged, and the overall figure clarity and color contrast have been enhanced to ensure accessibility and visual consistency throughout the manuscript.

Reviewer Technical corrections:

1. Correct minor grammatical errors (e.g., “profiles shows” → “profiles show”; “biase” → “biases”).

2. Maintain consistent notation for virtual potential temperature (θv) in equations and figure captions.

3. Define all acronyms (ERA5, RTTOV-gb, CLWC, WSM3) upon first mention in both abstract and text.

4. Add final acknowledgements before publication.

Author Response:

1. The grammatical errors “profiles shows” at Line 214 and “biase” at Line 411 has been corrected.

2. We confirm that the notation for virtual potential temperature is consistant in the revised manuscript.

3. Missing acronyms at Lines 21, 25, 77, and 290 have been added.

4. Acknowledgements have been added.