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Polynomial depth-duration-frequency curves

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Abstract. Depth-duration-frequency (DDF) curves depict how much precipitation occurs on average in a given location during various time intervals once in a given return period. The standard approach to the construction of these curves assumes that the parameters governing the scaling behaviour of rainfall intensity with duration remain constant. We show that in regions where different meteorological processes control short- and long-duration extreme precipitation events, this approach is applicable only in limited time intervals. If the range is as wide as several minutes to several days, three parameters are not sufficient for representing the complexity of the DDF curve shapes. In fact, the curves are wave-shaped because convective and cyclonic precipitation occur for limited lengths of up to several hours and several days, respectively. Thus, we suggest applying polynomial functions of the sixth degree to generate smooth DDF curves that fit design precipitation totals for individual time intervals. Nevertheless, return values need to be fitted against logarithmic time intervals instead of only time. These polynomial DDF curves suitably represent extreme precipitation statistics even in orographically influenced locations where already the precipitation maxima of several hours can be caused by cyclonic precipitation events.

1 Introduction

For water management purposes, it is essential to know heavy rainfall statistics at a given location (Overeem et al., 2008). To present statistics of abnormal precipitation, the relationship between duration (more precisely, the length of the considered time interval) and precipitation depth or intensity is usually expressed by depth-duration-frequency (DDF) or intensity-duration-frequency (IDF) curves, respectively. An individual curve characterizes a certain probability level and can be used to determine how much precipitation falls in a given location during a given time interval on average once in a given return period. The DDF and IDF curves increase and decrease, respectively, as the total precipitation generally increases, and its mean intensity decreases with increasing duration.

The primary goal of constructing both DDF and IDF curves is to establish a continuous and monotonic relationship between accumulation duration and return levels of precipitation totals/intensities (design precipitation) for a given return period. The construction process generally consists of two steps: (i) estimating design precipitation totals/intensities for selected time intervals and (ii) fitting these values to an empirical function to generate a smooth curve. Early methods relied on pure empirical relationships derived directly from observed data (e.g., Bernard, 1932; Bell, 1969; Pagliara and Viti, 1993). However, a key

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limitation of these approaches is the lack of a solid theoretical foundation, which could lead to inconsistencies, particularly for longer return periods (Koutsoyiannis et al., 1998).

A significant advancement in IDF and DDF curve modelling was introduced by Koutsoyiannis et al. (1998). Their rigorous IDF formula was built on a generalized formula that encapsulates insights from preceding traditional IDF studies. They developed a probabilistic framework that is consistent with extreme value theory and the scaling behaviour of rainfall intensity with duration. They derived the specific functional form of the IDF relationship using the underlying theoretical distribution of maximum intensities and employed robust statistical techniques for parameter estimation. Their formulation was validated using real data from a large region of Greece, demonstrating its reliability and applicability, including its ability to incorporate data from nonrecording stations. However, their approach assumes that the parameters governing the scaling behaviour of rainfall intensity with duration remain constant.

The approach introduced by Koutsoyiannis et al. (1998) has been widely adopted with various probability distribution functions. Most commonly, the 3-parametric Generalized Extreme Value (GEV) distribution (Coles, 2001) has been used (e.g., Mohymont et al., 2004; Sane et al., 2018; Shehu et al., 2023). Van de Vyver and Demarée (2010) modified the approach to incorporate the Peaks Over Threshold method and Generalized Pareto distribution in the tropical environment of the Congo, where high-frequency rain-gauge data are limited. Another modification was proposed by Endreny and Imbeah (2009), who applied the approach in Ghana by deriving regional parameters from ground-based rain gauges and estimating distribution parameters from satellite data. In a study by Fauer et al. (2021), a more flexible, duration-dependent IDF curve model was proposed, which is valid for a broader range of durations from 1 minute to 5 days. This model was achieved by applying different dependencies on duration for different return periods (multiscalling) and by deviating from the power law for long durations (flattening).

In addition to traditional empirical and statistical approaches for deriving IDF and DDF curves, both stochastic simulations of precipitation preserving observed precipitation attributes (e.g., Ritschel, 2017) and physically based simulations incorporating atmospheric (thermo)dynamics (e.g., Vu et al., 2017) have been employed. Although these methods are computationally demanding, they can, for example, provide means to assess changes in extreme precipitation under different climate scenarios (e.g., Mirhosseini et al., 2013).

The aims of this paper are (i) to demonstrate the limits of constructing DDF curves for a wide range of rainfall durations using the approach introduced by Koutsoyiannis et al. (1998), hereafter referred to as the standard approach; (ii) to present a novel approach using polynomial functions; and (iii) to explain the suitability of the novel approach by analysing rainfall intensities within a set of rainfall events. The data used and the standard approach are described in Sect. 2. In Sect. 3, DDF curves constructed by both approaches are compared, and an explanation is presented. The suitability and limitations of both approaches are discussed in Sect. 4.

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1 Data and methods

The present study is based on data from more than 160 Czech precipitation-recording stations with at least 20 years of 1-minute intensity data from the period 1951–2022 (Fig. 1). Stations Tábor (467 m a.s.l.) and Desná-Souš (772 m a.s.l.) with data series as long as 70 years were selected for presenting the results and highlighting the influence of orographic effects on the results. Unlike the Tábor station, Desná-Souš is significantly influenced by orography because it is surrounded by peaks up to 1000 m high.

For each station, annual rainfall maxima are selected within sliding time windows ranging from 2 minutes to 96 hours. The maxima for a specific length of time are then fitted by the GEV function (Coles, 2001), employing the L-moment-based index storm procedure (Hosking and Wallis, 1997). To improve the fit for individual stations, sample L-moments are adjusted by the region-of-influence (ROI) method introduced by Burn (1990). The ROI method utilizes statistically homogeneous pools of stations, in which all regional data, weighted by a dissimilarity measure (i.e., by geographical distance), are used for estimating the GEV distribution parameters at the station of interest. The suitability of this approach for the Czech station network was discussed in detail by Kyselý et al. (2011). The homogeneous pools around the Tábor and Desná-Souš stations are depicted in Fig. 1. The derived parameters are used to determine design rainfall totals for 25 different durations independently (Sect. 3.1). DDF curves are then constructed for each return period to identify a suitable approximation of the relationship between the duration and magnitude of the design precipitation. First, the standard approach with a three-parameter function is used to estimate the design rainfall intensity

$$I_N = \frac{a}{(c+d)^b} \qquad (1)$$

where I_N is the design precipitation intensity [l s^{-1} ha^{-1}] with a return period of *N* years; *d* is the duration [min]; and *a*, *b*, and *c* are parameters specific for individual stations and return periods derived by the least squares method using a modification of the Levenberg-Marquardt algorithm (Moré, 1978). Next, the design precipitation intensity I_N is transformed to the corresponding design precipitation total

$$R_{dN} = \frac{60\, I_N\, d}{10000} \qquad (2)$$

where R_{dN} is the design precipitation total [mm] with duration *d* [h] and return period *N* years. The results for different duration ranges for which the fitting is performed are presented in Sect. 3.2. Finally, polynomial functions are employed and interpreted (Sect. 3.3).

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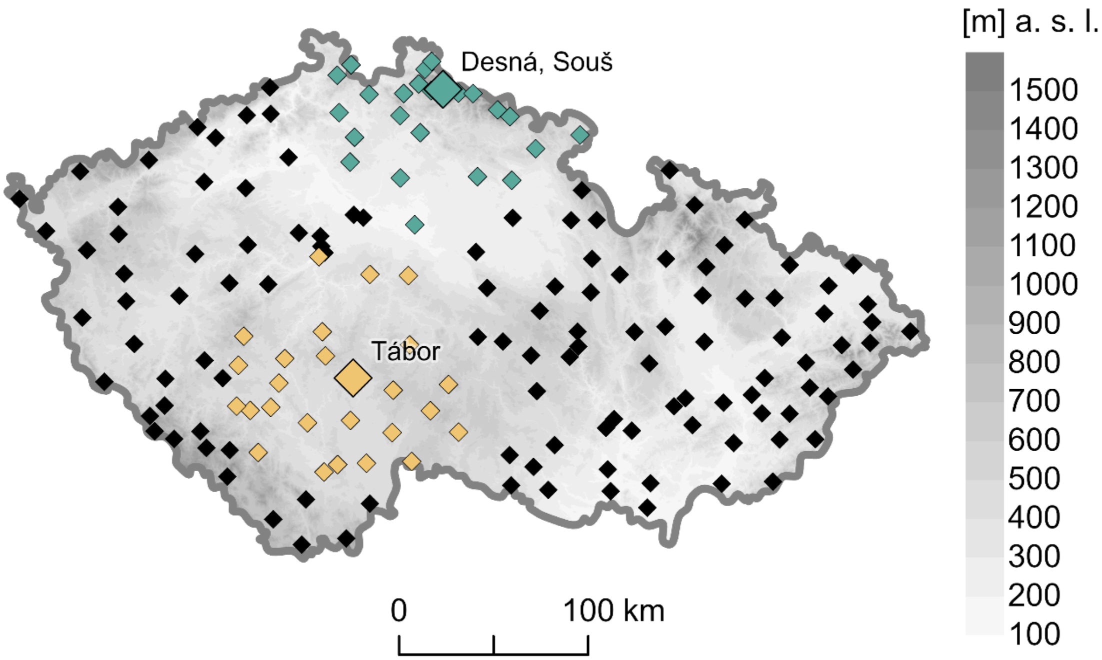

Figure 1: Locations of the precipitation-recording stations used in the study; the Tábor and Desná-Souš stations are denoted with large symbols. Small symbols of the same colour represent meteorological stations in pools employed to improve the design precipitation estimates for Tábor and Desná-Souš by the region-of-influence method.

3 Shapes of the DDF curves

3.1 Design precipitation totals

Despite the rather small area of Czechia (less than 80,000 km^2), its regions differ significantly in terms of the statistical distribution of precipitation maxima. The degree to which stations are influenced by orography plays a major role, as shown in Fig. 2, where two stations are compared from the viewpoint of precipitation maxima statistics. The maximum precipitation totals for durations of up to 2 hours are rather similar at both stations and even slightly greater at the lower station, which corresponds well with the findings of Bližňák et al. (2018). However, the opposite trend is observed for longer durations. For example, the 24-hour maxima are only approximately 150% of the hourly maxima at stations without significant orographic effects but are often more than three times greater at mountain stations, and the differences are even greater for longer durations.

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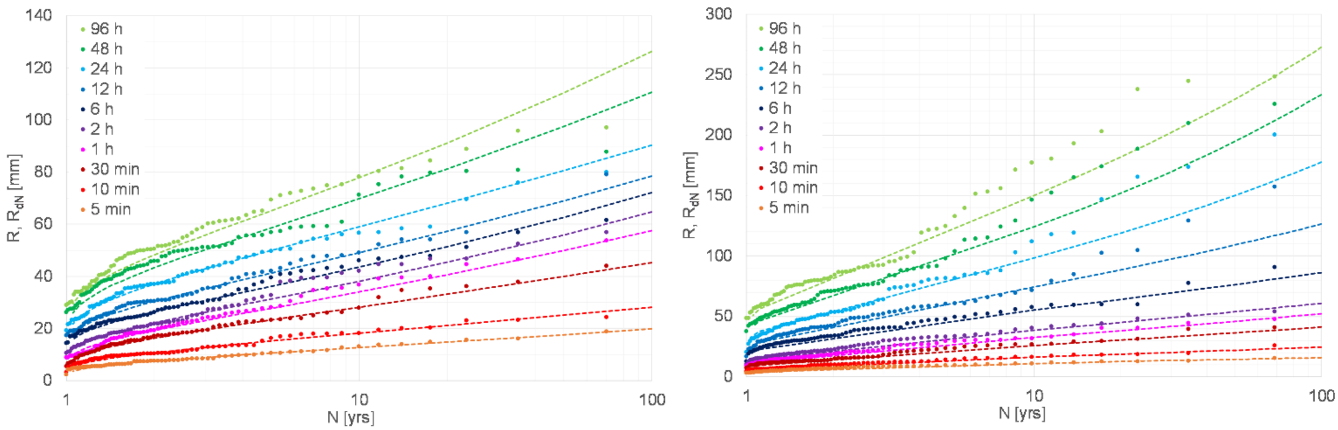

Figure 2: Annual maximum precipitation totals *R* for durations from 5 minutes to 96 hours recorded at the Tábor (left) and Desná-Souš (right) stations and local design totals R_{dN} with return periods *N*. Annual maxima depicted by signs are ranked by magnitude, with the horizontal axis showing the inverse of their order. The curves show estimates of design totals for the same durations, derived by the L-moment-based index storm procedure and the region-of-influence method.

The differences in the recorded precipitation maxima significantly influence the values of the design precipitation totals. The design totals are generally lower at mountain stations for short durations but greater for long durations, with increasing differences between the stations when the return period increases (Fig. 2).

3.2 DDF curves by the standard approach

To construct DDF curves, design totals are considered as a function of the precipitation duration for each return period independently. Because design totals are estimated only for a limited number of durations, the dependence is described by a series of individual values. It is clear from Fig. 3 that the shape of the dependence of the design total on duration is quite complex. In general, the growth of the design totals naturally slows with increasing duration but in different ways at different stations. At all the stations, this weakening of growth is relatively slow on the order of tens of minutes. For durations from several hours, there is another significant increase in the design totals with increasing logarithmic duration but only at orographically affected stations. At other stations, this increase starts at longer durations and is much less pronounced. The significance of all these features usually increases with increasing return period.

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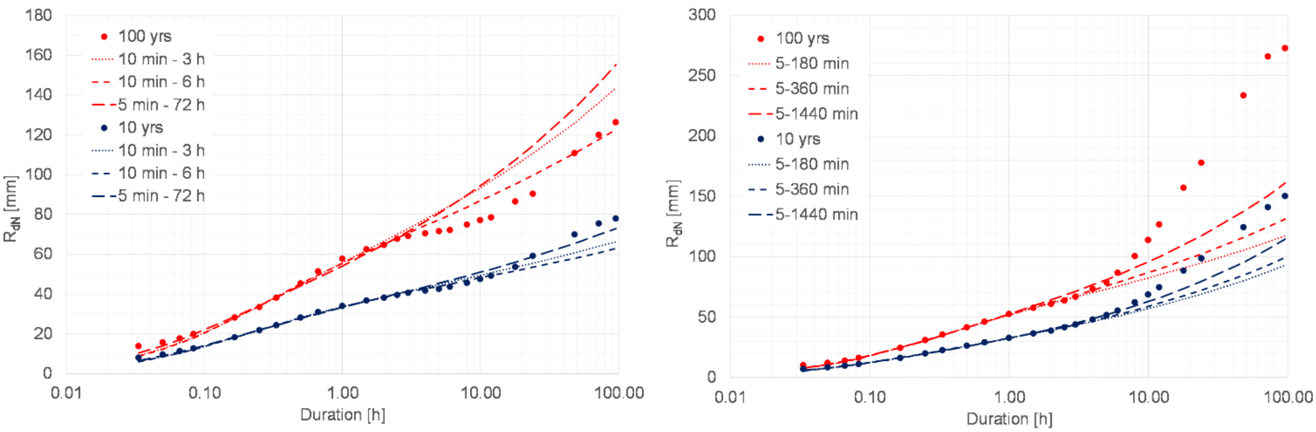

Figure 3: Estimates of design precipitation totals R_{dN} with return periods of 100 and 10 years for selected durations (signs), fitted with standard DDF curves for the Tábor (left) and Desná-Souš (right) stations. Three curves of the same colour in each graph differ from each other in the interval of durations to which they are fitted (see the legend).

The DDF curves created by the standard approach described in Sect. 2 are not complex enough to fit all the detected features. Figure 3 indicates that they fit only parts of the estimated values well, regardless of the range of durations from which the DDF curves are derived. For both presented stations, standard DDF curves fit the values in the interval of durations between 10 minutes and 2 hours rather well, with differences between the individual estimates and the theoretical function values less than 5%. For shorter durations, however, the theoretical functions generally underestimate the design totals (by more than 20% for 2 minutes). For longer durations, at least a part of each curve is completely beyond the estimated values. In general, the quality of the fit of the estimated values is worse at mountain stations and even decreases with increasing return period.

3.3 Polynomial DDF curves

The fact that standard DDF curves fit the estimates only within a limited range of durations motivated us to search for another theoretical function. Obviously, this function needs to be characterized by more than three parameters. Since the dependence of the design precipitation on the logarithm of the duration has a shape resembling a series of symmetrical waves, we decided to apply a polynomial function, namely, of the sixth degree

$$R_{dN} = p_N(\log d)^6 + q_N(\log d)^5 + r_N(\log d)^4 + s_N(\log d)^3 + t_N(\log d)^2 + u_N(\log d) + v_N \tag{3}$$

where R_{dN} is the design precipitation total with duration d [h] and return period N years, and p, q, r, s, t, u, and v are parameters specific for individual stations and return periods.

Fitting the estimates with the polynomial DDF curves is very accurate. For the two stations presented here (Fig. 4), the determinacy coefficient of all the curves is greater than 0.9995, with differences between all the individual estimates and the theoretical function values less than 5%.

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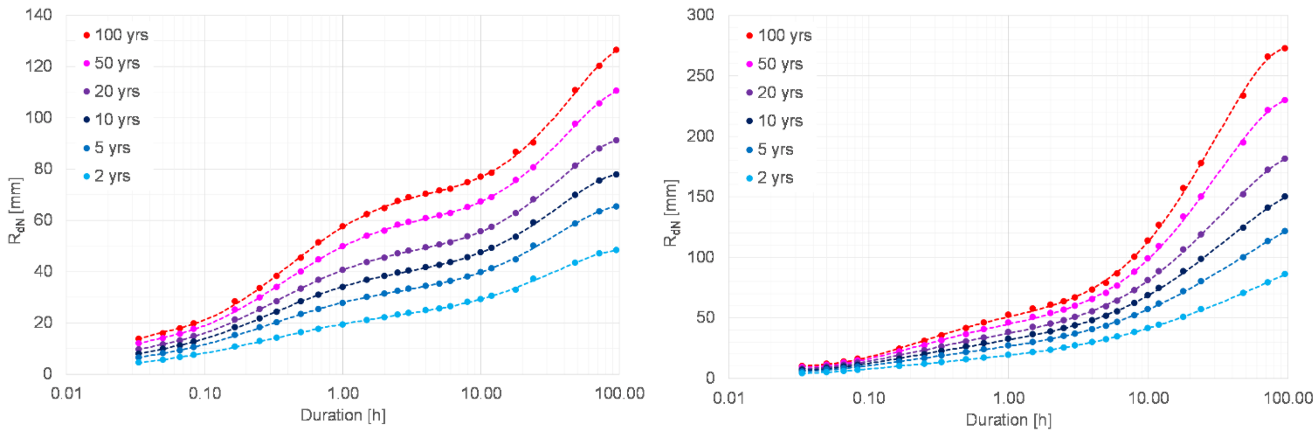

Figure 4: Estimates of design precipitation totals R_{dN} with six different return periods for selected durations (signs), fitted with polynomial DDF curves for the Tábor (left) and Desná-Souš (right) stations.

To explain the reasons for the polynomial shapes of the DDF curves, we analysed the recorded precipitation maxima for the considered durations (Fig. 5). At both presented stations, the set of precipitation events producing the maxima consists of two clearly separate groups that differ in their short-term precipitation time structures (Müller et al., 2018). One group is characterized by short-term rainfall intensity maxima higher than the corresponding mean annual maxima, whereas the other group includes events in which short-term rainfall intensities are well below the values of the mean annual maxima. These two groups can be identified with convective and stratiform precipitation events, respectively.

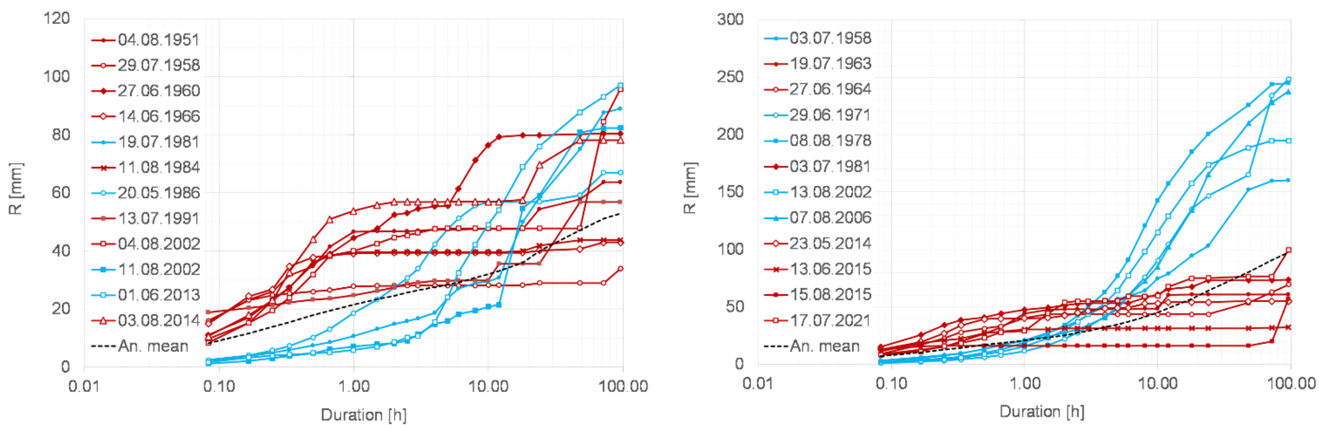

Figure 5: Precipitation totals *R* for durations from 5 minutes to 96 hours recorded at the Tábor (left) and Desná-Souš (right) stations during precipitation episodes when at least one of the values belonged to three absolute maxima for the given duration. Thus, at least three maximum values are depicted for each duration. Convective and stratiform events are distinguished by red and blue colors, respectively. The mean annual maxima are depicted by the black dashed line. Rainfall episodes are represented by the date (dd.mm.yyyy) when the highest one-day precipitation total was recorded.

The stations differ significantly in terms of the distribution of maximum intensities for different durations during extreme events. While convective events generally dominate for durations of up to 3 hours, the onset of stratiform events in the role of

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extreme events occurs for much shorter durations at orographically influenced stations than at other stations, which corresponds with the findings of Kašpar et al. (2021). Even multiday maxima include some convective events but only in areas without significant orographic influence; this can occur in the case where repeated instances of several hours of intense precipitation occurs on consecutive days.

The representation of the two types of precipitation among the precipitation maxima affects the shape of the derived DDF curves. For durations of tens of minutes, convective precipitation totals generally increase rapidly, but then the increase slows down or stops altogether due to the limited possible duration of thunderstorms over a single location. This result corresponds to the flattening of the DDF curves in their corresponding sections. The next increase in the slope of the curves is related to the stratiform precipitation maxima and to the possible recurrence of convective precipitation on the following day at stations without orographic influence. Because stratiform precipitation events generally last only a few days, the slopes of the DDF curves again decrease with increasing multiday duration.

4 Discussion and conclusions

As mentioned in Sect. 1, the standard approach to IDF and DDF curve modeling (Koutsoyiannis et al., 1998) assumes that the parameters governing the scaling behaviour of rainfall intensity with duration remain constant. This method works well in regions where both short- and long-duration extreme precipitation events have the same origin. However, this assumption may limit the effectiveness of the approach in regions where different meteorological processes control short- and long-duration extreme precipitation events (e.g., convective storms producing intense, short bursts of rainfall, whereas cyclonic systems generating steady, prolonged stratiform precipitation). For example, a comparative study on IDF curve derivation in Switzerland revealed that this approach may underperform compared with purely data-driven methods (Haruna et al., 2023). Nevertheless, the standard approach is obviously valid in general if the range of considered time intervals is not wide.

Czechia belongs to regions where design precipitation totals at the diurnal and hourly scales significantly differ (Hulec et al., 2024). In such regions, the six-parameter polynomial function appears to better fit the complex extreme precipitation statistics. Recently, the use of this function has been attempted by Parding et al. (2023) for the territory of Norway. The authors tried to use a weighted set of polynomials to represent the shapes of the DDF curves but rejected this approach because of the “wiggly shapes” of such curves. This problem did not occur in our study because, unlike those studies, we fitted return values against logarithmic time intervals instead of only time.

In our study, we demonstrate that the standard approach to IDF and DDF curve modelling is applicable only to a limited range of considered time intervals. If the range is as wide as several minutes to several days, we suggest polynomial functions of the sixth degree to generate smooth DDF curves fitting design precipitation totals with logarithmic lengths of time. We explain such complex shapes of the DDF curves as a result of the limited lengths of both convective and cyclonic precipitation, which dominate the maximum precipitation totals up to several hours and several days, respectively.

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Data availability

DDF curves designed by the presented method and respective design precipitation totals and intensities for more than 160 stations in the Czech Republic are available on the website https://www.perun-klima.cz/srazky/.

Author contribution

M. Müller proposed the method of polynomial DDF curves. L. Crhová and M. Kašpar determined the estimates of the design precipitation totals for individual durations. M. Müller and F. Hulec fitted these estimates with the DDF curves. M. Müller prepared the manuscript with contributions from all co-authors.

Competing interests

The authors declare that they have no conflict of interest.

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