Responses to Referee #1 comments:

Manuscript number: **egusphere-2025-4213**

Title: **Is earlier always better? A comparative assessment of rainfall timing for multiyear drought alleviation in China**

We sincerely thank you for the valuable comments and suggestions, which have greatly improved our manuscript. We have addressed all the comments and made the necessary revisions. Our responses to each comment are in blue text, while the referees' comments are in black.

This is my first review of the paper. The paper addresses an interesting and potentially important question: how the timing of rainfall replenishment affects the mitigation of multiyear droughts (MYDs). The authors propose a "drought mitigation quantitative model" (DMQM) to quantify how increases in precipitation at different stages of a drought affect drought severity, using historical events in China (1961–2020). The topic is relevant to understanding drought recovery dynamics and could make a valuable contribution if presented more clearly and conceptually coherently.

Response: Thank you very much for your clear summary and positive evaluation of our work. We appreciate your recognition of the importance of examining how the timing of rainfall replenishment influences the alleviation of multiyear droughts (MYDs). The development of the drought mitigation quantitative model (DMQM; now revised as the drought attenuation quantitative model, DAQM, to better reflect the hypothetical numerical experimental nature of the study) is indeed aimed at quantitatively assessing how hypothetical precipitation increments at different months of drought events affect drought evolution. We also appreciate the recognition of the potential contribution of this framework.

However, in its current form, the manuscript suffers from major issues, which I report below.

Response: Thank you very much for the constructive comments. Below we provide point-by-point responses and detailed clarifications for each of the concerns raised.

To facilitate understanding and provide a convenient reference, we first present Table S1, which lists the principal symbols and acronyms used throughout the manuscript.

In response to reviewer comments and to further improve conceptual consistency, expression precision, and readability, several key symbols and acronyms have been refined. The major adjustments are summarized below:

1. The original term Drought Mitigation Quantitative Model (DMQM) has been revised to Drought Attenuation Quantitative Model (DAQM). This modification better reflects the nature of this study, as the analysis is based on hypothetical increases in numerical experiments rather than actual precipitation modification or direct real-world intervention.

2. In the original manuscript, different symbols (t_a, t_b, t_c) were used to represent optimal timing under different conceptual models. To reduce notation complexity, these have been unified into a single symbol, $t_{optimal}$. In this study, the final interpretation focuses on the $t_{optimal}$ derived from the optimal solution model.

3. The acronym PIP has been revised to DIP (Drought Intensity Point of a drought event), defined as the PDSI value corresponding to the Drought Intensity (DI). This change improves terminological clarity and consistency with the event-based drought characterization used in this study.

Table S1: A list of principal symbols and acronyms

Symbols and Acronyms	Description
MYD	MultiYear Drought
PDSI	Palmer Drought Severity Index, a monthly drought index where each value represents the PDSI-defined "drought severity" for that month (the terminology is distinguished from the event-level drought severity, DS, used in this study)
DS	Drought Severity of a drought event, the cumulative deficiency of a drought index below the critical threshold throughout the event
DD	Drought Duration of a drought event, the period during which a drought index is continuously below the critical threshold during the event
DI	Drought Intensity of a drought event, the maximum deviation of the drought index below the critical threshold during the event
DDP	Drought Development Period of a drought event
DIP	DI Point of a drought event, the PDSI value corresponding to the DI
DAQM	Drought Attenuation Quantitative Model
CAFEC	Climatically Appropriate For Existing Conditions
$pr_{baseline}$	the actual precipitation in the targeted month under the baseline (no hypothetical additional precipitation is applied) scenario
$\overline{pr}$	the long-term mean monthly precipitation
pr_{rate}	a dimensionless proportion (0–1) that controls the relative magnitude of the hypothetical precipitation addition, the amount of hypothetical precipitation addition is given by $pr_{rate} \times \overline{pr}$
$pr_{forcing}$	the precipitation in the targeted month after applying hypothetical additional precipitation, $pr_{forcing} = pr_{baseline} + pr_{rate} \times \overline{pr}$, $pr_{rate} = 0, 5\%, 10\%, \ldots, 100\%$
$DS_{baseline}$	the actual DS under the baseline (no hypothetical additional precipitation is applied) scenario
$DS_{forcing}$	the DS after the hypothetical additional precipitation is applied
DS_{rate}	the relative change in DS, $DS_{rate} = \frac{DS_{baseline} - DS_{forcing}}{DS_{baseline}}$
k	MYD attenuation efficiency, k characterizes how rapidly additional precipitation is converted into reductions in DS
$k_{theoretical}$	the theoretical k value, $k_{theoretical}$ is obtained by fitting the DAQM analytical solution $DS_{rate} = 1 - exp(-k \cdot pr_{rate})$ to the DS_{rate}-pr_{rate} curve derived from the numerical experiments

k_{actual}	the actual k value, represent the empirical sensitivity of DS to small precipitation perturbations, $k_{actual}=\frac{DS_{rate}(pr_{rate}=0.05)}{0.05}$, pr_{rate}=0.05 is the step size in the numerical experiments
ε	relative error, $\varepsilon=\left(\frac{k_{theoretical}-k_{actual}}{k_{actual}}\right)\times 100\%$
k_1	$k_{theoretical}$ value at the first month of a MYD event
k_2	$k_{theoretical}$ value at the second month of a MYD event
k_3	$k_{theoretical}$ value at the third month of a MYD event
k_{max}	maximum $k_{theoretical}$ within a MYD event, obtained as the largest monthly k among all months of this event
Δk_{21}	the changes in $k_{theoretical}$ between the second and the first month of a MYD event
Δk_{31}	the changes in $k_{theoretical}$ between the third and the first month of a MYD event
$t_{forcing}$	the month of hypothetical additional precipitation is applied
t_1	the first month of a MYD event
t_2	the second month of a MYD event
t_3	the third month of a MYD event
t_{DIP}	the month corresponding to DIP within a MYD event
$t_{optimal}$	the optimal timing for drought alleviation

Note: To avoid ambiguity, the monthly values of the Palmer Drought Severity Index (PDSI) are referred to simply as "PDSI" in this study, whereas DS is used exclusively to denote event-level drought severity.

1. The term mitigation is central to the paper but is never clearly or consistently defined.

Throughout the introduction and abstract, "drought mitigation" is used in the general sense of reducing drought impacts, which typically implies human or management intervention. Later, in Section 3.4, it becomes apparent that mitigation actually refers to the modelled reduction in drought severity (DS) resulting from hypothetical precipitation forcing in numerical experiments. In this way, readers are left uncertain whether the study concerns physical hydrological recovery, operational drought management, or synthetic model sensitivity.

I suggest defining drought mitigation explicitly in the introduction as a numerical or index-based construct (i.e., "the reduction in DS under hypothetical precipitation increases"). In practice, clarify early that no human intervention or feasible rainfall modification is implied. Terms such as index-based recovery or drought attenuation efficiency would be more accurate and less misleading.

This also impacts other terms. Indeed, the manuscript repeatedly refers to "precipitation-based drought mitigation" and "early intervention," giving the impression that rainfall timing can be controlled or managed (e.g., through artificial replenishment). Yet, the study merely perturbs precipitation values in the PDSI model. This language conflates natural rainfall timing with anthropogenic mitigation, generating unnecessary conceptual ambiguity. I suggest rephrasing these expressions to reflect that the experiments involve idealized precipitation scenarios, not management interventions. For example: "The model explores how the timing of hypothetical precipitation increases affects index-based drought recovery."

Response: Thank you for raising this important point. We agree that the previous use of the term "drought mitigation" lacked conceptual precision and could generate confusion among physical hydrological recovery, operational drought management, and synthetic model sensitivity. To avoid this ambiguity, we now remove this term and clarify the conceptual scope of the study. In our framework, the reduction in drought severity (DS) of a multiyear drought event results solely from hypothetical precipitation increases imposed within numerical experiments, rather than from real-world management action or rainfall modification. Accordingly, we now refer to this process as index-based drought alleviation and describe the DS response to hypothetical precipitation forcing using drought attenuation efficiency (k). We also clarify that the experiments involve idealized precipitation scenarios generated by perturbing the water-input variable (precipitation) inside the PDSI framework. These scenarios are designed to evaluate how the timing of a hypothetical single-month additional water input influences the reduction of DS, rather than to imply controllable or anthropogenic "intervention". Expressions such as "precipitation-based drought mitigation", "early intervention", and other potentially misleading wording are therefore replaced with conceptually accurate descriptions such as "drought attenuation quantitative model" (Page 1, Line 18), "drought alleviation" (Page 4, Line 97), "drought attenuation efficiency" (Page 8, Line 190), and "timing of hypothetical precipitation increases" (Page 8, Line 196). These clarifications ensure that the terminology consistently reflects the synthetic nature of the numerical experiments and avoids any implication that rainfall timing can be operationally controlled. The aim of our study is to isolate how the PDSI index responds to controlled water-input perturbations, rather than to evaluate practical drought management strategies. Although the numerical experiments are idealized, the conceptual patterns revealed by the model may still be relevant to broader discussions about drought alleviation, since many real-world water-management practices (such as managed redistribution of available water resources or exploratory cloud-seeding trails) ultimately aim to reduce water deficits caused by prolonged precipitation shortages.

The manuscript has been systematically revised throughout to ensure conceptual consistency. Relevant modifications have been made in the Abstract (Page 1, Lines 26 to 27), Introduction (Page 4, Lines 107 to 108), and Section 3.4 (Page 8, Lines 196 to 198) among other corresponding locations in the manuscript.

2. The central relationship of the DMQM model, that is, the DS rate equation, is introduced without theoretical justification or empirical testing. The exponential law is simply postulated at the beginning, not derived from data or hydrological reasoning. Consequently, the "goodness of fit" ($R^2 > 0.9$) is largely tautological, since k is defined within this same assumed functional form. I think that the auhtors, shall provide evidence that the exponential function is appropriate, or test alternative relationships (linear, logistic, power-law). Explain whether k has a physical interpretation (e.g., storage or recovery constant) or is purely a curve-fitting parameter. Without this, the DMQM remains a mathematical construct lacking process basis.

Response: Thank you for this constructive comment. We acknowledge that the exponential relationship in DAQM needed clearer justification. Below we provide a detailed explanation of its empirical basis, theoretical relevance, and comparative performance relative to alternative formulations.

To clarify our approach, the exponential formulation in DAQM is not arbitrarily postulated. Our modeling framework was developed by performing numerical experiments driven by observations from meteorological stations. Before specifying any functional structure, we first examined how drought severity (DS) of a multiyear drought event responds to incremental precipitation increases within controlled numerical experiment. Across events and stations, the numerical experiments consistently demonstrated a clear diminishing return pattern (Fig. 5). When hypothetical additional precipitation was incrementally increased at the same month of a drought event, the reduction in DS showed progressively weaker marginal benefits. The initial portion of added water input produced the strongest alleviation, while subsequent increments at that same month yielded progressively weaker effects. This empirical feature motivated us to seek a formulation capable of representing a bounded recovery process. To formalize this behavior, we drew an analogy with systems exhibiting saturation-type adjustment, similar to first-order kinetic expressions (e.g. $C_t=C_0 \cdot exp(-k \cdot t)$, where C_t denotes the remaining intensity at time t, C_0 is the initial state at $t=0$, and k is the first-order rate constant controlling the decay speed. The instantaneous decay rate $\frac{dC}{dt}$ is proportional to the current magnitude C_t.). Following this reasoning, we adopted the formulation: $DS_{rate}=1-exp(-k \cdot pr_{rate})$. This formulation captures how incremental water input progressively reduces the residual DS (Fig. 1). In this framework, $exp(-k \cdot pr_{rate})$ represents the proportion of remaining DS after a given additional precipitation increment. When the additional water supply is sufficient, the relative reduction in DS approaches a practical upper limit, a pattern similar to physical constraints such as finite soil water storage capacity or vegetation water use limits.

To demonstrate that the exponential structure represents real system behavior rather than tautology, we compared formulations inspired by zero-order, first-order, and second-order kinetic processes using historical multiyear drought events (MYDs) during 1991-2020. Across most stations and events, the first-order structure more accurately reproduced the curvature observed in the numerical experiments at different month of MYDs. We further extended this comparison to an n-order formulation and found that the estimated n converged toward 0.98 across MYDs, which is essentially unity. This outcome strengthens the conclusion that a first-order structure is the most appropriate representation of the observed pattern. The results are summarized in Table S2. We also compared the exponential form against alternative empirical relationships. Logistic curves did not match the observed pattern. Power-law functions were unable to satisfy the required boundary conditions. We note that the same exponential adjustment structure has been adopted in other fields. For instance, Childs et al. (2025) applied a similar functional pattern in modeling climate-driven disease dynamics. The recurrence of this structure across domains further supports its suitability for representing systems with constrained recovery capacity.

Table S2: Comparison of R^2 values for different functional models

Model order	Differential form	Integral form	t_1	t_2	t_3
Zero-order	$\frac{\partial DS_{rate}}{\partial pr_{rate}} = k$	$DS_{rate}=k \cdot pr_{rate}$	0.992	0.984	0.989
First-order	$\frac{\partial DS_{rate}}{\partial pr_{rate}} = k \cdot (1 - DS_{rate})$	$DS_{rate}=1-exp(-k \cdot pr_{rate})$	0.985	0.996	0.993

Second-order	$\frac{\partial DS_{rate}}{\partial pr_{rate}} = k \cdot (1 - DS_{rate})^2$	$DS_{rate}=1-1/(k \cdot pr_{rate}+1)$	0.982	0.976	0.961
N-order	$\frac{\partial DS_{rate}}{\partial pr_{rate}} = k \cdot (1 - DS_{rate})^n$	$DS_{rate}=1-[1-k \cdot (1-n) \cdot pr_{rate}]^{1/1-n}$	0.989	0.996	0.998

Note: t_1, t_2, and t_3 represent the first to the third month of the MultiYear Drought event.

Furthermore, although parameter k is empirically estimated, it is not merely a curve-fitting constant. The parameter quantifies the system's drought attenuation efficiency, namely how rapidly additional precipitation is converted into reductions in DS. Formally, in the differential form $\frac{\partial DS_{rate}}{\partial pr_{rate}}=k \cdot (1-DS_{rate})$ (eq. 3), k represents the initial response rate when $DS_{rate}=0$. Higher k values correspond to faster alleviation, while lower k values reflect slower hydrological recovery or more persistent drought conditions.

Overall, the exponential law in DAQM reflects empirically observed behavior, aligns with physically meaningful adjustment processes, and performs more effectively than linear, logistic, power-law, and higher-order alternatives.

The manuscript has been revised accordingly, with the relevant additions incorporated into Section 3.4 (Pages 8 to 12, Lines 192 to 287), particularly on Pages 10 to 11, Lines 241 to 271.

References

Childs, M.L., Lyberger, K., Harris, M.J., Burke, M., and Mordecai, E.A.: Climate warming is expanding dengue burden in the Americas and Asia, Proc. Natl. Acad. Sci., 122, e2512350122, doi: https://doi.org/10.1073/pnas.2512350122, 2025.

3. Although the analysis uses real meteorological data, the results remain statistical and abstract. The study would benefit from at least some discussion on what the "optimal timing" might mean in real hydrological terms, e.g., soil moisture recharge, vegetation response, or water resource management, instead of only index sensitivity. Otherwise, the practical implications of $t_{optimal}$ remain unclear.

Response: Thank you for this valuable comment. We agree that without process-level interpretation, the practical significant of $t_{optimal}$ may appear abstract. To clarify its conceptual significance, we provide additional discussion on how the "optimal timing" can be interpreted within the broader context of drought propagation and system-level response efficiency quantified by parameter k in the DAQM.

In our framework, $t_{optimal}$ refers to the month within a multiyear drought (MYD) event when additional precipitation produces the most efficient reduction in drought severity (DS) of this event. This is determined by identifying the month at which the parameter k reaches its maximum within a drought event. As described in our response to the Comment 2, k characterizes how rapidly additional precipitation is converted into reductions in DS. Higher k values correspond to faster alleviation, while lower k values reflect slower hydrological recovery or more persistent drought conditions.

In the original manuscript, different symbols (t_a, t_b, and t_c) were used to represent optimal timing under different conceptual models. To reduce notation complexity, these have now been unified into a single symbol, $t_{optimal}$. In this study, the final interpretation focuses on the $t_{optimal}$ derived from the optimal solution model. The corresponding revisions have been implemented in the manuscript. (Page 8, Lines 194 to 214)

Although these symbols have been unified in the revised manuscript, we briefly explain the conceptual meaning of t_a, t_b, and t_c in this response document to clarify how the conceptual models are connected.

From our numerical experiments, we observed that in most cases (58.79%), the maximum *k* occurs at the first month (t_1) of the MYD event (Fig. 6b). However, a non-negligible fraction occurs at the second (t_2) and the third (t_3) months, accounting for 22.11% and 11.06%, respectively (Fig. 6b). To interpret these patterns, we introduced two conceptual models.

Conceptual Model 1 (Fig. 4b): For a single MYD event, earlier precipitation perturbation exerts influence over a longer fraction of the event duration, leading to stronger drought attenuation efficiency at early month. In this model, $t_{optimal}$ tends to t_1, denoted as t_a, representing the optimal timing under the early-intervention scenario.

Conceptual Model 2 (Fig. 4c): Considering different PDSI values at the same month, standardized indices are more sensitive to extreme one. Thus, perturbations near peak intensity may yield stronger effects. In this model, $t_{optimal}$ tends to the month at Peak Intensity Point of one MultiYear Drought event (t_{DIP}), denoted as t_b, representing the optimal timing under standardization-induced sensitivity.

These two conceptual models illustrate opposing tendencies. Conceptual Model 1 favors earlier perturbation, whereas Conceptual Model 2 favors timing near peak intensity. To reconcile these, we propose the Optimal Solution Model (Fig. 4d), which balances the advantages of both conceptual models. The resulting $t_{optimal}$, denoted as t_c, is the timing identified by our study.

This t_c has two main hydrological and ecological meaning. From the perspective of drought propagation, meteorological drought typically occurs first in the system, so understanding its optimal timing is essential for interpreting how precipitation signals subsequently influence soil moisture conditions and vegetation responses. By identifying the month when the system responds most efficiently to precipitation (maximum *k*), t_c provides a reference for real-world water-management practices that can effectively influence the subsequent stages of drought propagation. Regarding the potential extension of our framework to other drought types, t_c may also have broader value. If this framework can be validated for hydrological or agricultural drought, which would require incorporating additional relevant variables beyond those considered for meteorological drought, t_c could similarly guide real-world water-management practices in these types of drought directly. In ecological terms, Applying water around t_c can optimize soil water recharge while the root zones still have high infiltration capacity, and provide vegetation with water at a stage when its physiological recovery potential remains high. Both perspectives represent meaningful directions for future research. In practical terms, t_c can also guide real-world water-management decisions. It helps determine when irrigation provides the greatest drought relief per unit of water applied, guides the timing of managed water recharge or emergency ecological water delivery for maximal effectiveness. It also helps prevent

cascading losses and further degradation in prolonged droughts. So that $t_{optimal}$ is presented not solely as an attenuation efficiency parameter but as a concept with hydrological, ecological, and management relevance, grounded in both system response efficiency and the dual conceptual considerations of drought progression.

In addition, we would like to clarify why our study focus on meteorological drought. Drought types such as hydrological and agricultural drought are strongly influenced by factors such as vegetation types (e.g., forest consuming more water than grass or shrubs), periods (e.g., dry seasons or warm years), and soil moisture. These factors bring substantial uncertainty when attempting to isolate the system's response to precipitation perturbation alone. Meteorological drought, by contrast, is driven primarily by precipitation deficits and atmospheric evaporative demand (Wei et al., 2025). It therefore provides a clearer and more physically controlled condition for establishing a quantitative framework of drought attenuation efficiency. Meteorological drought provides a necessary starting point for the present analysis before extending the framework to other drought types in future work.

The revised manuscript includes a concise and conceptually integrated discussion of these point in Section 5.1 (Page 21, Lines 483 to 489).

References

Wei, Y., Huang, W., and Zhu, S.: Characterization of the propagation of two types of meteorological drought events, insufficient precipitation and excessive evaporative demand, into hydrology and agriculture, J. Hydrol., 650, 132555, doi: https://doi.org/10.1016/j.jhydrol.2024.132555, 2025.

4. While the paper contains extensive methodological detail, the presentation is overly technical and textually dense, often making it difficult to follow the main line of reasoning. Equations, parameters, and symbols are introduced in rapid succession without sufficient intuitive explanation, and transitions between conceptual discussion, mathematical derivation, and numerical experiments are often abrupt.

As a result, even readers with a strong hydrological background may struggle to understand the logical progression from the conceptual models (Section 3.4) to the construction of the DMQM and its application. The heavy use of notation (DS, DD, DI, DDP, PIP, k, t_1–t_8, toptimal) without repeated restatement of their physical meaning contributes to confusion. I strongly suggest providing a short, intuitive summary before or after key equations and reducing unnecessary algebraic detail or moving it to supplementary material and restating the physical meaning of variables (e.g., PIP, DDP, k) when they reappear in later sections.

Response: Thank you for this constructive comment. We understand that the methodological presentation in current version is dense and that the rapid introduction of equations, symbols, and terminology may hinder readers from following the conceptual progression of the DAQM framework. Below, we provide a detailed explanation of how we will address these concerns in revision.

1. We recognize the need for clearer and more intuitive transitions between conceptual reasoning, mathematical formulation, and numerical experiments. In several parts of the paper, equations appear in quick succession without sufficient explanation of their physical meaning. In the revised version,

each major equation is accompanied by a concise and intuitive explanation placed immediately before or after its presentation. These explanations clarify why the equation is introduced, what physical reasoning it represents, and how it connects to the drought evolution process.

Furthermore, the methodological workflow and the logic connecting the numerical experiments with the analytical formulation have been clarified. As illustrated in Fig. 3, our analysis begins with a controllable hypothetical additional precipitation numerical experiment designed on the basis of the PDSI. For each MYD event, we perturb precipitation month bu month while keeping all other months unchanged. To ensure no bias, the imposed precipitation perturbation is expressed as a percentage of the multiyear mean precipitation for the corresponding month, which is formalized in $pr_{forcing}=pr_{baseline}+pr_{rate}\times\overline{pr}$, pr_{rate}=0, 5%, 10%, ..., 100% (Eq. 2). pr_{rate} denotes a dimensionless proportion (0–1) that controls the relative magnitude of the hypothetical precipitation addition, the amount of hypothetical precipitation addition is given by $pr_{rate}\times\overline{pr}$ ($\overline{pr}$ is the long-term mean monthly precipitation). For each prescribed pr_{rate} we compute the corresponding relative change in DS (DS_{rate}). Consistent with the observation described in our response to Comment 2, the DS_{rate}-pr_{rate} relationship exhibits a clear diminishing-return pattern. To formalize this behavior, we drew an analogy with systems exhibiting saturation-type adjustment and adopted a form similar to a first-order kinetic expression. After testing several functional forms, this structure provided a better performance in capturing this pattern (Table S2). Accordingly, we represent the response rate as $\frac{\partial DS_{rate}}{\partial pr_{rate}}=k\cdot(1-DS_{rate})$ (Eq. 3), where k characterizes how rapidly additional precipitation is converted into reductions in DS. The boundary and limiting conditions described in Eq. 4 and Eq. 5 then allow us to derive the analytical solution presented in Eq. 6: $DS_{rate}=1-exp(-k\cdot pr_{rate})$. Additionally, the structure of Eq. 3 further enables us to quantify the empirical sensitivity of DS to small precipitation perturbations. This sensitivity is approximated using the initial slope of the DS_{rate}-pr_{rate} curve as $k_{actual}=\frac{DS_{rate}(pr_{rate}=0.05)}{0.05}$, with 0.05 representing the step size in our numerical experiment. The $k_{theoretical}$ is obtained by fitting the analytical solution Eq. 6 to the numerically derived DS_{rate}-pr_{rate} curve. To evaluate the consistency between the empirical behavior and the analytical framework, we compute the relative error using Eq.7: $\varepsilon=\left(\frac{k_{theoretical}-k_{actual}}{k_{actual}}\right)\times100\%$. These additions will substantially improve the transparency of the formulation and ensure that readers can clearly follow the logical progression from conceptual reasoning to numerical experimentation and analytical derivation.

These clarification have been incorporated into Section 3.4 of the revised manuscript. (Pages 8 to 12, Lines 192 to 287)

2. We agree that readers may experience cognitive burden due to the heavy algebraic detail included in the current manuscript. To reduce unnecessary complexity in the main narrative, material not essential for understanding the DAQM framework has been provided in the supplementary material. Specifically, Table S2 (Comparison of R^2 values for different functional models) has been added to present the comparative results in a more concise form. This allows the main text to focus on the conceptual logic and the key components of the framework, while still ensuring that all methodological details remain available for readers who wish to examine them more deeply.

3. We acknowledge that the frequent use of symbols and acronyms such as DS, DD, DI, DDP, DIP, k, t_i–t_s, and $t_{optimal}$ can be overwhelming when their meaning is not periodically reiterated. These symbols and acronyms play central roles across multiple sections, and readers unfamiliar with the notation may lose track of their physical interpretation. To address this, brief reminders of the meaning of the most important variables have been restated when they reappear in later discussion. In addition, a consolidated table listing principal symbols, acronyms and their descriptions has been provided so that readers can easily consult definitions at any point (Table S1).

Overall, we greatly appreciate the suggestion to improve clarity and accessibility. The revisions described above are intended to strengthen the logical continuity between conceptual models, mathematical construction, and numerical experiments. They will also ensure that readers with hydrological expertise but without prior exposure to this modelling framework can follow the development of ideas more comfortably and gain a more intuitive understanding of the DAQM.

Responses to Referee #2 comments:

Manuscript number: **egusphere-2025-4213**

Title: **Is earlier always better? A comparative assessment of rainfall timing for multiyear drought alleviation in China**

We sincerely thank you for the valuable comments and suggestions, which have greatly improved our manuscript. We have addressed all the comments and made the necessary revisions. Our responses to each comment are in blue text, while the referees' comments are in black.

This manuscript investigates the role of precipitation timing on reducing the severity of multiyear droughts. The topic is generally well suited for this journal, and the objective clearly identified.

Response: Thank you sincerely for the careful assessment of our work. We are grateful that the study's objective and relevance were recognized. We also truly appreciate the time taken to evaluate the manuscript. Your comments are highly valuable and have guided us in clarifying the conceptual framework, methodology, and implication of our work.

I found, however, a few major drawbacks that limit the scope of the proposed research.

Response: Thank you for pointing out several major issues that currently limit the clarity and scope of the study. We greatly appreciate these constructive comments. In response, we have carefully reconsidered the conceptual framework, the methodological approach, and the interpretation of the results. Detailed, point-by-point responses to each comment are provided below.

To facilitate understanding and provide a convenient reference, we first present Table S1, which lists the principal symbols and acronyms used throughout the manuscript.

In response to reviewer comments and to further improve conceptual consistency, expression precision, and readability, several key symbols and acronyms have been refined. The major adjustments are summarized below:

1. The original term Drought Mitigation Quantitative Model (DMQM) has been revised to Drought Attenuation Quantitative Model (DAQM). This modification better reflects the nature of this study, as the analysis is based on hypothetical increases in numerical experiments rather than actual precipitation modification or direct real-world intervention.

2. In the original manuscript, different symbols (t_a, t_b, t_c) were used to represent optimal timing under different conceptual models. To reduce notation complexity, these have been unified into a single symbol, $t_{optimal}$. In this study, the final interpretation focuses on the $t_{optimal}$ derived from the optimal solution model.

3. The acronym PIP has been revised to DIP (Drought Intensity Point of a drought event), defined as the PDSI value corresponding to the Drought Intensity (DI). This change improves terminological clarity and consistency with the event-based drought characterization used in this study.

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Symbols and Acronyms	Description
MYD	MultiYear Drought
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DS	Drought Severity of a drought event, the cumulative deficiency of a drought index below the critical threshold throughout the event
DD	Drought Duration of a drought event, the period during which a drought index is continuously below the critical threshold during the event
DI	Drought Intensity of a drought event, the maximum deviation of the drought index below the critical threshold during the event
DDP	Drought Development Period of a drought event
DIP	DI Point of a drought event, the PDSI value corresponding to the DI
DAQM	Drought Attenuation Quantitative Model
CAFEC	Climatically Appropriate For Existing Conditions
$pr_{baseline}$	the actual precipitation in the targeted month under the baseline (no hypothetical additional precipitation is applied) scenario
$\overline{pr}$	the long-term mean monthly precipitation
pr_{rate}	a dimensionless proportion (0–1) that controls the relative magnitude of the hypothetical precipitation addition, the amount of hypothetical precipitation addition is given by $pr_{rate} \times \overline{pr}$
$pr_{forcing}$	the precipitation in the targeted month after applying hypothetical additional precipitation, $pr_{forcing} = pr_{baseline} + pr_{rate} \times \overline{pr}$, $pr_{rate} = 0, 5\%, 10\%, \ldots, 100\%$
$DS_{baseline}$	the actual DS under the baseline (no hypothetical additional precipitation is applied) scenario
$DS_{forcing}$	the DS after the hypothetical additional precipitation is applied
DS_{rate}	the relative change in DS, $DS_{rate}=\frac{DS_{baseline}-DS_{forcing}}{DS_{baseline}}$
k	MYD attenuation efficiency, k characterizes how rapidly additional precipitation is converted into reductions in DS
$k_{theoretical}$	the theoretical k value, $k_{theoretical}$ is obtained by fitting the DAQM analytical solution $DS_{rate}=1-exp(-k \cdot pr_{rate})$ to the DS_{rate}-pr_{rate} curve derived from the numerical experiments
k_{actual}	the actual k value, represent the empirical sensitivity of DS to small precipitation perturbations, $k_{actual}=\frac{DS_{rate}(pr_{rate}=0.05)}{0.05}$, $pr_{rate}=0.05$ is the step size in the numerical experiments
ε	relative error, $\varepsilon=\left(\frac{k_{theoretical}-k_{actual}}{k_{actual}}\right)\times 100\%$
k_1	$k_{theoretical}$ value at the first month of a MYD event
k_2	$k_{theoretical}$ value at the second month of a MYD event
k_3	$k_{theoretical}$ value at the third month of a MYD event

k_{max}	maximum $k_{theoretical}$ within a MYD event, obtained as the largest monthly k among all months of this event
Δk_{21}	the changes in $k_{theoretical}$ between the second and the first month of a MYD event
Δk_{31}	the changes in $k_{theoretical}$ between the third and the first month of a MYD event
$t_{forcing}$	the month of hypothetical additional precipitation is applied
t_1	the first month of a MYD event
t_2	the second month of a MYD event
t_3	the third month of a MYD event
t_{DIP}	the month corresponding to DIP within a MYD event
$t_{optimal}$	the optimal timing for drought alleviation

Note: To avoid ambiguity, the monthly values of the Palmer Drought Severity Index (PDSI) are referred to simply as "PDSI" in this study, whereas DS is used exclusively to denote event-level drought severity.

1. The term mitigation is, in my opinion, improperly used or, at least, not well described. The term mitigation is usually associated with anthropogenic action to reduce the impacts of a natural hazard. Here, terms such as "amelioration" or "alleviation" may be more appropriate. In general, the authors discuss about precipitation as it is usually done for irrigation in agricultural districts. While there are some similarities, precipitation over large areas cannot be discussed in the same terms. I suggest a deep revision of the text to avoid any confusion on the strategy discussed here.

Response: We sincerely thank you for this important and constructive comment. We fully agree that the term "mitigation" can create conceptual ambiguity, as it is commonly associated with anthropogenic actions to reduce the impacts of natural hazards. Following this suggestion, we have performed a systematic revision throughout the manuscript to avoid any anthropogenic-intervention interpretation. In our study, this term specifically refers to the reduction in drought severity (DS) of a multiyear drought (MYD) event under hypothetical precipitation increases applied within numerical experiments, rather than to any direct human intervention. To avoid confusion, we now consistently use drought "attenuation" to describe the system's response and drought "alleviation" to describe the event-level reduction in DS.

To clarify, we explicitly define this process as index-based drought alleviation and describe the system's internal response using drought attenuation efficiency (k). The experiments involve idealized precipitation forcing, implemented by perturbing precipitation in the PDSI model for a single month while keeping other months unchanged. This design isolates how the system responds to controlled water-input perturbations at different drought stages and does not imply operational control over rainfall.

Expressions that could suggest direct management, such as "precipitation-based drought mitigation" or "drought mitigation quantitative model", are replaced with conceptually precise terminology, including "drought attenuation quantitative model" (Page 1, Line 18), "drought alleviation" (Page 4, Line 97), "drought attenuation efficiency" (Page 8, Line 190), and "timing of hypothetical precipitation increases" (Page 8, Line 196), as representative examples. These revision ensure that the language consistently reflects the synthetic nature of the numerical experiments and avoids giving the impression that precipitation timing can be artificially controlled.

Although the findings are derived from idealized experiments, they provide useful insights into how the system responds to additional water input. These insights can conceptually inform discussions about real-world drought alleviation, since many water management practices, such as reservoir releases, ecological flow restoration, irrigation, or experimental cloud-seeding trials, aim to reduce water deficits caused by prolonged precipitation shortages. Such actions influence soil moisture, hydrological storage, and overall water availability. Because the PDSI framework treats precipitation as the sole water-input variable, the additional precipitation applied in our numerical experiments functions as an equivalent representation of increased available water, regardless of the specific real-world mechanism through which this water is supplied. This allows the numerical experiments to isolate the system's intrinsic sensitivity to controlled water-input perturbations without tying the analysis to any particular management measure. Our results therefore highlight the potential timing and effectiveness of increased water availability, even though the model input is expressed as hypothetical precipitation. Furthermore, the hypothetical perturbations are constructed based on historical MYDs, ensuring that the imposed conditions are grounded in real drought evolution rather than artificially invented scenarios. By consistently emphasizing attenuation for system response and alleviation for event-level improvement, we ensure conceptual clarity and avoid ambiguity in interpreting the model outcomes.

These revisions have been implemented throughout the manuscript.

2. The focus on multiyear drought is never clearly explained nor justified. Why the analysis focuses specifically on multiyear drought and not on drought in general? What is the reasoning behind such choice?

Response: We sincerely thank you for this comment. The decision to focus specially on multiyear drought (MYD) is motivated by both scientific and practical considerations.

MYDs represents the most consequential category of drought events because their impacts accumulate across seasons and years. Shorter droughts often allow for partial natural recovery through subsequent rainfall or seasonal hydrological recharge, whereas MYDs progressively deplete soil moisture, groundwater storage, and ecological resilience. In these long-duration events, the timing of additional water input becomes far more critical for drought alleviation, making MYDs an ideal research subjects for examining how hypothetical precipitation increases influence the drought severity (DS) of MYD events. Events of short duration tend to have limit impacts, are not extreme or typical, and often end naturally before any intervention could meaningful influence their progress.

The cumulative and multi-stage nature of MYDs brings uncertainty regarding the most effective timing of additional water input. Because these events occur across several hydrological cycles, the system's response to water-input perturbations is not uniform across months. This provides a meaningful opportunity to investigate drought attenuation efficiency (k) at different months of a prolonged drought, and to identify which stages are most sensitive to additional water input.

MYDs are also the events where index-based drought alleviation is most relevant. The PDSI framework accumulates deficits over time, and alleviating a deep, long-lasting deficit requires understanding how the system converts additional water input into reductions in DS. Our numerical

experiments are therefore particularly well suited to MYDs, where the relative importance of water-input timing is magnified.

This point has now been explicitly clarified in the revised manuscript. (Page 4, Lines 100 to 102)

Beyond the modeling perspective, these persistent, expansive, and difficult-to-recover MYDs have shown an escalating trend in severity, duration, and frequency under climate change (Dai, 2013; Stevenson et al., 2022; Chen et al., 2025). This further highlights the importance of understanding their dynamics and potential alleviation strategies.

These considerations collectively justify why MYDs serve as the focus of this study. In addition, several of the Specific comments (29, 37, and 38) raise related questions about the definition and treatment of MYDs in our framework, and detailed responses to these comments are provided in their respective locations.

References

Chen, L., Brun, P., Buri, P., Fatichi, S., Gessler, A., McCarthy, M.J., Pellicciotti, F., Stocker, B. and Karger, D.N.: Global increase in the occurrence and impact of multiyear droughts, Science, 387, 278-284, doi: 10.1126/science.ado4245, 2025.

Dai, A.: Increasing drought under global warming in observations and models, Nat. Clim. Change, 3, 52-58, doi: https://doi.org/10.1038/nclimate1633, 2013.

Stevenson, S., Coats, S., Touma, D., Cole, J., Lehner, F., Fasullo, J., and Otto-Bliesner, B.: Twenty-first century hydroclimate: A continually changing baseline, with more frequent extremes, Proc. Natl. Acad. Sci., 119, e2108124119, doi: https://doi.org/10.1073/pnas.2108124119, 2022.

3. The issue of non-linear response is not well discussed in the probabilistic context of drought indicators. The fact that drought severity changes non-linearly with the usual indicators is a well know factor, well reflected by the commonly used classifications of drought indices. In this context, it is not possible to simply compare a unitary change in severity between events occurring at different severity levels, as (example) a change in SPI from 0 to -1 and from -1 to -2 have completely different probabilistic implications. I suggest the authors to carefully revisit their considerations on the "mitigation" efficiency by explicitly taking into account for these disparities.

Response: Thank you very much for this thoughtful and important comment. We fully acknowledge the point that standardized drought indices exhibit nonlinear and uneven probabilistic spacing, and that a one-unit change (e.g., from 0 to -1 and from -1 to -2 in SPI) carries fundamentally different statistical implications, and our framework is built precisely on this understanding. We appreciate this reminder, as it highlights a key conceptual issue that we realized was not clearly explained in our original manuscript. We did not clearly distinguish between the instantaneous drought severity given by PDSI and the event-level drought severity (DS) that is actually used in our study.

To clarify our approach, the "drought severity" used in our analysis is not the instantaneous severity value provided directly by an index such as PDSI, SPI, or SPEI. Instead, DS is an event-based

cumulative indicator, computed using run theory as the integral of PDSI series over the drought duration (DD). DS therefore represents the accumulated water deficit over the entire event rather than a single index value. Importantly, the unequal spacing and probabilistic nonlinearity of PDSI are inherently preserved in the calculation of DS. Thus, our method does not rely on comparing a uniform one-unit change of the index across different severity levels. Instead, DS represents the cumulative effect of all PDSI values that naturally vary in magnitude, spacing, and probabilistic meaning.

Moreover, in light of your suggestion, we have carefully re-examined our assessment of attenuation efficiency (k). Because DS integrates PDSI over the entire MYD event, the calculation inherently accounts for the unequal probabilistic spacing and nonlinear response of the underlying index. These nonlinear characteristics are central to understanding why the timing of hypothetical water input does not always follow a monotonic "earlier is better" pattern. The selection of an optimal timing in our results is precisely driven by the nonlinear feature of PDSI and DS, rather than any assumption of linear response. This comment helped us recognize that our explanation of this conceptual link needed improvement.

We sincerely appreciate your insightful observations. They have prompted us to clarify key definitions and improve the conceptual rigor of the manuscript (Table S1). We have incorporated clarifications into the responses to the Specific comments (21, 22, 40, 41, 50 and 52), and to the Specific comment surrounding Eq. 3, explaining the difference between instantaneous PDSI severity and event-level DS, and reinforcing that our framework is fully compliant with the probabilistic structure of standardized drought indices.

4. The terminology used in the paper is sometime rather confusing. Different terms are used to define the same quantities in different part of the paper (e.g., t_optimal, t_a, t_b, t_c, etc.).

Response: We sincerely thank you for pointing out potential confusion in the terminology. We fully acknowledge that multiple labels for related quantities can be confusing if not clearly explained.

In the original manuscript, different symbols (t_a, t_b, and t_c) were used to represent optimal timing under different conceptual models. To reduce notation complexity, these have now been unified into a single symbol, $t_{optimal}$. In this study, the final interpretation focuses on the $t_{optimal}$ derived from the optimal solution model. The corresponding revisions have been implemented in the manuscript. (Page 8, Lines 199 to 214)

Although these symbols have been unified in the revised manuscript, we briefly explain the conceptual meaning of t_a, t_b, and t_c in this response document to clarify how the conceptual models are connected:

In our study, $t_{optimal}$ represents the general optimal timing of hypothetical additional precipitation at the event scale. The other symbols (t_a, t_b, and t_c) are introduced only to denote the optimal timing implied by different conceptual perspectives, as summarized in Fig. 4:

1. t_a corresponds to the optimal timing in Conceptual Model 1 (Figure 4b), which emphasizes the advantage of earlier water input within a drought event. Because the accumulated deficit is still low during the earlier month of a drought event, hypothetical precipitation added at this month produces a

larger proportional reduction in drought severity (DS) of a multiyear drought (MYD) event. Under this perspective, t_a tends to approach the first month of a MYD event (t_1).

2. t_b corresponds to Conceptual Model 2 (Figure 4c), which emphasizes the statistical sensitivity of standardized indices. For example, when PDSI values are far below the drought threshold (e.g., PDSI = -4), a small shift in cumulative probability produces a large numerical change in the index compared with values closer to the threshold (e.g., PDSI = -1) under the same perturbation (Fig. 1). Since PDSI values approach their maximum deviation below the threshold when moving toward the drought intensity point (DIP), this perspective indicates a potential drought alleviation advantage for hypothetical precipitation added timing near the month corresponding to DIP within a MYD event (t_{DIP}).

3. t_c is the optimal solution from the Optimal Solution Model (Figure 4d) and represents the balanced timing that reconciles the opposing tendencies of Conceptual Model 1 and 2. t_c denotes the overall optimal timing that our study aims to identify.

To reduce the readers' cognitive burden, we provide a consolidated reference table of principal symbols and acronyms. This table lists $t_{optimal}$, t_{DIP}, and other frequently used symbols, and gives a short description of each symbol (Table S1).

Additionally, Fig. 1 introduces the problem by contrasting event-level characteristics and standardized-index statistical features. Figures 1a and 1b illustrate event-level progression, while Figures 1c and 1d illustrate the probabilistic properties of standardized indices. Fig. 4 follows logically from Fig. 1 by converting the problem posed in Fig. 1 into analytical perspectives (Figures 4b and 4c) and proposed solutions (Fig. 4d). Conceptual Models 1 and 2 in Fig. 4 emphasize the two different mechanisms that drive alleviation advantage, and the Optimal Solution Model integrates them to determine $t_{optimal}$.

Several specific comments (7, 23, 24, 30, 31, 41 and 44) are directly related to this terminology issue and will be addressed in detail in their corresponding replies.

5. Overall, while a recognize some potential in this research, I suggest a careful revision of the text before considering it for publication. I added some further specific comments below, in the hope that they will be useful to improve the quality of the manuscript.

Response: We sincerely thank you for the overall assessment and for recognizing the potential of our research. We fully acknowledge the importance of clarity and conceptual consistency. In response to the specific comments, we have provided detailed clarifications and explanations in this response document, including revised phrasing, figure descriptions, and terminology definitions. We believe these clarifications improve the clarity and conceptual rigor of the study.

Specific comments

6. L12. Remove "especially".

Response: Thank you for this comment. The word "especially" has been removed as recommended. (Page 1, Line 12)

7. L13. The concept of "optimal rainfall replenishment timing (T_optimal)" needs to be better established here.

Response: Thank you for pointing out this issue. To address this concern, we have revised the manuscript to more clearly establish this concept by defining it as the timing that maximizes drought attenuation efficiency under additional precipitation ($t_{optimal}$). (Page 1, Lines 13 to 14)

8. L15. Calibration period. I suggest rewording.

Response: Thank you for the suggestion. In our experiment, the baseline period must remain unchanged to avoid affecting the hydrological parameters derived from historical conditions. Although the dataset covers 1961-2020, the calibration period (baseline) is set to 1961-1990, and the numerical experiments adding hypothetical water input are applied only to 1991-2020. This ensures that adding hypothetical precipitation to specific months affects only the experiment period and does not retroactively alter the baseline conditions or the associated hydrological parameters.

The manuscript has been reworded accordingly to clarify this description. (Page 1, Lines 14 to 18)

Subsequent comments related to this issue in this document, including Comments 10, 33, 36, and 43, are addressed accordingly in their respective responses.

9. L16. "351 MYD". These are obviously not droughts, but cells under drought. Many of these cells are under the same MYD contemporarily. Please reword, as this statement seems to suggest that 351 MYD were observed in a period of only 60 years.

Response: Thank you for pointing out this potential misunderstanding. We clarify that our analysis is conducted at the grid scale. For each of the 199 grids during 1991-2020, multiyear drought (MYD) occurrences are identified independently, and 351 grid-level MYD instances were identified in total. Some of these grid-level instances may correspond to the same regional MYD event, but our study focuses on the grid-level drought periods as the statistical units for attenuation efficiency analysis, rather than aggregating drought events at the regional scale.

We also clarify why the grid-based treatment is necessary and appropriate for our framework. Grid identification allows us to directly link drought characteristics to the underlying station observations within each grid. When multiple stations fall inside one grid, their meteorological records are first averaged to represent the grid box, and the drought attenuation efficiency (k) is subsequently calculated at the grid level. This helps reduce biases caused by the uneven spatial distribution of stations across China and ensures that gridded results remain observation-based. Working at the grid scale also

provides a consistent spatial structure that aligns with the way most hydroclimatic datasets are distributed worldwide. Many datasets used for drought assessment, especially global and regional data products, are available exclusively in gridded format. Mapping station-derived k onto grids therefore makes our approach more easily reproducible and transferable to other regions or for future global-scale applications. Additionally, because run theory, as applied in this study, quantifies drought characteristics based on a single time series record, its formulation is inherently suited to station-level or single-grid calculations, making the grid an appropriate methodological unit for analyzing k. At the same time, we also recognize that beyond the station and grid scales, drought processes exhibit coherent dynamics at the regional scale. We are therefore exploring clustering-based approaches to identify and analyze regional drought events. This direction represents an important extension of the present work and is being developed as part of our subsequent research. Given that this study focuses on the patterns and statistics at the grid scale, the regional scale clustering analysis will be presented in a separate follow-up study rather than included here.

The manuscript has been revised accordingly to clarify the interpretation of the MYD count. (Page 1, Lines 16 to 18).

Subsequent comments related to this issue in this document, including Comments 39, 50, and 53, are addressed accordingly in their respective responses.

10. L17. Is the study period 1991-2020 or 1961-2020? The different periods are confusing.

Response: Thank you for pointing this out. The two periods serve different purposes in our framework. The full dataset covers 1961-2020, but the baseline hydrological parameters are derived from 1961-1990, while the numerical experiments adding hypothetical precipitation are conducted only for 1991-2020. This design ensures that the experiments do not retroactively alter the baseline conditions or the associated hydrological parameters, clarifying the distinction between the calibration and study periods.

The manuscript has been revised accordingly to clarify this distinction. (Page 1, Lines 14 to 18).

11. L17-18. A key coefficient (k)… This does not clarify what "k" is.

Response: Thank you for pointing this out. In our framework, k quantifies the system's drought attenuation efficiency, namely how rapidly hypothetical additional precipitation is converted into reductions in drought severity (DS) of multiyear drought (MYD) events. Higher k values correspond to faster alleviation, while lower k values reflect slower hydrological recovery or more persistent drought conditions.

Given the central role of k in our study, we provide a more complete explanation below.

To clarify our approach, the exponential formulation in DAQM is not arbitrarily postulated. Our modeling framework was developed by performing numerical experiments driven by observations from meteorological stations. Before specifying any functional structure, we first examined how drought severity (DS) of a multiyear drought event responds to incremental precipitation increases within

controlled numerical experiment. Across events and stations, the numerical experiments consistently demonstrated a clear diminishing return pattern (Fig. 5). When hypothetical additional precipitation was incrementally increased at the same month of a drought event, the reduction in DS showed progressively weaker marginal benefits. The initial portion of added water input produced the strongest alleviation, while subsequent increments at that same month yielded progressively weaker effects. This empirical feature motivated us to seek a formulation capable of representing a bounded recovery process. To formalize this behavior, we drew an analogy with systems exhibiting saturation-type adjustment, similar to first-order kinetic expressions (e.g. $C_t=C_0 \cdot exp(-k \cdot t)$, where C_t denotes the remaining intensity at time t, C_0 is the initial state at $t=0$, and k is the first-order rate constant controlling the decay speed. The instantaneous decay rate $\frac{dC}{dt}$ is proportional to the current magnitude C_t.). Following this reasoning, we adopted the formulation: $DS_{rate}=1-exp(-k \cdot pr_{rate})$. This formulation captures how incremental water input progressively reduces the residual DS (Fig. 1). In this framework, $exp(-k \cdot pr_{rate})$ represents the proportion of remaining DS after a given additional precipitation increment. When the additional water supply is sufficient, the relative reduction in DS approaches a practical upper limit, a pattern similar to physical constraints such as finite soil water storage capacity or vegetation water use limits.

To demonstrate that the exponential structure represents real system behavior, we compared formulations inspired by zero-order, first-order, and second-order kinetic processes using historical multiyear drought events (MYDs) during 1991-2020. Across most stations and events, the first-order structure more accurately reproduced the curvature observed in the numerical experiments at different month of MYDs. We further extended this comparison to an n-order formulation and found that the estimated n converged toward 0.98 across MYDs, which is essentially unity. This outcome strengthens the conclusion that a first-order structure is the most appropriate representation of the observed pattern. The results are summarized in Table S2. We also compared the exponential form against alternative empirical relationships. Logistic curves did not match the observed pattern. Power-law functions were unable to satisfy the required boundary conditions. We note that the same exponential adjustment structure has been adopted in other fields. For instance, Childs et al. (2025) applied a similar functional pattern in modeling climate-driven disease dynamics. The recurrence of this structure across domains further supports its suitability for representing systems with constrained recovery capacity.

Table S2: Comparison of R^2 values for different functional models

Model order	Differential form	Integral form	t_1	t_2	t_3
Zero-order	$\frac{\partial DS_{rate}}{\partial pr_{rate}} = k$	$DS_{rate}=k \cdot pr_{rate}$	0.992	0.984	0.989
First-order	$\frac{\partial DS_{rate}}{\partial pr_{rate}} = k \cdot (1 - DS_{rate})$	$DS_{rate}=1-exp(-k \cdot pr_{rate})$	0.985	0.996	0.993
Second-order	$\frac{\partial DS_{rate}}{\partial pr_{rate}} = k \cdot (1 - DS_{rate})^2$	$DS_{rate}=1-1/(k \cdot pr_{rate}+1)$	0.982	0.976	0.961
N-order	$\frac{\partial DS_{rate}}{\partial pr_{rate}} = k \cdot (1 - DS_{rate})^n$	$DS_{rate}=1-[1-k \cdot (1-n) \cdot pr_{rate}]^{1/1-n}$	0.989	0.996	0.998

Note: t_1, t_2, and t_3 represent the first to the third month of the MultiYear Drought event.

The manuscript has been revised accordingly to explicitly define *k* as the coefficient quantifying MYD attenuation efficiency. (Page 1, Lines 18 to 20)

References

Childs, M.L., Lyberger, K., Harris, M.J., Burke, M., and Mordecai, E.A.: Climate warming is expanding dengue burden in the Americas and Asia, Proc. Natl. Acad. Sci., 122, e2512350122, doi: https://doi.org/10.1073/pnas.2512350122, 2025.

Subsequent comments related to this issue in this document, including Comments 13, 32, 44, 46, and 47, are addressed in their respective replies.

12. L18. T_optimal should be a subscript.

Response: Thank you for the suggestion. We have carefully checked the notation and ensured that $t_{optimal}$ is consistently presented with a subscript throughout the manuscript. (Page 1, Line 19)

13. L20. This sentence is not clear if you do not clarify what k is.

Response: Thank you for this comment. As clarified in our response to Comment 11, *k* quantifies the system's drought attenuation efficiency, namely how rapidly hypothetical additional precipitation is converted into reductions in drought severity (DS) of multiyear drought (MYD) events. Higher *k* values correspond to faster alleviation, while lower *k* values reflect slower hydrological recovery or more persistent drought conditions. k_{max} refers to the maximum monthly *k* within a MYD event and thus identifies the optimal timing for drought alleviation under the same perturbation.

This has been clarified in the revised manuscript. (Page 1, Lines 18 to 22)

14. L24. T_optimal should be subscript.

Response: Thank you for the suggestion. We have carefully checked the notation and ensured that $t_{optimal}$ is consistently presented with a subscript throughout the manuscript. (Page 1, Line 25)

15. L24-25. This sentence needs to be clarified, as it is not clear to me how these findings can be used for "intervention".

Response: Thank you for this comment. As clarified in Comment 1 in this document, although the findings are derived from idealized experiments, they provide insights into how the system intrinsically responds to additional water input. The hypothetical additional precipitation applied represents increased water availability, allowing us to identify the timing ($t_{optimal}$) when drought severity (DS) of a multiyear drought (MYD) event is most effectively reduced. By consistently distinguishing MYD attenuation efficiency (system response) and drought alleviation (event-level improvement), the results conceptually inform discussions on real-world MYD alleviation strategies, without implying direct control over rainfall.

This has been clarified in the revised manuscript. (Page 1, Lines 25 to 27)

16. L30. I was not able to check the reported number of "3000 deaths" in the US, beside the Science paper cited in the text. I suggest checking the source of this information.

Response: Thank you for this comment. We have rechecked this figure and confirmed that the reported number of approximately 3,000 deaths in the United States was obtained from the National Integrated Drought Information System, an official U.S. government platform that coordinates drought monitoring, forecasting, planning, and information at national, state, and local scales (Smith, 2020). The data are available at: https://www.drought.gov/news/high-cost-drought. The source provides the basis for the reported number.

The corresponding citation has been verified and further supported by an additional citation in the revised manuscript. (Page 1, Line 33)

Smith, A. B.: 2010-2019: A landmark decade of U.S. billion-dollar weather and climate disasters, National Oceanic and Atmospheric Administration, 2020.

17. L55. "three key dimensions". Please reword, as the term "dimension" is misused here.

Response: Thank you for pointing this out. The "term" is replaced with "characteristics" to more accurately reflect that drought duration (DD), drought intensity (DI), and drought severity (DS) describe distinct features of a drought event rather than mathematical or spatial dimensions. (Page 2, Line 57)

18. L55. "the lowest". This is true only for indicator where droughts are negative. Please clarify (largest in absolute value).

Response: Thank you for this comment. In our study, we focus only on drought events. During these events, the standardized drought indices take negative values, and greater severity corresponds to larger departures of the index below the critical threshold. Therefore, drought intensity (DI) is now defined as the maximum deviation of the drought index below the critical threshold during a drought event. This revised wording provides a clearer and more general interpretation of DI. (Page 2, Line 58)

19. L57. These are not indicators but rather drought features.

Response: Thank you for this comment. We agree that these are features describing drought event characteristics rather than drought indicators. The wording has been revised accordingly in the manuscript. (Page 2, Line 60)

20. L60. "the four stages". These stages have not been introduced yet.

Response: Thank you for this comment. We agree that the four stages had not yet been explicitly stated at this point in the original manuscript. To improve clarity, we have now added a brief definition at the first mention, specifying that the four stages refer to the onset, development, recovery, and termination of drought events. (Page 2, Lines 63 to 64)

21. L68-70. This sentence is misleading. A large numerical shift in anomalies (e.g., from 4 to 2) does not necessarily means a large change in drought conditions, as conditions still remains extreme. This is due to the non-linearity in the drought definition according to the indices, but not to a non-linear response of drought to precipitation. I think that this needs to be better clarified, as it has huge implications on your analyses.

Response: Thank you for this valuable comment. The original sentence may have caused a misleading interpretation. Our intention was not to suggest that a numerical change in a standardized drought index (e.g., from 4 to 2) necessarily represents a proportional improvement in actual drought events. Our original sentence was intended to describe the statistical behavior of standardized indices.

Standardized drought indices follow a normal distribution, so values in the tail region are numerically more sensitive to small shifts in cumulative probability. Consequently, when the standardized index is far below its drought threshold, even a small hydrological improvement can lead to a relatively large numerical change in the index. This sensitivity arises from the statistical construction of the standardized index. This is the specific meaning we aimed to convey in the original sentence.

This characteristic was already considered in our framework, which is why we focus on drought severity (DS) derived through run theory to capture the accumulated PDSI values over a drought event rather than a single anomaly value. Here we clarify this point and emphasize that under the same perturbation, PDSI values far below the drought threshold exhibit relatively large numerical changes compared with values closer to the threshold, reflecting the probabilistic structure of standardized indices. At the same time, DS provides a meaningful evaluation of event-level drought response. These two considerations together motivate the design of our numerical experiments.

The manuscript has been revised to clarify this point accordingly. (Page 3, Lines 72 to 75)

22. L71. This example is incorrect, as a 1-unit change does not have the same physical and probabilistic meaning at, e.g., -1 or -4. Maybe I misunderstand your statement, and we are saying the same think, but changes in anomalies cannot be treated linearly but they need to account for the non-linearity.

Response: Thank you for this comment. We agree that a 1-unit change in a standardized drought index does not have the same physical and probabilistic meaning across the range of values, and that the non-linearity of the index must be considered. This is exactly the conceptual basis of our study.

In the manuscript, the example using PDSI values of -4 and -1 (Fig. 1c and 1d) was intended to illustrate the statistical properties of standardized indices. Specifically, for PDSI values that are far below the drought threshold (PDSI = -4), a small shift in cumulative probability results in a larger numerical change in index compared with values closer to the threshold (PDSI = -1) under the same perturbation. This demonstrates the non-linear probabilistic spacing of the index, and it is this standardized feature that underlies our approach to analyzing drought attenuation efficiency.

We recognize that the arrangement of Fig. 1 may have led to some confusion. Fig. 1a and 1b describe event-level characteristics. During the early months of a drought event, the PDSI deviation is relatively

small and the accumulated deficit is low (Fig. 1a). As the event progresses toward the drought intensity (DI) point, defined as the maximum deviation of the index below the critical threshold during the event, the PDSI deviation increases and the accumulated deficit becomes larger (Fig. 1b). Fig. 1c and 1d focus solely on the statistical behavior of the standardized index and the relative sensitivity of PDSI values under the same perturbation, independent of event-level cumulative deficits. The separation of these panels is intentional because it distinguishes between the standardized-index response (Fig. 1c and 1d) and the event-level progression (Fig. 1a and 1b).

The potential misinterpretation that Fig. 1c and 1d reflect cumulative event-level drought severity is understandable given the layout. Our key point is that early-stage PDSI deviations are smaller and correspond to less accumulated deficit, while later-stage, more extreme deviations are more sensitive to identical perturbations but occur when substantial deficit has already been accumulated. This distinction explains why, in Fig. 4, Conceptual Model 1 and 2 are separated to illustrate the opposite alleviation advantages. The standardized index is more responsive at extreme deviations, whereas the event-level deficit has already progressed significantly.

In summary, the numerical example and Fig. 1c-d are intended to illustrate the statistical, non-linear behavior of the standardized index, which is distinct from the event-level cumulative drought severity discussed in Fig. 1a-b.

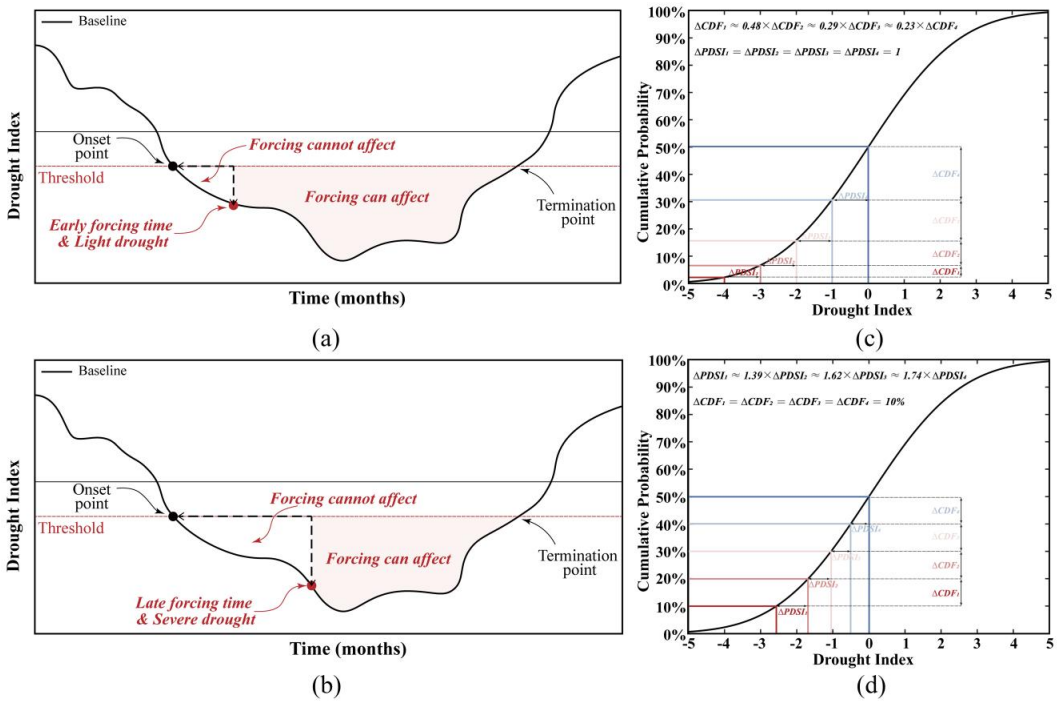


Figure 1: Conceptual and statistical illustrations related to drought alleviation and the normal distribution characteristics of commonly used drought indices (with PDSI as an example). Here, "forcing" refers to a hypothetical incremental water input applied at a given month within a drought event for conceptual illustration. Panels (a) and (b) illustrate the event-level progression of a drought and the associated accumulation of deficit. (a) During the early stage of a drought event, PDSI deviation is relatively small and the accumulated deficit is low. (b) As the drought progresses toward the drought intensity (DI) point, which is defined as the maximum deviation of the index below the critical threshold during the event, the PDSI deviation increases and the accumulated deficit becomes larger. Panels (c) and (d) illustrate the statistical

behavior of the standardized index, independent of event-level cumulative deficit. (c) Change in cumulative probability induced by a one-unit shift in PDSI at different levels of PDSI deviation below the drought threshold. (d) PDSI shift required to achieve an equivalent change in cumulative probability at different levels of PDSI deviation.

The relevant text has been checked and slightly refined for clarity to avoid potential misunderstanding. (Page 3, Lines 75 to 77)

23. Fig. 1. The term forcing is used without a proper definition.

Response: Thank you for this comment. In our study, "forcing" specifically refers to a hypothetical incremental water input applied at a given month within a drought event for conceptual illustration. This definition has now been explicitly stated in the manuscript and clarified in relation to Fig. 1. (Page 4, Line 86)

24. L85. This concept is here introduced without a proper definition.

Response: Thank you for pointing out this issue. Here we clarify that $t_{optimal}$ refers to the month during a multiyear drought event when additional precipitation (implemented as hypothetical water input in our numerical framework) achieves the greatest drought attenuation efficiency (i. e., corresponding to the maximum value of k). This definition has now been explicitly clarified in the manuscript. (Page 4, Lines 95 to 96)

25. L90. Fig. 3 is cited before fig. 2.

Response: Thank you for the comment. We have adjusted the figure citations so that all figures are cited in numerical order. The corresponding adjustment has been made in the revised manuscript. (Page 4, Line 102)

26. L104. Remove and.

Response: Thank you for this comment. The sentence has been adjusted accordingly in the revised manuscript. (Page 4, Lines 117 to 118)

27. L105. Some additional details on the selection procedure are needed.

Response: Thank you for this comment. Daily records from 1960-2020 were examined for each station, and stations with more than 5% missing values were excluded. For the remaining stations, short gaps ($\leq$ 5 days) were filled by linear interpolation, while longer gaps were replaced using the multi-year mean for the corresponding day. This procedure ensures continuity of the time series while minimizing bias in subsequent analyses.

The corresponding details have been added in the revised manuscript. (Page 4, Lines 118 to 121)

28. Fig. 2. Add region names instead of only numbers.

Response: Thank you for the suggestion. The region names have been now added to Fig.2 in the revised manuscript. (Page 5, Fig. 2)

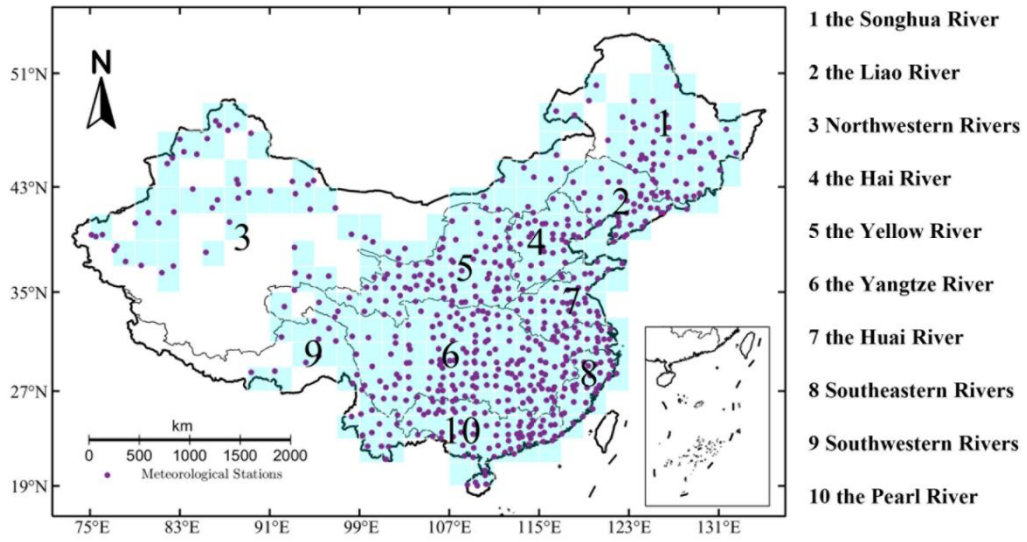


Figure 2: The study area covers 10 large river basins (regions). No. 1, the Songhua River; No. 2, the Liao River; No. 3, Northwestern Rivers; No. 4, the Hai River; No. 5, the Yellow River; No. 6, the Yangtze River; No. 7, the Huai River; No. 8, Southeastern Rivers; No. 9, Southwestern Rivers and No. 10, Pearl River. A total of 199 ($2° \times 2°$) grid boxes (color: cyan) were identified as containing at least one station with continuous instrumental meteorological records. In total, 620 meteorological stations (purple dots) were used.

29. L121. "representative". How?

Response: Thank you for this comment. The representativeness of MYDs in this study is ensured through both duration and intensity criteria. Previous studies have applied different duration thresholds to define MYDs, ranging from at least 13 months (Brunner and Tallaksen, 2019; Mourik et al., 2025) to at least two years (Chen et al., 2025). Considering the objectives of our study and the need to retain sufficient samples for statistical analysis, we adopted the widely used at least 13 months criterion and supplemented it with an intensity requirement.

Specifically, MYDs were identified at the $2° \times 2°$ grid level as events where PDSI remained below -1 for at least 13 consecutive months. To further ensure representativeness and consistency with established MYD characteristics, only events with $DD > 12$ and $DI < -3$ were retained. This combination of duration and intensity criteria guarantees that the selected MYDs reflect significant, prolonged drought events rather than short or marginal deficits, thereby capturing events that are physically meaningful.

The corresponding clarification has been incorporated into the revised manuscript. In Section 3.1, representative MYDs are described as events capturing persistent and severe drought characteristics to maintain the conciseness of the overview description. (Page 5, Lines 139 to 142) The detailed identification criteria are provided separately in Section 3.3. (Page 7, Lines 183 to 186)

References

Brunner, M.I. and Tallaksen, L.M.: Proneness of European catchments to multiyear streamflow droughts, Water Resour. Res., 55, 8881-8894, doi: https://doi.org/10.1029/2019WR025903, 2019.

van Mourik, J., Ruijsch, D., van der Wiel, K., Hazeleger, W., and Wanders, N.: Regional drivers and characteristics of multi-year droughts, Weather Clim. Extremes, 48, 100748, doi: https://doi.org/10.1016/j.wace.2025.100748, 2025.

Chen, L., Brun, P., Buri, P., Fatichi, S., Gessler, A., McCarthy, M.J., Pellicciotti, F., Stocker, B. and Karger, D.N.: Global increase in the occurrence and impact of multiyear droughts, Science, 387, 278-284, doi: 10.1126/science.ado4245, 2025.

30. L122. PIP is not well defined. Is it the value of the PDSI or the time?

Response: Thank you for the comment. We clarify that the Drought Intensity Point (DIP) refers to the PDSI value at the drought intensity (DI) point of a drought event. DI is defined as the maximum deviation of the PDSI below the critical threshold during the event. The month at which this DIP occurs is denoted as t_{DIP}. (Page 6, Line 141)

To improve clarity and consistency of notation, the symbol DIP is now used throughout the manuscript instead of DIP.

31. L125-128. This paragraph is confusing, as it relies on several concepts not introduced yet.

Response: Thank you for this helpful comment. We agree that this paragraph could be confusing if key concepts were not sufficiently introduced when first mentioned. To improve clarity, we have refined the wording in this paragraph to better define the key concepts (such as k, k_{max}, and $t_{optimal}$) within the methodological framework, so that the paragraph functions as a concise overview of the framework and is clear on its own. (Page 6, Lines 145 to 153)

In addition, a consolidated table listing principal symbols, acronyms and their descriptions has been provided for reference (Table S1 of the Supplement).

32. Fig. 3. Many terms are not explained here (pr_mean, is it the long term of the mean of each month?), k_theoretical and k_actual.

Response: Thank you for this helpful comment. Figure 3 is intended to serve as a workflow and method overview, and therefore remains placed early in the Methods section to provide readers with a structural understanding of the study design. (Page 6, Fig. 3) To address the concern that some terms were not sufficiently explained, we have revised both Fig. 3 and the corresponding textual description in Section 3.1. (Page 6, Lines 137 to 153) In the revised manuscript, the key concepts are first introduced and explained in the text, followed by Fig. 3, which provides a structural visualization of the workflow and methodological framework. In addition, we have improved symbol descriptions throughout the manuscript to enhance readability and consistency. A consolidated table listing principal symbols, acronyms and their descriptions has also been provided for easy reference (Table S1).

Specifically, $\overline{pr}$ is the long-term mean monthly precipitation. The amount of hypothetical precipitation added in the numerical experiments is calculated as $pr_{rate} \times \overline{pr}$, ensuring comparability across grids and months with different climatological precipitation levels.

$k_{theoretical}$ is obtained by fitting the DAQM analytical solution $DS_{rate} = 1 - exp(-k \cdot pr_{rate})$ to the full series of data-based experimental outcomes across all perturbation steps. It represents the theoretical efficiency implied by the DAQM framework, capturing the nonlinear, bounded response of DS to hypothetical additional precipitation. $k_{theoretical}$ also integrating the combined effects of both Conceptual Model 1 and 2 (as clarified in our response to Comment 4 at the beginning of this document).

k_{actual} is directly calculated from the numerical experiments. It represents the empirical (data-based) sensitivity of DS to small precipitation perturbations, approximated from the initial slop of the DS_{rate} and pr_{rate} relationship. Specifically, because the model expansion for small pr_{rate} gives $DS_{rate} \approx k \cdot pr_{rate}$ for $pr_{rate} \ll 1$, we approximate $k_{actual} = \frac{DS_{rate}(pr_{rate}=0.05)}{0.05}$, where 0.05 is the smallest perturbation step used in the experiments. This provides a direct, local estimate of the initial slope.

Our main analysis focuses on $k_{theoretical}$ because it characterizes the system-level response of DS to hypothetical additional precipitation and suitable for spatial and temporal comparison. k_{actual} serves as an independent reference to evaluate the internal consistency and numerical credibility of the DAQM formulation, but it is not used as the primary analysis indicator ($\varepsilon = \left(\frac{k_{theoretical} - k_{actual}}{k_{actual}}\right) \times 100\%$).

33. L132. PDSI requires calilbration. How was this performed? Add details.

Response: Thank you for this comment. The PDSI used in this study was calculated using the Climatically Appropriate For Existing Conditions (CAFEC) formulation (Jacobi et al., 2013), which includes a full calibration process to ensure regional applicability. The calibration was performed over the baseline period 1961-1990. For each grid derived from gridded station-based observations, monthly climatological values of the water balance components (including precipitation, potential evapotranspiration, recharge, runoff and losses) were first derived over the calibration period. From these, the CAFEC precipitation and the monthly calibration coefficients were computed following the algorithm. Departures between observed precipitation and CAFEC were then multiplied by the calibrated weighting factor to obtain the Z-index and PDSI time series.

The corresponding methodological clarification has been incorporated into the revised manuscript. (Page 7, Lines 158 to 165)

References

Jacobi, J., Perrone, D., Duncan, L.L., and Hornberger, G.: A tool for calculating the Palmer drought indices, Water Resour. Res., 49, 6086-6089, doi: https://doi.org/10.1002/wrcr.20342, 2013.

34. L135. Was the FAO-56 applied directly on monthly data? Clarify how, as daily data are previously discussed.

Response: Thank you for this comment. The FAO-56 method was not applied directly to monthly data. Instead, daily potential evapotranspiration (PET) was first calculated using the FAO-56 formulation based on daily meteorological observations. The resulting daily PET values were then aggregated to obtain monthly PET, which was subsequently used as input for the PDSI calculation.

The corresponding clarification has been incorporated into the revised manuscript. (Page 7, Lines 169 to 170)

35. L138. How is G modelled?

Response: Thank you for the comment. In this study, because potential evapotranspiration (PET) was calculated at the daily timescale, and the magnitude of the day soil heat flux (G) beneath the grass reference surface is relatively small, it may be ignored and thus $G \approx 0$ (Allen et al., 1998).

The corresponding clarification has been incorporated into the revised manuscript. (Page 7, Lines 172 to 173)

References

Allen, R. G., Pereira, L. S., Raes, D., and Smith, M.: FAO Irrigation and Drainage Paper No. 56 – Crop Evapotranspiration, FAO, Rome, Italy, 1998.

36. L142. Why is the period 1961-1990 used for reference? If the analysis focuses on 1991-2020, why wasn't this period (recommended by WMO) used instead.

Response: Thank you for this thoughtful comment. Consistent with our responses to Comments 8 and 10, in our study, 1991-2020 serves as the simulation period for applying hypothetical additional precipitation in the numerical experiments. Using the same period as the baseline would alter the reference climatology and the derived hydrological parameters, potentially confounding the experimental results. To maintain an independent and stable reference, 1961-1990 is adopted as the calibration period, while all numerical experiments are applied only to 1991-2020. This approach ensures that the effects of hypothetical additional precipitation are evaluated consistently against a fixed climatological reference.

The corresponding clarification has been incorporated into the revised manuscript. (Page 7, Lines 158 to 163)

37. L148. What is a "typical" MYD? Please clarify.

Response: Thank you for the comment. In this study, a "typical" MYD is defined as an event that captures the key characteristics of persistent and severe droughts, ensuring both temporal continuity and intensity. Following the approach described in our response to Comment 29, at each 2° × 2° grid,

drought events are first identified where PDSI remains below -1 for at least 13 consecutive months. To ensure representativeness and focus on significant events, only those with $DD > 12$ and $DI <- 3$ are retained.

These criteria guarantee that the selected MYDs are temporally continuous, sufficiently severe, and physically meaningful. By combining both duration and intensity thresholds, the definition of "typical" MYDs aligns with previous studies (Brunner and Tallaksen, 2019; Mourik et al., 2025; Chen et al., 2025) and ensures that the events included in the analysis reflect the most consequential and characteristic prolonged droughts in the study region.

The corresponding clarification has been incorporated into the revised manuscript. (Page 7, Lines 183 to 185)

References

Brunner, M.I. and Tallaksen, L.M.: Proneness of European catchments to multiyear streamflow droughts, Water Resour. Res., 55, 8881-8894, doi: https://doi.org/10.1029/2019WR025903, 2019.

van Mourik, J., Ruijsch, D., van der Wiel, K., Hazeleger, W., and Wanders, N.: Regional drivers and characteristics of multi-year droughts, Weather Clim. Extremes, 48, 100748, doi: https://doi.org/10.1016/j.wace.2025.100748, 2025.

Chen, L., Brun, P., Buri, P., Fatichi, S., Gessler, A., McCarthy, M.J., Pellicciotti, F., Stocker, B. and Karger, D.N.: Global increase in the occurrence and impact of multiyear droughts, Science, 387, 278-284, doi: 10.1126/science.ado4245, 2025.

38. L149. Did you perform any pooling? A single month of interruption was enough to stop a MYD? Clarify

Response: Thank you for this comment. In this study, identified MYDs are strictly continuous, meaning that the PDSI remains below -1 for the entire duration without interruption. A single month with PDSI above this threshold would terminate the event. No pooling across non-continuous periods was performed. This strict continuity criterion ensures that all selected events represent uninterrupted multiyear droughts with consistent intensity (DI = -3), allowing us to accurately assess the timing of hypothetical precipitation increases on drought alleviation. By focusing on continuous events, we avoid conflating separate drought stages (including onset, development, recovery, and termination) and ensure that the analysis captures the full temporal structure of prolonged droughts, which is essential for evaluating system responses and attenuation efficiency.

The corresponding clarification has been incorporated into the revised manuscript. (Page 8, Lines 185 to 186)

39. L151-152. These are not events, but drought periods for each given grid cells. Speaking about "at least 11" multiyear drought events in each basin over 30 years is misleading.

Response: Thank you for the comment. As clarified in our response to Comment 9, our analysis is conducted at the grid scale. For each of the 199 grids during 1991-2020, multiyear drought (MYD) occurrences are identified independently. Across all grids, a total of 351 grid-level MYD instances were identified. Some of these may belong to the same regional MYD event, but our study focuses on the grid-level response rather than regional aggregation. This grid-scale approach is appropriate because it captures spatial heterogeneity in drought severity and timing, which is essential for addressing our research questions.

When summarizing basin-level characteristics, aggregation is performed across grid-level MYD samples within each basin, providing a statistically meaningful representation of basin-scale drought attenuation efficiency (k). In this framework, the statement that each basin contained at least 11 MYD samples refers to the number of valid grid-level MYD periods rather than independent basin-level drought events. This clarification aligns with the explanation provided in our responses to Comments 9 and 50.

To avoid possible confusion between regional drought events and grid-level drought events, corresponding clarifications have been incorporated into the revised manuscript. Specifically, MYD counts are explicitly described as grid-level MYD samples rather than basin-level independent drought events, and the grid-level definition of MYDs has been clarified in the Introduction. (Page 4, Lines 103 to 105; Page 8, Lines 186 to 189)

40. L164. "greater negative values". Even if this is true, the relationship is not linear, and the same DS change may mean very different things depending on the conditions. As an example, if you use the corresponding probability rather than the standardize anomalies, the results will be very different. The "mitigation" effect should be independent from the way the metric is presented.

Response: Thank you for this comment. We clarify that in our study, drought severity (DS) represents the sum of monthly PDSI values over the duration of a drought event (DD), capturing both the severity and persistent of drought rather than being a single PDSI value. Therefore, all analyses of drought attenuation efficiency are conducted at the event level, which represents the integrated impact of hypothetical additional precipitation across the drought period.

In our framework, the standardized nature of drought indices is fully considered. As discussed in our response to Comment 22, PDSI values that deviate further below the drought threshold exhibit relatively larger numerical changes under the same perturbation compared with values closer to the threshold. This characteristic affects how a single month of hypothetical additional precipitation contributes to reductions in DS over the event, as illustrated by Conceptual Model 2. In other words, the non-linearity and tail sensitivity of the standardized index influence the conversion efficiency of added precipitation into DS reduction, linking the single-month PDSI response to the event-level DS response.

Accordingly, the original sentence contained a misleading description. The intended meaning is that PDSI values further below the drought threshold exhibit larger numerical changes under the same perturbation, as explained above.

The corresponding clarification has been incorporated into the revised manuscript. (Page 8, Lines 204 to 208)

41. L166. I do not follow this logic. Maybe it is still a consequence of the misunderstanding on the meaning of a unitary change in DS. Please reword and clarify, ad this is a key assumption of your methodology.

Response: Thank you for this comment. Before clarifying the logic of Conceptual Model 2, we note that the drought severity (DS) used in our framework is an event-level cumulative quantity, not the instantaneous value of PDSI. DS is calculated as the integral of the PDSI series over the drought duration using run theory. Therefore, the unequal spacing and probabilistic non-linearity of PDSI are inherently preserved in DS. Then the logic under Conceptual Model 2 is rooted in the statistical sensitivity of standardized drought indices such as PDSI. PDSI values that fall far below the drought threshold respond more strongly to the same hypothetical precipitation perturbation. As illustrated in Fig. 1, for example, a shift in cumulative probability produces a much larger numerical change when the PDSI is -4 compared with -1 under identical perturbation.

Within a multiyear drought (MYD) event, PDSI values progressively deviate further below the threshold when approaching the drought intensity point (DIP), which corresponds to the maximum deviation of the PDSI below the critical threshold during the event. Additional hypothetical water input applied at the month corresponding to DIP (t_{DIP}) is expected to produce the greatest reduction in the event-level drought severity (DS) of MYD event under Conceptual Model 2. Consequently, the optimal timing predicted by Conceptual Model 2 tends to coincide with t_{DIP}.

The corresponding clarification has been incorporated into the revised manuscript. (Page 8, Lines 203 to 209)

42. Fig. 4a is the same as in Fig. 3. Avoid repeating the same plot multiple times.

Response: Thank you for this comment. We would like to clarify that in the original structure of the manuscript, Fig. 3 and Fig. 4a served different purposes. Fig. 3 presents a workflow-style overview of the methodological framework, with its subpanels designed to enhance the readability of the framework. In contrast, Fig. 4a illustrated drought event under baseline pattern before applying hypothetical additional precipitation. Because Fig. 4a supports the conceptual derivation of the DAQM, it is not a duplicate of Fig. 3 but plays a distinct explanatory role.

However, after considering this comment together with other suggestions in this document, we have revised the figure content to reduce overlapping information and to further clarify the distinct roles of the two figures. Specifically, Fig. 3 now focuses on the methodological workflow overview, while Fig. 4 emphasizes the baseline pattern and its role in the conceptual derivation of DAQM. The revised figures are presented in Page 6 (Fig. 3) and Page 9 (Fig. 4a).

43. L185. Why the absence of overlap between the baseline period and the experiment period is needed?

Response: Thank you for the comment. Consistent with our responses to Comments 8, 10, 33, and 36, the numerical experiments add hypothetical additional precipitation only after the start of the simulation period. If the baseline period overlapped with the experiment period, the added hypothetical precipitation perturbations would also influence the calculation of the reference climate and related hydrological parameters. Having no overlap ensures that the baseline remains fixed and unaffected, so the response to hypothetical additional precipitation can be evaluated relative to a stable and independent reference.

The corresponding clarification has been incorporated into the revised manuscript. (Page 9, Lines 229 to 232)

44. L194. Clarify the acronym. Also, why a theoretical model is needed? As seen later, this introduced some unneeded approximations. Cannot be the same analyses made on the empirically computed values Delta_DS and Delta_pr?

Response: Thank you for this comment. DAQM stands for drought attenuation quantitative model. As clarified in our response to Comment 11, in our framework, k quantifies the system's drought attenuation efficiency, namely how rapidly hypothetical additional precipitation is converted into reductions in drought severity (DS) of multiyear drought (MYD) events. DAQM was introduced to provide a consistent and system-level parameterization of the DS-precipitation response, integrating the effects of Conceptual Models 1 and 2 (as clarified in our response to Comment 4 at the beginning of this document) and capturing the nonlinear attenuation behavior observed in the numerical experiments. While empirical ΔDS and Δpr values can be used directly, they are more variable and less comparable across grids, forcing timings, and MYD samples. The DAQM model allows this nonlinear attenuation behavior to be summarized into a physically interpretable attenuation efficiency parameter (k), enabling systematic comparison across different drought conditions.

The corresponding clarification has been incorporated into the revised manuscript. (Page 10, Lines 241 to 253)

45. Eq. 3. As stated before, a change in DS can have different meaning depending on the severity itself. How did you account for that?

Response: Thank you for this comment. We clarify that that Eq. 3 represents the differential form of the drought alleviation process at the event level, using drought severity (DS) calculated as the cumulative sum of PDSI values over the duration of a multiyear drought. It is not derived from Conceptual Model 2 alone. In our numerical experiments, DS exhibited a consistent nonlinear diminishing-return response to hypothetical added precipitation (Fig. 5). This empirically observed pattern resembles first-order kinetics, where the instantaneous rate of DS reduction is proportional to the remaining DS. Eq. 3 captures this behavior and links the instantaneous rate of DS reduction to the remaining DS. Our testing confirms that this formulation provides the best fit to the observed response pattern (Table S2). The formulation therefore integrates the mechanisms represents in both conceptual

models (Fig. 4b and 4c). Within this unified system representation, the parameter k characterizes the efficiency with which hypothetical additional precipitation reduces DS, and comparing k across different intervention timings provides the basis for identifying the optimal timing of drought alleviation.

The corresponding clarification has been incorporated into the revised manuscript. (Page 10, Lines 241 to 259)

46. L209. The terms "actual" and "theoretical" are poorly explained. Is the former based on the empirical values and the latter on the model? If you have K_actual, why you should base your analysis on the theoretical?

Response: Thank you for this important comment. As clarified in our response to Comments 11, 32, and 44, k quantifies the system's drought attenuation efficiency, namely how rapidly hypothetical additional precipitation is converted into reductions in drought severity (DS) of multiyear drought (MYD) events. Formally, the differential form is $\frac{\partial DS_{rate}}{\partial pr_{rate}}=k\cdot(1-DS_{rate})$ (eq. 3), k also represents the initial response rate when $DS_{rate}=0$.

In this study, k_{actual} is directly calculated from the numerical experiments. It represents the empirical (data-based) sensitivity of DS to small precipitation perturbations, approximated from the initial slop of the DS_{rate} and pr_{rate} relationship. Specifically, because the model expansion for small pr_{rate} gives $DS_{rate} \approx k \cdot pr_{rate}$ for $pr_{rate} \ll 1$, we approximate $k_{actual} = \frac{DS_{rate}(pr_{rate}=0.05)}{0.05}$, where 0.05 is the smallest perturbation step used in the experiments. This provides a direct, local estimate of the initial slope. In contrast, $k_{theoretical}$ is obtained by fitting the DAQM analytical solution $DS_{rate} = 1 - exp(-k \cdot pr_{rate})$ to the full series of data-based experimental outcomes across all perturbation steps. It represents the theoretical efficiency implied by the DAQM framework, capturing the nonlinear, bounded response of DS to hypothetical additional precipitation. $k_{theoretical}$ also integrating the combined effects of both Conceptual Model 1 and 2 (as clarified in our response to Comment 4 at the beginning of this document).

Our main analysis focuses on $k_{theoretical}$ because it characterizes the system-level response of DS to hypothetical additional precipitation and suitable for spatial and temporal comparison. k_{actual} serves as an independent reference to evaluate the internal consistency and numerical credibility of the DAQM formulation, but it is not used as the primary analysis indicator.

The corresponding clarification has been incorporated into the revised manuscript. (Page 12, Lines 273 to 287)

47. Fig. 5. It is not clear to me how these plots were derived. Are these from eq. 6. Using which k? This section of the results and the previous methodology need to be restructured.

Response: Thank you for this comment. In Fig. 5, the DS_{rate} and pr_{rate} response curves are derived from the numerical experiments aggregated over all grids in the study region. The horizontal axis represents

pr_{rate} (a dimensionless proportion that controls the relative magnitude of the hypothetical precipitation addition, the amount of hypothetical precipitation addition is given by $pr_{rate} \times \overline{pr}$, $\overline{pr}$ is the long-term mean monthly precipitation), and the vertical axis represents DS_{rate} (the corresponding relative change in DS). For each month of a multiyear drought event during the drought development period (Month 1 to 8), we used the experimental outcomes to fit the DAQM analytical form: $DS_{rate} = 1 - exp(-k \cdot pr_{rate})$, which yields a single $k_{theoretical}$ for the entire study region for that month. This parameter represents the system-level drought attenuation efficiency and captures the nonlinear dependence implied by the differential formulation of DAQM, $\frac{\partial DS_{rate}}{\partial pr_{rate}} = k \cdot (1 - DS_{rate})$. The resulting curves therefore represents the nonlinear, diminishing-return behavior observed in the experiments: strong attenuation at low DS_{rate} followed by progressively weaker gains as $1 - DS_{rate}$ weakens ($DS_{rate} = \frac{DS_{baseline} - DS_{forcing}}{DS_{baseline}}$, $DS_{baseline}$ is the actual DS under the baseline scenario that no hypothetical additional precipitation is applied, and $DS_{forcing}$ is the DS after the hypothetical additional precipitation is applied).

For comparison, a linear reference line is plotted for Month 1. Its slope is computed from the marginal response at very small hypothetical precipitation perturbation ($pr_{rate} = 0.05$, the step size in the numerical experiments): $k_{actual} = \frac{DS_{rate}(pr_{rate}=0.05)}{0.05}$. This linear reference represents the hypothetical scenario where attenuation efficiency remains constant with additional water. The clear divergence between this linear reference and the fitted nonlinear DAQM curves represents the intrinsic non-linearity of the drought attenuation process, which cannot be captured by a constant-slope formulation.

The corresponding clarification has been incorporated into the revised manuscript. (Page 12, Lines 290 to 318)

48. Fig. 6. Errors in the theoretical seem larger in the North-east, which is the same region where with several Kmax at month >= 2. Is there an effect of the error in the theoretical model in these results?

Response: Thank you for the comment. We would first like to clarify that in Fig. 6a, the relative errors of $k_{theoretical}$ are not larger in the North-east. Instead, they are generally smaller than in many parts of the study area. This indicates that the DAQM model performs well in this region. Therefore, the results in these North-east region are more reliable.

To further avoid misunderstanding, we provide additional clarification regarding the spatial distribution of relative error. In northern China more broadly, especially in the northwestern arid region, relatively larger errors do appear. This pattern can be explained by the physical environment of these areas. In arid regions, drought variability is strongly influenced by factors other than precipitation, most notably the high evaporative demand. Because precipitation amounts are very low and potential evapotranspiration (PET) is large, changes in precipitation alone cannot fully capture the main drivers of drought development. Our numerical experiment perturbs only hypothetical precipitation while keeping PET fixed. Consequently, in arid regions, the influence of non-precipitation factors can be reflected in the estimation of drought attenuation efficiency (k). It is important to note that evaporative

demand is not directly controllable in practical drought management, whereas water availability is. This is precisely the motivation for focusing on hypothetical water-input perturbations. The slightly higher relative errors in arid regions are therefore physically reasonable and consistent with expectations. In the humid southern region, the larger relative errors also occur in some grids. The underlying cause of these localized large relative errors is different from that in arid regions. As clarified in Comments 32 and 46, k_{actual} represents the initial slope. In humid regions, baseline precipitation is high. Increasing hypothetical precipitation by the minimum experiment step of 0.05 therefore corresponds to a relatively large increment of hypothetical water input. This leads to a greater deviation from the initial slope and produces relative larger differences between $k_{theoretical}$ and k_{actual}. Even in these arid and humid grids, however, the relative errors remain modest (mostly around 6%), suggesting that the DAQM still performs reasonably well across the study area. In semi-arid regions (North-eastern region of our study area), precipitation is neither extremely low nor excessively high, and it remains the dominant factor governing drought development (Feng et al., 2025). The 0.05 increment represents a relatively small amount of hypothetical water input compared with the local baseline precipitation, and the agreement between theoretical and actual results is therefore the strongest in these regions.

The corresponding clarification has been incorporated into the revised manuscript. (Page 14, Lines 337 to 347)

Subsequent comments related to this issue in this document, including Comments 49 and 55, are addressed accordingly in their respective responses.

References

Feng, Y., Sun, F. and Deng, X.: Attributing the divergent changes of drought from humid to dry regions across China, J. Hydrol., 660, 133363, doi: https://doi.org/10.1016/j.jhydrol.2025.133363, 2025

49. Fig. 7. Same as before. Most of the large differences seem to be in the north-east.

Response: Thank you for the comment. As clarified in our response to Comment 48, the North-east region does not exhibit the large differences. Instead, the relative errors between $k_{theoretical}$ and k_{actual} in this region are generally smaller than in many other parts of the study area. This indicates that the DAQM performs well in the North-east, and the corresponding results are therefore reliable. The detailed explanation provided in our response to Fig. 6 applies here as well.

The corresponding clarification has been incorporated into the revised manuscript. (Page 15, Lines 368 to 370)

50. Fig. 8. Why reporting the map of the duration. Is this the median duration of the droughts (as in Table 1)? How are the probability plots made? Based on how many data? Is there any smoothing applied?

Response: Thank you for the constructive comment. Fig. 8 presents the spatial distribution of the median drought duration (DD) for each grid cell, consistent with Table 1. In our framework, drought

severity (DS) is the dependent variable, computed using run theory as the integral of PDSI series over DD. DS therefore represents the accumulated water deficit over the entire event rather than a single PDSI value. Since DS is defined as the cumulative PDSI over DD, and k is derived from DS, DD represents a key factor influencing k in addition to the standardized characteristics of PDSI itself. Therefore, we present the DD map to help interpret differences in k across basins.

To remain consistent with our clarification in response to Comment 9, we emphasize that our analysis is conducted at the grid scale. The probability plots are constructed using k estimates from all 351 grid-level multiyear drought (MYD) samples across China. Here, a "MYD sample" refers to a grid-level MYD period during 1991-2020. For each of the 199 grids, MYD occurrences are identified independently. Across all grids, a total of 351 grid-level MYD samples were identified. Some of these samples may belong to the same regional MYD event, but our analysis focuses on the grid-level response as the statistical unit rather than aggregating events regionally. Each river basin contains at least 11 valid grid-level MYDs, ensuring basic statistical robustness. For each grid, k_1, k_2, and k_3 represent how rapidly hypothetical additional precipitation is converted into reductions in DS at Month 1, 2, and 3. The empirical distributions (i.e., probability distributions derived directly from the observed Δk) of $\Delta k_{21} = k_2 - k_1$ and $\Delta k_{31} = k_3 - k_1$ are then evaluated for each basin and China as a whole. The probability curves are generated using kernel density estimation (ksdensity in MATLAB), which provides a smooth empirical approximation of the underlying probability distribution. No further smoothing, filtering, or curve fitting is applied beyond this standard KDE procedure.

The corresponding clarification has been incorporated into the revised manuscript. (Page 16, Lines 383 to 386)

51. Table 1. Add the number of pixels for each basin, and also the DS.

Response: Thank you for the comment. We have added the number of grids for each basin to Table 1.

In our framework, DS is the dependent variable, computed using run theory as the integral of PDSI series over the drought duration (DD). DS therefore represents the accumulated water deficit over the entire event rather than a single PDSI value.

Because Table 1 is intended to summarize basin characteristics relevant to interpreting the variations in Δk (the changes in drought attenuation efficiency k between different months of a MYD event), and DS is the response variable, it is not conceptually appropriate to include DS in this table.

Table 1: Summary of the number of grids, median values of DD and Δk, including $\Delta k_{21}(=k_2-k_1)$, $\Delta k_{31}(=k_3-k_1)$, and $\Delta k_{32}(=k_3-k_2)$, across the ten river basins and all of China.

River basin	Number of grids	Median DD (Months)	P_{50} (Δk_{21})	P_{50} (Δk_{31})	P_{50} (Δk_{32})
Songhua River	26	22	-0.091	-0.235	***-0.142***
Liao River	10	28	**0.022**	**0.003**	**-0.024**

Northwestern Rivers	44	17	-0.071	-0.152	-0.087
Hai River	8	29	-0.083	-0.110	-0.053
Yellow River	20	21	-0.069	-0.111	-0.090
Yangtze River	39	19	-0.145	***-0.250***	-0.127
Huai River	11	23	-0.081	-0.141	-0.042
Southeastern Rivers	8	14	-0.024	-0.196	-0.077
Southwestern Rivers	15	21	-0.148	-0.148	-0.083
Pearl River	18	19	***-0.162***	-0.200	-0.063
All of China	199	21	-0.080	-0.166	-0.094

Note: The bold value marks the maximum P_{50} among the ten river basins in China, and the bold-italic value marks the minimum P_{50}.

The corresponding revision has been incorporated into the revised manuscript. (Page 18, Table 1)

52. L305. Why not DS? Is it possible that this result is just the effect of DS as mentioned before (not all the delta have the same effect, as they depend on DS itself).

Response: Thank you for the comment. We note that there may be some confusion between DS (the drought severity characteristic of each MYD) and PDSI. In our framework, DS is the dependent variable, computed using run theory as the integral of PDSI series over the drought duration (DD). DS therefore represents the accumulated water deficit over the entire event rather than a single index value.

The analysis in this section focus on Δk (the changes in drought attenuation efficiency k between different months of a MYD event) variation with respect to the drought duration (DD), which is an independent event characteristic potentially influencing sensitivity. Using DS itself as a grouping factor would be inappropriate because DS is the quantity whose response to hypothetical precipitation perturbations we are evaluating. Allowing DS to serve simultaneously as both response and grouping variable would confound the interpretation. By contrast, binning by DD allows us to isolate the effect of DD on Δk, consistent with the event-based framework of our study.

53. L325. Some basins are just one pixel, or close to. How these values can be compared among basins?

Response: Thank you for this comment. We clarify that each river basin in our research contains at least eight girds (Table 1 in this document), with each grid having at least one identified grid-level multiyear drought (MYD) sample. The parameter k_{max}/k_1 is computed independently for each grid-level MYD sample. The basin-level k_{max}/k_1 is then obtained as the median of the grid-level k_{max}/k_1 values within the basin. This approach captures the spatial variability within the basin while

providing a representative indicator for the overall basin response. This explanation is consistent with our clarifications in response to Comments 9, 39, and 50.

54. L339. "precipitation-based drought mitigation". What does this mean?

Response: Thank you for the comment. In this place, "precipitation-based drought mitigation" refers to studies focusing on the contribution of precipitation to drought recovery, which generally dominates over other factors.

The corresponding clarification has been incorporated into the revised manuscript. (Page 21, Lines 455 to 456)

55. L355-356. How much of this result is due to the error in the theoretical values?

Response: Thank you for the comment. As clarified in our response to Comment Fig. 6, although Figure 6a prensents the relative error at the grid level, a more integrated view from the river basin perspective (Figures 6c-6e) indicates that $k_{theoretical}$ closely match k_{actual} across the ten major river basins. The red 1:1 line and black fitted line illustrate the strong overall agreement, and the orange crosses indicate that individual basins are also well captured. This suggests that the results are more robust when considered at the basin scale, and the observed regional patterns in attenuation efficiency are not driven by error in the theoretical values.

The corresponding clarification has been incorporated into the revised manuscript. (Page 21, Lines 473 to 474)

56. L366. This is not true everywhere in the world.

Response: Thank you for the comment. Our original statement implied a global uniformity that is not accurate. The sentence has been revised to clarify that regional differences exist. Our study focuses on China, and no conclusion regarding global uniformity is intended.

The corresponding clarification has been incorporated into the revised manuscript. (Page 22, Lines 491 to 492)

Responses to Referee #3 comments:

Manuscript number: **egusphere-2025-4213**

Title: **Is earlier always better? A comparative assessment of rainfall timing for multiyear drought alleviation in China**

We sincerely thank you for the valuable comments and suggestions, which have greatly improved our manuscript. We have addressed all the comments and made the necessary revisions. Our responses to each comment are in blue text, while the referees' comments are in black.

This manuscript develops a Drought Mitigation Quantitative Model (DMQM) based on the Palmer Drought Severity Index (PDSI) and examines how the timing of rainfall increases influence drought mitigation efficiency. The research question is reasonable, and the modeling framework provides a structured way to evaluate drought mitigation sensitivity. The results have the potential to contribute to the understanding of drought mitigation processes and to inform strategic decisions regarding drought response planning.

Response: We sincerely appreciate your positive and encouraging evaluation of our study, including the research motivation, the modeling framework, and its potential implications for drought alleviation planning. Your recognition that the DMQM (now revised as the drought attenuation quantitative model, DAQM, to better reflect the hypothetical numerical experimental nature of the study) provides a structured and reasonable approach is very encouraging for us.

At the same time, several conceptual assumptions underlying the framework require more explicit clarification to enhance the interpretability and generality of the findings. In particular, the reliance on a single drought index, the interpretation of the key coefficient (k), and the mechanisms driving regional differences in optimal rainfall replenishment timing require further discussion. Addressing these points would improve the clarity of the conceptual narrative and broaden the potential applicability of the DMQM. The specific issues are reported below:

Response: Thank you for highlighting these important conceptual issues. We agree that providing clearer explanations of the underlying assumptions, the use of a single drought index, the interpretation of the key coefficient k, and the mechanisms behind regional differences in optimal hypothetical rainfall replenishment timing will strengthen the interpretability and generality of our findings. Accordingly, below we clarify these conceptual foundations and expand the related discussion to strengthen the applicability of the DMQM (now revised as the drought attenuation quantitative model, DAQM, to better reflect the hypothetical numerical experimental nature of the study).

Below we provided detailed, point-to-point responses to each comment.

To facilitate understanding and provide a convenient reference, we first present Table S1, which lists the principal symbols and acronyms used throughout the manuscript.

In response to reviewer comments and to further improve conceptual consistency, expression precision, and readability, several key symbols and acronyms have been refined. The major adjustments are summarized below:

1. The original term Drought Mitigation Quantitative Model (DMQM) has been revised to Drought Attenuation Quantitative Model (DAQM). This modification better reflects the nature of this study, as the analysis is based on hypothetical increases in numerical experiments rather than actual precipitation modification or direct real-world intervention.

2. In the original manuscript, different symbols (t_a, t_b, t_c) were used to represent optimal timing under different conceptual models. To reduce notation complexity, these have been unified into a single symbol, $t_{optimal}$. In this study, the final interpretation focuses on the $t_{optimal}$ derived from the optimal solution model.

3. The acronym PIP has been revised to DIP (Drought Intensity Point of a drought event), defined as the PDSI value corresponding to the Drought Intensity (DI). This change improves terminological clarity and consistency with the event-based drought characterization used in this study.

Table S1: A list of principal symbols and acronyms

Symbols and Acronyms	Description
MYD	MultiYear Drought
PDSI	Palmer Drought Severity Index, a monthly drought index where each value represents the PDSI-defined "drought severity" for that month (the terminology is distinguished from the event-level drought severity, DS, used in this study)
DS	Drought Severity of a drought event, the cumulative deficiency of a drought index below the critical threshold throughout the event
DD	Drought Duration of a drought event, the period during which a drought index is continuously below the critical threshold during the event
DI	Drought Intensity of a drought event, the maximum deviation of the drought index below the critical threshold during the event
DDP	Drought Development Period of a drought event
DIP	DI Point of a drought event, the PDSI value corresponding to the DI
DAQM	Drought Attenuation Quantitative Model
CAFEC	Climatically Appropriate For Existing Conditions
$pr_{baseline}$	the actual precipitation in the targeted month under the baseline (no hypothetical additional precipitation is applied) scenario
$\overline{pr}$	the long-term mean monthly precipitation
pr_{rate}	a dimensionless proportion (0–1) that controls the relative magnitude of the hypothetical precipitation addition, the amount of hypothetical precipitation addition is given by $pr_{rate} \times \overline{pr}$
$pr_{forcing}$	the precipitation in the targeted month after applying hypothetical additional precipitation, $pr_{forcing} = pr_{baseline} + pr_{rate} \times \overline{pr}$, $pr_{rate} = 0, 5\%, 10\%, \ldots, 100\%$
$DS_{baseline}$	the actual DS under the baseline (no hypothetical additional precipitation is applied)

	scenario
$DS_{forcing}$	the DS after the hypothetical additional precipitation is applied
DS_{rate}	the relative change in DS, $DS_{rate}=\frac{DS_{baseline}-DS_{forcing}}{DS_{baseline}}$
k	MYD attenuation efficiency, k characterizes how rapidly additional precipitation is converted into reductions in DS
$k_{theoretical}$	the theoretical k value, $k_{theoretical}$ is obtained by fitting the DAQM analytical solution $DS_{rate}=1-exp(-k\cdot pr_{rate})$ to the DS_{rate}-pr_{rate} curve derived from the numerical experiments
k_{actual}	the actual k value, represent the empirical sensitivity of DS to small precipitation perturbations, $k_{actual}=\frac{DS_{rate}(pr_{rate}=0.05)}{0.05}$, $pr_{rate}=0.05$ is the step size in the numerical experiments
ε	relative error, $\varepsilon=\left(\frac{k_{theoretical}-k_{actual}}{k_{actual}}\right)\times 100\%$
k_1	$k_{theoretical}$ value at the first month of a MYD event
k_2	$k_{theoretical}$ value at the second month of a MYD event
k_3	$k_{theoretical}$ value at the third month of a MYD event
k_{max}	maximum $k_{theoretical}$ within a MYD event, obtained as the largest monthly k among all months of this event
Δk_{21}	the changes in $k_{theoretical}$ between the second and the first month of a MYD event
Δk_{31}	the changes in $k_{theoretical}$ between the third and the first month of a MYD event
$t_{forcing}$	the month of hypothetical additional precipitation is applied
t_1	the first month of a MYD event
t_2	the second month of a MYD event
t_3	the third month of a MYD event
t_{DIP}	the month corresponding to DIP within a MYD event
$t_{optimal}$	the optimal timing for drought alleviation

Note: To avoid ambiguity, the monthly values of the Palmer Drought Severity Index (PDSI) are referred to simply as "PDSI" in this study, whereas DS is used exclusively to denote event-level drought severity.

1. The model framework is built entirely on PDSI, yet standardized drought indices vary substantially in their input variables, water balance assumptions, and how auto-correlation processes are incorporated. Because PDSI embeds a two-layer soil moisture storage structure, the mitigation efficiency patterns derived here may reflect this structure rather than a general characteristic of drought mitigation. A discussion of whether similar outcomes would be expected when using SPI (precipitation based) or SPEI (precipitation and evapotranspiration based) would clarify the extent to which the coefficient k reflects a broader drought response characteristic compared with the behavior of a particular index formulation. Even a conceptual comparison would enhance confidence in the applicability of the framework.

Response: Thank you for this insightful comment. We agree that standardized drought indices differ in their input variables, water balance assumptions, and how auto-correlation processes are incorporated. However, several considerations motivated the use of PDSI in the present numerical experiment.

Although different in formulation, most standardized drought indices share a common statistical characteristic. The standardization process produces an approximately normal distribution. PDSI, despite embedding a two-layer soil moisture storage structure, ultimately generates a standardized index that also conforms to an approximately normal distribution. This implies that changes occurring in the distribution tails commonly lead to disproportionate shifts in PDSI value. Therefore, the sensitivity patterns detected here are not unique to PDSI and may conceptually appear in other standardized indices as well.

More importantly, the numerical experiment requires that hypothetical additional precipitation imposed in a target month affects only the drought evolution from that target month forward, without altering the the reference climatology and the derived parameters. In our study, this constraint can be met with PDSI because its calibration is performed over a fixed historical period (1961-1990) while the perturbation numerical experiments operate in 1991-2020. The baseline hydrological parameters therefore remain stable, and the experiment does not retroactively change the reference climatology and the derived parameters. Standardized indices such as SPEI and SPI do not incorporate a separate calibration period. Their standardization parameters are computed directly from the data period being modified, which means that applying hypothetical additional precipitation would inevitably alter the baseline itself. Under such circumstances, the numerical results would no longer represent a controlled perturbation to drought development but instead a shifting statistical reference frame.

Another consideration is that PDSI is a physically based drought index. Its two-layer soil moisture storage structure provides a mechanistic foundation for analyzing drought alleviation processes. Indices such as SPEI, which rely on the direct P-PET difference, contain far fewer physical constraints, and SPI/SPEI's flexible timescales may lead to divergent drought evolution depending on the selected temporal window. Such variability would bring additional uncertainty when attempting to interpret mitigation efficiency in a mechanistic sense. Using PDSI, a widely adopted and physically based index, therefore offers a stable and representative framework for our numerical experiment.

For these reasons, PDSI provides a consistent calibration structure, an internally closed baseline period, and a physically meaningful representation of drought dynamics, all of which are essential for designing controlled hypothetical precipitation-perturbation experiments. Although PDSI is more suitable for our numerical experiment design due to its fix calibration framework, the standardized nature of these standardized drought indices means that similar qualitative sensitivity patterns could also emerge under other indices such as SPI and SPEI, provided that their statistical reference frames can be held constant. We acknowledge the importance of understanding how the drought attenuation efficiency (k) behaves across different standardized drought indices. This issue will be addressed in our next study, where we plan to systematically compare PDSI, SPEI, and SPI and examine the mechanisms responsible for their differences.

2. Standardized drought indices are designed to approximate a normal distribution, which can make them particularly sensitive to the treatment of tails and extremes in hydroclimatic data. Since the core results of DMQM concern changes in drought severity under incremental rainfall increases, it would be worthwhile to examine whether the statistical normalization process itself influences the magnitude of the estimated mitigation efficiency. A brief consideration of whether alternative representations that

are less sensitive to extremes might produce similar mitigation patterns would help delineate the robustness of the conclusions.

Response: Thank you for this valuable comment. We fully agree that standardized drought indices are inherently sensitive to the treatment of extreme values, and this is central to both the motivation and the design of our study. Our framework is not only built upon this characteristic, but is designed to explicitly examine how such statistical characteristics influence the estimated drought attenuation efficiency (k).

In current drought research and operational monitoring, standardized indices remain the most widely applied tools. Therefore, understanding whether this normalization affects the inferred efficiency of drought attenuation is an important and practical question. Our results show that this influence is real and measurable. Conceptual Model 2 demonstrates that, because standardized indices respond more strongly to distribution tails, a severe PDSI values can exhibit larger sensitivity when hypothetical additional rainfall is applied closer to peak intensity. This contrasts with the traditional expectation (represented by Conceptual Model 1) that earlier water input is always more effective because they operate over a longer remaining duration of the event. These two mechanisms exert opposite influences. To reconcile these competing effects, our Optimal Solution Model identifies the balance point between them. The numerical experiments confirm that both mechanisms are present in historical multiyear drought (MYD) events. While the maximum efficiency occurs at the first month in most events (58.79%), non-negligible proportions of events reach their maximum sensitivity in the second (22.11%) or third (11.06%) month. This implies that the statistical characteristic of standardized indices can, in some regions, partially offset the disadvantages of delayed hypothetical water input, thereby producing alleviation advantages at the event scale.

It is also important note that PDSI, while not perfectly normal, is designed to approximate a normal distribution. This property ensures that extreme index values occur much less frequently than mild index values. This is an essential feature of real-world drought occurrence. A uniform distribution would contradict this fundamental hydrological reality, as it would imply equal probabilities for mild and extreme droughts, which is inconsistent with observations. Using PDSI as an example, to induce the same one-unit change in the index, the required shift in cumulative probability under extreme drought (PDSI = -4) is only 48% of that under milder conditions (PDSI = -1) (Fig. 1c and 1d). This demonstrates that drought severity cannot be treated as a linear process. To further evaluate whether the normalization procedure itself drives our findings, we conducted additional comparisons using a linear models. The results, summarized in our response to Comment 3 (Table S2), show that the optimal model is not the linear model. This reinforces that the statistical properties of standardized indices capture essential characteristics of drought occurrence and development. This consideration has also been incorporated into the revised manuscript. (Page 11, Lines 265 to 271)

3. The coefficient k is positioned as a measure of mitigation efficiency, yet its physical interpretation remains implicit. Clarifying whether k should be understood primarily as an empirical sensitivity parameter or whether it reflects an underlying hydrological recovery rate would help readers understand how it should be compared across regions and timescales. Further elaboration on this point would strengthen the conceptual coherence of the framework. At the same time, the spatial patterns of k and the optimal rainfall replenishment timing suggest systematic regional differences,

especially between northern and southern basins. These differences may reflect variations in climate seasonality, soil water storage capacity, and land-atmosphere feedbacks. A more detailed discussion of which factors are most likely to drive the observed spatial differences would improve the interpretability of the results. It would also help clarify the extent to which the framework may be transferable to regions with different hydroclimatic conditions.

Response: Thank you for this constructive comment. Although parameter k is empirically estimated, it is not merely a curve-fitting constant. In our framework, k quantifies the system's drought attenuation efficiency, namely how rapidly hypothetical additional precipitation is converted into reductions in drought severity (DS) of multiyear drought (MYD) events. Higher k values correspond to faster alleviation, while lower k values reflect slower hydrological recovery or more persistent drought conditions.

Given the central role of k in our study, we provide a more complete explanation below.

To clarify our approach, the exponential formulation in DAQM is not arbitrarily postulated. Our modeling framework was developed by performing numerical experiments driven by observations from meteorological stations. Before specifying any functional structure, we first examined how drought severity (DS) of a multiyear drought event responds to incremental precipitation increases within controlled numerical experiment. Across events and stations, the numerical experiments consistently demonstrated a clear diminishing return pattern (Fig. 5). When hypothetical additional precipitation was incrementally increased at the same month of a drought event, the reduction in DS showed progressively weaker marginal benefits. The initial portion of added water input produced the strongest alleviation, while subsequent increments at that same month yielded progressively weaker effects. This empirical feature motivated us to seek a formulation capable of representing a bounded recovery process. To formalize this behavior, we drew an analogy with systems exhibiting saturation-type adjustment, similar to first-order kinetic expressions (e.g. $C_t=C_0 \cdot exp(-k \cdot t)$, where C_t denotes the remaining intensity at time t, C_0 is the initial state at $t=0$, and k is the first-order rate constant controlling the decay speed. The instantaneous decay rate $\frac{dC}{dt}$ is proportional to the current magnitude C_t.). Following this reasoning, we adopted the formulation: $DS_{rate}=1-exp(-k \cdot pr_{rate})$. This formulation captures how incremental water input progressively reduces the residual DS (Fig. 1). In this framework, $exp(-k \cdot pr_{rate})$ represents the proportion of remaining DS after a given additional precipitation increment. When the additional water supply is sufficient, the relative reduction in DS approaches a practical upper limit, a pattern similar to physical constraints such as finite soil water storage capacity or vegetation water use limits.

To demonstrate that the exponential structure represents real system behavior, we compared formulations inspired by zero-order, first-order, and second-order kinetic processes using historical multiyear drought events (MYDs) during 1991-2020. Across most stations and events, the first-order structure more accurately reproduced the curvature observed in the numerical experiments at different month of MYDs. We further extended this comparison to an n-order formulation and found that the estimated n converged toward 0.98 across MYDs, which is essentially unity. This outcome strengthens the conclusion that a first-order structure is the most appropriate representation of the observed pattern. The results are summarized in Table S2. We also compared the exponential form against alternative

empirical relationships. Logistic curves did not match the observed pattern. Power-law functions were unable to satisfy the required boundary conditions. We note that the same exponential adjustment structure has been adopted in other fields. For instance, Childs et al. (2025) applied a similar functional pattern in modeling climate-driven disease dynamics. The recurrence of this structure across domains further supports its suitability for representing systems with constrained recovery capacity.

Table S2: Comparison of R^2 values for different functional models

Model order	Differential form	Integral form	t_1	t_2	t_3
Zero-order	$\frac{\partial DS_{rate}}{\partial pr_{rate}} = k$	$DS_{rate}=k \cdot pr_{rate}$	0.992	0.984	0.989
First-order	$\frac{\partial DS_{rate}}{\partial pr_{rate}} = k \cdot (1 - DS_{rate})$	$DS_{rate}=1-exp(-k \cdot pr_{rate})$	0.985	0.996	0.993
Second-order	$\frac{\partial DS_{rate}}{\partial pr_{rate}} = k \cdot (1 - DS_{rate})^2$	$DS_{rate}=1-1/(k \cdot pr_{rate}+1)$	0.982	0.976	0.961
N-order	$\frac{\partial DS_{rate}}{\partial pr_{rate}} = k \cdot (1 - DS_{rate})^n$	$DS_{rate}=1-[1-k \cdot (1-n) \cdot pr_{rate}]^{1/1-n}$	0.989	0.996	0.998

Note: t_1, t_2, and t_3 represent the first to the third month of the MultiYear Drought event.

Regarding regional differences, our study focuses primarily on event-intrinsic characteristics rather than external environmental controls. Because DS is computed using run theory as the integral of PDSI series over drought duration (DD), we centered our analysis on DD. Our results demonstrate a positive correlation between DD and k in MYD (Fig. 8 and Fig. 9). Specifically, basins in northern China, characterized by longer DD, tend to have a higher probability of t_2 or t_3 being the $t_{optimal}$ in contrast to southern basins with shorter DD (Fig. 8 and Table 1). As discussed in Section 5.3, environment controls on k are complex. Identifying the dominant drivers is therefore a key direction for future work, especially for assessing the transferability of the framework to other hydroclimatic regions.

These clarifications have been incorporated into the revised manuscript. (Page 11, Lines 265 to 271; Page 16, Lines 383 to 386)

References

Childs, M.L., Lyberger, K., Harris, M.J., Burke, M., and Mordecai, E.A.: Climate warming is expanding dengue burden in the Americas and Asia, Proc. Natl. Acad. Sci., 122, e2512350122, doi: https://doi.org/10.1073/pnas.2512350122, 2025.