Responses to Referee #3 comments:

Manuscript number: **egusphere-2025-4213**

Title: **Is earlier always better? A comparative assessment of rainfall replenishment timing for multiyear drought mitigation**

We sincerely thank you for the valuable comments and suggestions, which have greatly improved our manuscript. We have addressed all the comments and made the necessary revisions. Our responses to each comment are in blue text, while the referees' comments are in black.

This manuscript develops a Drought Mitigation Quantitative Model (DMQM) based on the Palmer Drought Severity Index (PDSI) and examines how the timing of rainfall increases influence drought mitigation efficiency. The research question is reasonable, and the modeling framework provides a structured way to evaluate drought mitigation sensitivity. The results have the potential to contribute to the understanding of drought mitigation processes and to inform strategic decisions regarding drought response planning.

Response: We sincerely appreciate your positive and encouraging evaluation of our study, including the research motivation, the modeling framework, and its potential implications for drought alleviation planning. Your recognition that the DMQM provides a structured and reasonable approach is very encouraging for us.

At the same time, several conceptual assumptions underlying the framework require more explicit clarification to enhance the interpretability and generality of the findings. In particular, the reliance on a single drought index, the interpretation of the key coefficient (k), and the mechanisms driving regional differences in optimal rainfall replenishment timing require further discussion. Addressing these points would improve the clarity of the conceptual narrative and broaden the potential applicability of the DMQM. The specific issues are reported below:

Response: Thank you for highlighting these important conceptual issues. We agree that providing clearer explanations of the underlying assumptions, the use of a single drought index, the interpretation of the key coefficient *k*, and the mechanisms behind regional differences in optimal hypothetical rainfall replenishment timing will strengthen the interpretability and generality of our findings. Accordingly, below we clarify these conceptual foundations and expand the related discussion to strengthen the applicability of the DMQM.

Below we provided detailed, point-to-point responses to each comment.

1. The model framework is built entirely on PDSI, yet standardized drought indices vary substantially in their input variables, water balance assumptions, and how auto-correlation processes are incorporated. Because PDSI embeds a two-layer soil moisture storage structure, the mitigation efficiency patterns derived here may reflect this structure rather than a general characteristic of drought mitigation. A discussion of whether similar outcomes would be expected when using SPI (precipitation based) or SPEI (precipitation and evapotranspiration based) would clarify the extent to *which the coefficient k reflects a broader drought response characteristic compared with the behavior of a particular index formulation. Even a conceptual comparison would enhance confidence in the applicability of the framework.*

Response: Thank you for this insightful comment. We agree that standardized drought indices differ in their input variables, water balance assumptions, and how auto-correlation processes are incorporated. However, several considerations motivated the use of PDSI in the present numerical experiment.

Although different in formulation, most standardized drought indices share a common statistical characteristic. The standardization process produces an approximately normal distribution. PDSI, despite embedding a two-layer soil moisture storage structure, ultimately generates a standardized index that also conforms to an approximately normal distribution. This implies that changes occurring in the distribution tails commonly lead to disproportionate shifts in PDSI value. Therefore, the sensitivity patterns detected here are not unique to PDSI and may conceptually appear in other standardized indices as well.

More importantly, the numerical experiment requires that hypothetical additional precipitation imposed in a target month affects only the drought evolution from that target month forward, without altering the the reference climatology and the derived parameters. In our study, this constraint can be met with PDSI because its calibration is performed over a fixed historical period (1961-1990) while the perturbation numerical experiments operate in 1991-2020. The baseline hydrological parameters therefore remain stable, and the experiment does not retroactively change the reference climatology and the derived parameters. Standardized indices such as SPEI and SPI do not incorporate a separate calibration period. Their standardization parameters are computed directly from the data period being modified, which means that applying hypothetical additional precipitation would inevitably alter the baseline itself. Under such circumstances, the numerical results would no longer represent a controlled perturbation to drought development but instead a shifting statistical reference frame.

Another consideration is that PDSI is a physically based drought index. Its two-layer soil moisture storage structure provides a mechanistic foundation for analyzing drought alleviation processes. Indices such as SPEI, which rely on the direct P-PET difference, contain far fewer physical constraints, and SPI/SPEI's flexible timescales may lead to divergent drought evolution depending on the selected temporal window. Such variability would bring additional uncertainty when attempting to interpret mitigation efficiency in a mechanistic sense. Using PDSI, a widely adopted and physically based index, therefore offers a stable and representative framework for our numerical experiment.

For these reasons, PDSI provides a consistent calibration structure, an internally closed baseline period, and a physically meaningful representation of drought dynamics, all of which are essential for designing controlled hypothetical precipitation-perturbation experiments. Although PDSI is more suitable for our numerical experiment design due to its fix calibration framework, the standardized nature of these standardized drought indices means that similar qualitative sensitivity patterns could also emerge under other indices such as SPI and SPEI, provided that their statistical reference frames can be held constant. We acknowledge the importance of understanding how the drought attenuation efficiency (k) behaves across different standardized drought indices. This issue will be addressed in our next study, where we plan to systematically compare PDSI, SPEI, and SPI and examine the mechanisms responsible for their differences.

2. Standardized drought indices are designed to approximate a normal distribution, which can make them particularly sensitive to the treatment of tails and extremes in hydroclimatic data. Since the core results of DMQM concern changes in drought severity under incremental rainfall increases, it would be worthwhile to examine whether the statistical normalization process itself influences the magnitude of the estimated mitigation efficiency. A brief consideration of whether alternative representations that are less sensitive to extremes might produce similar mitigation patterns would help delineate the robustness of the conclusions.

Response: Thank you for this valuable comment. We fully agree that standardized drought indices are inherently sensitive to the treatment of extreme values, and this is central to both the motivation and the design of our study. Our framework is not only built upon this characteristic, but is designed to explicitly examine how such statistical characteristics influence the estimated drought attenuation efficiency (k).

In current drought research and operational monitoring, standardized indices remain the most widely applied tools. Therefore, understanding whether this normalization affects the inferred efficiency of drought attenuation is an important and practical question. Our results show that this influence is real and measurable. Conceptual Model 2 demonstrates that, because standardized indices respond more strongly to distribution tails, a severe PDSI values can exhibit larger sensitivity when hypothetical additional rainfall is applied closer to peak intensity. This contrasts with the traditional expectation (represented by Conceptual Model 1) that earlier water input is always more effective because they operate over a longer remaining duration of the event. These two mechanisms exert opposite influences. To reconcile these competing effects, our Optimal Solution Model identifies the balance point between them. The numerical experiments confirm that both mechanisms are present in historical multiyear drought (MYD) events. While the maximum efficiency occurs at the first month in most events (58.79%), non-negligible proportions of events reach their maximum sensitivity in the second (22.11%) or third (11.06%) month. This implies that the statistical characteristic of standardized indices can, in some regions, partially offset the disadvantages of delayed hypothetical water input, thereby producing alleviation advantages at the event scale.

It is also important note that PDSI, while not perfectly normal, is designed to approximate a normal distribution. This property ensures that extreme index values occur much less frequently than mild index values. This is an essential feature of real-world drought occurrence. A uniform distribution would contradict this fundamental hydrological reality, as it would imply equal probabilities for mild and extreme droughts, which is inconsistent with observations. Using PDSI as an example, to induce the same one-unit change in the index, the required shift in cumulative probability under extreme drought (PDSI = -4) is only 48% of that under milder conditions (PDSI = -1) (Fig. 1c and 1d). This demonstrates that drought severity cannot be treated as a linear process. To further evaluate whether the normalization procedure itself drives our findings, we conducted additional comparisons using a linear models. The results, summarized in our response to Comment 3 (Table S1), show that the optimal model is not the linear model. This reinforces that the statistical properties of standardized indices capture essential characteristics of drought occurrence and development.

3. The coefficient k is positioned as a measure of mitigation efficiency, yet its physical interpretation remains implicit. Clarifying whether k should be understood primarily as an empirical sensitivity parameter or whether it reflects an underlying hydrological recovery rate would help readers understand how it should be compared across regions and timescales. Further elaboration on this point would strengthen the conceptual coherence of the framework. At the same time, the spatial patterns of k and the optimal rainfall replenishment timing suggest systematic regional differences, especially between northern and southern basins. These differences may reflect variations in climate seasonality, soil water storage capacity, and land-atmosphere feedbacks. A more detailed discussion of which factors are most likely to drive the observed spatial differences would improve the interpretability of the results. It would also help clarify the extent to which the framework may be transferable to regions with different hydroclimatic conditions.

Response: Thank you for this constructive comment. Although parameter k is empirically estimated, it is not merely a curve-fitting constant. In our framework, k quantifies the system's drought attenuation efficiency, namely how rapidly hypothetical additional precipitation is converted into reductions in drought severity (DS) of multiyear drought (MYD) events. Higher k values correspond to faster alleviation, while lower k values reflect slower hydrological recovery or more persistent drought conditions.

Given the central role of k in our study, we provide a more complete explanation below.

To clarify our approach, the exponential formulation in DMQM is not arbitrarily postulated. Our modeling framework was developed by performing numerical experiments driven by observations from meteorological stations. Before specifying any functional structure, we first examined how drought severity (DS) of a multiyear drought event responds to incremental precipitation increases within controlled numerical experiment. Across events and stations, the numerical experiments consistently demonstrated a clear diminishing return pattern (Fig. 5). When hypothetical additional precipitation was incrementally increased at the same month of a drought event, the reduction in DS showed progressively weaker marginal benefits. The initial portion of added water input produced the strongest alleviation, while subsequent increments at that same month yielded progressively weaker effects. This empirical feature motivated us to seek a formulation capable of representing a bounded recovery process. To formalize this behavior, we drew an analogy with systems exhibiting saturation-type adjustment, similar to first-order kinetic expressions (e.g. $C_t=C_0 \cdot exp(-k \cdot t)$, where C_t denotes the remaining intensity at time t, C_0 is the initial state at t=0, and k is the first-order rate constant controlling the decay speed. The instantaneous decay rate $\frac{dC}{dt}$ is proportional to the current magnitude C_t.). Following this reasoning, we adopted the formulation: $DS_{rate}=1-exp(-k \cdot pr_{rate})$. This formulation captures how incremental water input progressively reduces the residual DS (Fig. 1). In this framework, $exp(-k \cdot pr_{rate})$ represents the proportion of remaining DS after a given additional precipitation increment. When the additional water supply is sufficient, the relative reduction in DS approaches a practical upper limit, a pattern similar to physical constraints such as finite soil water storage capacity or vegetation water use limits.

To demonstrate that the exponential structure represents real system behavior, we compared formulations inspired by zero-order, first-order, and second-order kinetic processes using historical multiyear drought events (MYDs) during 1991-2020. Across most stations and events, the first-order structure more accurately reproduced the curvature observed in the numerical experiments at different month of MYDs. We further extended this comparison to an n-order formulation and found that the estimated n converged toward 0.98 across MYDs, which is essentially unity. This outcome strengthens the conclusion that a first-order structure is the most appropriate representation of the observed pattern. The results are summarized in Table S1. We also compared the exponential form against alternative empirical relationships. Logistic curves did not match the observed pattern. Power-law functions were unable to satisfy the required boundary conditions. We note that the same exponential adjustment structure has been adopted in other fields. For instance, Childs et al. (2025) applied a similar functional pattern in modeling climate-driven disease dynamics. The recurrence of this structure across domains further supports its suitability for representing systems with constrained recovery capacity.

Table S1: Comparison of R^2 values for different functional models

Formulation	t_1	t_2	t_3
Zero-order: $DS_{rate}=k{\cdot}pr_{rate}$	0.992	0.984	0.989
First-order: $DS_{rate}=1-exp(-k{\cdot}pr_{rate})$	0.985	0.996	0.993
Second-order: $DS_{rate}=1-1/(k{\cdot}pr_{rate}+1)$	0.982	0.976	0.961
N-order: $DS_{rate}=1-[1-\mathrm{k}{\cdot}(1-\mathrm{n}){\cdot}pr_{rate}]^{1/1-\mathrm{n}}$	0.989	0.996	0.998

Note: t_1, t_2, and t_3 represent the first to the third month of the MultiYear Drought event.

Regarding regional differences, our study focuses primarily on event-intrinsic characteristics rather than external environmental controls. Because DS is computed using run theory as the integral of PDSI series over drought duration (DD), we centered our analysis on DD. Our results demonstrate a positive correlation between DD and k in MYD (Fig. 8 and Fig. 9). Specifically, basins in northern China, characterized by longer DD, tend to have a higher probability of t_2 or t_3 being the $t_{optimal}$ in contrast to southern basins with shorter DD (Fig. 8 and Table 1). As discussed in Section 5.3, environment controls on k are complex. Identifying the dominant drivers is therefore a key direction for future work, especially for assessing the transferability of the framework to other hydroclimatic regions.

References

Childs, M.L., Lyberger, K., Harris, M.J., Burke, M., and Mordecai, E.A.: Climate warming is expanding dengue burden in the Americas and Asia, Proc. Natl. Acad. Sci., 122, e2512350122, doi: https://doi.org/10.1073/pnas.2512350122, 2025.

In addition, a consolidated table listing principal symbols, acronyms and their description will be provided so that readers can easily consult definitions at any point (Table S2).

Table S2: A list of principal symbols and acronyms

Symbols and Acronyms	**Description**
MYD	MultiYear Drought
PDSI	Palmer Drought Severity Index, a monthly drought index where each value represents the PDSI-defined "drought severity" for that month (the terminology is distinguished from the event-level drought severity, DS, used in this study)
DS	Drought Severity of a drought event, the cumulative deficiency of a drought index below the critical threshold throughout the event
DD	Drought Duration of a drought event, the period during which a drought index is continuously below the critical threshold during the event
DI	Drought Intensity of a drought event, the maximum deviation of the drought index below the critical threshold during the event
DDP	Drought Development Period of a drought event
PIP	DI Point of a drought event, the PDSI value corresponding to the DI
DMQM	Drought Mitigation Quantitative Model
CAFEC	Climatically Appropriate For Existing Conditions
$pr_{baseline}$	the actual precipitation in the targeted month under the baseline (no hypothetical additional precipitation is applied) scenario
$\overline{pr}$	the long-term mean monthly precipitation
pr_{rate}	a dimensionless proportion (0–1) that controls the relative magnitude of the hypothetical precipitation addition, the amount of hypothetical precipitation addition is given by $pr_{rate} \times \overline{pr}$
$pr_{forcing}$	the precipitation in the targeted month after applying hypothetical additional precipitation, $\mathrm{pr_{forcing}} = \mathrm{pr_{baseline}} + \mathrm{pr_{rate}} \times \overline{\mathrm{pr}}$, $\mathrm{pr_{rate}} = 0, 5\%, 10\%, \ldots, 100\%$
$DS_{baseline}$	the actual DS under the baseline (no hypothetical additional precipitation is applied) scenario
$DS_{forcing}$	the DS after the hypothetical additional precipitation is applied
DS_{rate}	the relative change in DS, $DS_{rate}=\frac{DS_{baseline}-DS_{forcing}}{DS_{baseline}}$
k	MYD attenuation efficiency, k characterizes how rapidly additional precipitation is converted into reductions in DS
$k_{theoretical}$	the theoretical k value, $k_{theoretical}$ is obtained by fitting the DMQM analytical solution $\mathrm{DS_{rate}}=1-\exp(-\mathrm{k}\cdot\mathrm{pr_{rate}})$ to the DS_{rate}-pr_{rate} curve derived from the numerical experiments
k_{actual}	the actual k value, represent the empirical sensitivity of DS to small precipitation perturbations, $k_{actual}=\frac{DS_{rate}(pr_{rate}=0.05)}{0.05}$, $pr_{rate}=0.05$ is the step size in the numerical experiments
ε	relative error, $\varepsilon=\left(\frac{k_{theoretical}-k_{actual}}{k_{actual}}\right)\times 100\%$

k_1	$k_{theoretical}$ value at the first month of a MYD event
k_2	$k_{theoretical}$ value at the second month of a MYD event
k_3	$k_{theoretical}$ value at the third month of a MYD event
k_{max}	maximum $k_{theoretical}$ within a MYD event, obtained as the largest monthly k among all months of this event
Δk_{21}	the changes in $k_{theoretical}$ between the second and the first month of a MYD event
Δk_{31}	the changes in $k_{theoretical}$ between the third and the first month of a MYD event
$t_{forcing}$	the month of hypothetical additional precipitation is applied
t_1	the first month of a MYD event
t_2	the second month of a MYD event
t_3	the third month of a MYD event
t_{PIP}	the month at PIP of a MYD event
$t_{optimal}$	the optimal timing for drought alleviation
t_a	$t_{optimal}$ in Conceptual Model 1
t_b	$t_{optimal}$ in Conceptual Model 2
t_c	$t_{optimal}$ in Optimal Solution Model

Note: To avoid ambiguity, the monthly values of the Palmer Drought Severity Index (PDSI) are referred to simply as "PDSI" in this study, whereas DS is used exclusively to denote event-level drought severity.