Responses to Referee #1 comments:

Manuscript number: **egusphere-2025-4213**

Title: **Is earlier always better? A comparative assessment of rainfall replenishment timing for multiyear drought mitigation**

We sincerely thank you for the valuable comments and suggestions, which have greatly improved our manuscript. We have addressed all the comments and made the necessary revisions. Our responses to each comment are in blue text, while the referees' comments are in black.

This is my first review of the paper. The paper addresses an interesting and potentially important question: how the timing of rainfall replenishment affects the mitigation of multiyear droughts (MYDs). The authors propose a "drought mitigation quantitative model" (DMQM) to quantify how increases in precipitation at different stages of a drought affect drought severity, using historical events in China (1961–2020). The topic is relevant to understanding drought recovery dynamics and could make a valuable contribution if presented more clearly and conceptually coherently.

Response: Thank you very much for your clear summary and positive evaluation of our work. We appreciate your recognition of the importance of examining how the timing of rainfall replenishment influences the alleviation of multiyear droughts (MYDs). The development of the drought mitigation quantitative model (DMQM) is indeed aimed at quantitatively assessing how hypothetical precipitation increments at different months of drought events affect drought evolution. We also appreciate the recognition of the potential contribution of this framework.

However, in its current form, the manuscript suffers from major issues, which I report below.

Response: Thank you very much for the constructive comments. Below we provide point-by-point responses and detailed clarifications for each of the concerns raised.

1. The term mitigation is central to the paper but is never clearly or consistently defined.

Throughout the introduction and abstract, "drought mitigation" is used in the general sense of reducing drought impacts, which typically implies human or management intervention. Later, in Section 3.4, it becomes apparent that mitigation actually refers to the modelled reduction in drought severity (DS) resulting from hypothetical precipitation forcing in numerical experiments. In this way, readers are left uncertain whether the study concerns physical hydrological recovery, operational drought management, or synthetic model sensitivity.

I suggest defining drought mitigation explicitly in the introduction as a numerical or index-based construct (i.e., "the reduction in DS under hypothetical precipitation increases"). In practice, clarify early that no human intervention or feasible rainfall modification is implied. Terms such as index-based recovery or drought attenuation efficiency would be more accurate and less misleading.

This also impacts other terms. Indeed, the manuscript repeatedly refers to "precipitation-based drought mitigation" and "early intervention," giving the impression that rainfall timing can be controlled or managed (e.g., through artificial replenishment). Yet, the study merely perturbs precipitation values in the PDSI model. This language conflates natural rainfall timing with anthropogenic mitigation, generating unnecessary conceptual ambiguity. I suggest rephrasing these expressions to reflect that the experiments involve idealized precipitation scenarios, not management interventions. For example: "The model explores how the timing of hypothetical precipitation increases affects index-based drought recovery."

Response: Thank you for raising this important point. We agree that the previous use of the term "drought mitigation" lacked conceptual precision and could generate confusion among physical hydrological recovery, operational drought management, and synthetic model sensitivity. To avoid this ambiguity, we now explicitly clarify the meaning of this term in the context of our study. In our framework, the reduction in drought severity (DS) of a multiyear drought event results solely from hypothetical precipitation increases imposed within numerical experiments, rather than from real-world management action or rainfall modification. Accordingly, we now define this process as index-based drought alleviation and describe the system's internal response using drought attenuation efficiency (k). We also clarify that the experiments involve idealized precipitation scenarios generated by perturbing the water-input variable (precipitation) inside the PDSI framework. These scenarios are designed to evaluate how the timing of a hypothetical single-month additional water input influences the evolution of DS, rather than to imply controllable or anthropogenic "intervention". Expressions such as "precipitation-based drought mitigation", "early intervention", and other potentially misleading wording are therefore replaced with conceptually accurate descriptions such as "timing of hypothetical precipitation increases", "idealized precipitation forcing", or "model-derived attenuation efficiency." These clarification ensure that the terminology consistently reflects the synthetic nature of the numerical experiments and avoids any implication that rainfall timing can be operationally controlled. The aim of our study is to isolate how the PDSI-based system responds to controlled water-input perturbations, rather than to evaluate practical drought management strategies. Although the numerical experiments are idealized, the conceptual patterns revealed by the model may still be relevant to broader discussions about drought alleviation, since many real-world water-management practices (such as managed redistribution of available water resources or exploratory cloud-seeding trails) ultimately aim to reduce water deficits caused by prolonged precipitation shortages.

2. The central relationship of the DMQM model, that is, the DS rate equation, is introduced without theoretical justification or empirical testing. The exponential law is simply postulated at the beginning, not derived from data or hydrological reasoning. Consequently, the "goodness of fit" ($R^2 > 0.9$) is largely tautological, since k is defined within this same assumed functional form. I think that the auhtors, shall provide evidence that the exponential function is appropriate, or test alternative relationships (linear, logistic, power-law). Explain whether k has a physical interpretation (e.g., storage or recovery constant) or is purely a curve-fitting parameter. Without this, the DMQM remains a mathematical construct lacking process basis.

Response: Thank you for this constructive comment. We acknowledge that the exponential relationship in DMQM needed clearer justification. Below we provide a detailed explanation of its empirical basis, theoretical relevance, and comparative performance relative to alternative formulations.

To clarify our approach, the exponential formulation in DMQM is not arbitrarily postulated. Our modeling framework was developed by performing numerical experiments driven by observations from meteorological stations. Before specifying any functional structure, we first examined how drought severity (DS) of a multiyear drought event responds to incremental precipitation increases within controlled numerical experiment. Across events and stations, the numerical experiments consistently demonstrated a clear diminishing return pattern (Fig. 5). When hypothetical additional precipitation was incrementally increased at the same month of a drought event, the reduction in DS showed progressively weaker marginal benefits. The initial portion of added water input produced the strongest alleviation, while subsequent increments at that same month yielded progressively weaker effects. This empirical feature motivated us to seek a formulation capable of representing a bounded recovery process. To formalize this behavior, we drew an analogy with systems exhibiting saturation-type adjustment, similar to first-order kinetic expressions (e.g. $C_t = C_0 \cdot exp(-k \cdot t)$, where C_t denotes the remaining intensity at time t, C_0 is the initial state at t=0, and k is the first-order rate constant controlling the decay speed. The instantaneous decay rate $\frac{dC}{dt}$ is proportional to the current magnitude C_t.). Following this reasoning, we adopted the formulation: $DS_{rate} = 1 - exp(-k \cdot pr_{rate})$. This formulation captures how incremental water input progressively reduces the residual DS (Fig. 1). In this framework, $exp(-k \cdot pr_{rate})$ represents the proportion of remaining DS after a given additional precipitation increment. When the additional water supply is sufficient, the relative reduction in DS approaches a practical upper limit, a pattern similar to physical constraints such as finite soil water storage capacity or vegetation water use limits.

To demonstrate that the exponential structure represents real system behavior rather than tautology, we compared formulations inspired by zero-order, first-order, and second-order kinetic processes using historical multiyear drought events (MYDs) during 1991-2020. Across most stations and events, the first-order structure more accurately reproduced the curvature observed in the numerical experiments at different month of MYDs. We further extended this comparison to an n-order formulation and found that the estimated n converged toward 0.98 across MYDs, which is essentially unity. This outcome strengthens the conclusion that a first-order structure is the most appropriate representation of the observed pattern. The results are summarized in Table S1. We also compared the exponential form against alternative empirical relationships. Logistic curves did not match the observed pattern. Power-law functions were unable to satisfy the required boundary conditions. We note that the same exponential adjustment structure has been adopted in other fields. For instance, Childs et al. (2025) applied a similar functional pattern in modeling climate-driven disease dynamics. The recurrence of this structure across domains further supports its suitability for representing systems with constrained recovery capacity.

Table S1: Comparison of R^2 values for different functional models

Formulation	t_1	t_2	t_3
Zero-order: $DS_{rate}=k \cdot pr_{rate}$	0.992	0.984	0.989
First-order: $DS_{rate}=1-exp(-k \cdot pr_{rate})$	0.985	0.996	0.993
Second-order: $DS_{rate}=1-1/(k \cdot pr_{rate}+1)$	0.982	0.976	0.961
N-order: $DS_{rate}=1-[1-\mathrm{k} \cdot (1-\mathrm{n}) \cdot pr_{rate}]^{1/1-\mathrm{n}}$	0.989	0.996	0.998

Note: t_1, t_2, and t_3 represent the first to the third month of the MultiYear Drought event.

Furthermore, although parameter k is empirically estimated, it is not merely a curve-fitting constant. The parameter quantifies the system's drought attenuation efficiency, namely how rapidly additional precipitation is converted into reductions in DS. Formally, in the differential form $\frac{\partial DS_{rate}}{\partial pr_{rate}}=k \cdot (1-DS_{rate})$ (eq. 3), k represents the initial response rate when $DS_{rate}=0$. Higher k values correspond to faster alleviation, while lower k values reflect slower hydrological recovery or more persistent drought conditions.

Overall, the exponential law in DMQM reflects empirically observed behavior, aligns with physically meaningful adjustment processes, and performs more effectively than linear, logistic, power-law, and higher-order alternatives.

References

Childs, M.L., Lyberger, K., Harris, M.J., Burke, M., and Mordecai, E.A.: Climate warming is expanding dengue burden in the Americas and Asia, Proc. Natl. Acad. Sci., 122, e2512350122, doi: https://doi.org/10.1073/pnas.2512350122, 2025.

3. Although the analysis uses real meteorological data, the results remain statistical and abstract. The study would benefit from at least some discussion on what the "optimal timing" might mean in real hydrological terms, e.g., soil moisture recharge, vegetation response, or water resource management, instead of only index sensitivity. Otherwise, the practical implications of $t_{optimal}$ remain unclear.

Response: Thank you for this valuable comment. We agree that without process-level interpretation, the practical significant of $t_{optimal}$ may appear abstract. To clarify its conceptual significance, we provide additional explanation regarding what the "optimal timing" represents in hydrological and ecological terms, and how it relates to the system-level response efficiency quantified by parameter k in the DMQM.

In our framework, $t_{optimal}$ refers to the month within a multiyear drought (MYD) event when additional precipitation produces the most efficient reduction in drought severity (DS) of this event. This is determined by identifying the month at which the parameter k reaches its maximum within a drought event. As described in our response to the Comment 2, k characterizes how rapidly additional precipitation is converted into reductions in DS. Higher k values correspond to faster alleviation, while lower k values reflect slower hydrological recovery or more persistent drought conditions.

From our numerical experiments, we observed that in most cases (58.79%), the maximum k occurs at the first month (t_1) of the MYD event (Fig. 6b). However, a non-negligible fraction occurs at the second (t_2) and the third (t_3) months, accounting for 22.11% and 11.06%, respectively (Fig. 6b). To interpret these patterns, we introduced two conceptual models.

Conceptual Model 1 (Fig. 4b): For a single MYD event, earlier precipitation perturbation exerts influence over a longer fraction of the event duration, leading to stronger alleviation efficiency at early month. In this model, $t_{optimal}$ tends to t_1, denoted as t_a, representing the optimal timing under the early-intervention scenario.

Conceptual Model 2 (Fig. 4c): Considering different PDSI values at the same month, standardized indices are more sensitive to extreme one. Thus, perturbations near peak intensity may yield stronger effects. In this model, $t_{optimal}$ tends to the month at Peak Intensity Point of one MultiYear Drought event (t_{PIP}), denoted as t_b, representing the optimal timing under standardization-induced sensitivity.

These two conceptual models illustrate opposing tendencies. Conceptual Model 1 favors earlier perturbation, whereas Conceptual Model 2 favors timing near peak intensity. To reconcile these, we propose the Optimal Solution Model (Fig. 4d), which balances the advantages of both conceptual models. The resulting $t_{optimal}$, denoted as t_c, is the timing identified by our study.

This t_c has two main hydrological and ecological meaning. From the perspective of drought propagation, meteorological drought typically occurs first in the system, so understanding its optimal timing is essential for interpreting how precipitation signals subsequently influence soil moisture conditions and vegetation responses. By identifying the month when the system responds most efficiently to precipitation (maximum k), t_c provides a reference for real-world water-management practices that can effectively influence the subsequent stages of drought propagation. Regarding the potential extension of our framework to other drought types, t_c may also have broader value. If this framework can be validated for hydrological or agricultural drought, which would require incorporating additional relevant variables beyond those considered for meteorological drought, t_c could similarly guide real-world water-management practices in these types of drought directly. In ecological terms, Applying water around t_c can optimize soil water recharge while the root zones still have high infiltration capacity, and provide vegetation with water at a stage when its physiological recovery potential remains high. Both perspectives represent meaningful directions for future research. In practical terms, t_c can also guide real-world water-management decisions. It helps determine when irrigation provides the greatest drought relief per unit of water applied, guides the timing of managed water recharge or emergency ecological water delivery for maximal effectiveness. It also helps prevent cascading losses and further degradation in prolonged droughts. So that $t_{optimal}$ is presented not solely as an attenuation efficiency parameter but as a concept with hydrological, ecological, and management relevance, grounded in both system response efficiency and the dual conceptual considerations of drought progression.

In addition, we would like to clarify why our study focus on meteorological drought. Drought types such as hydrological and agricultural drought are strongly influenced by factors such as vegetation types (e.g., forest consuming more water than grass or shrubs), periods (e.g., dry seasons or warm years), and soil moisture. These factors bring substantial uncertainty when attempting to isolate the system's response to precipitation perturbation alone. Meteorological drought, by contrast, is driven primarily by precipitation deficits and atmospheric evaporative demand (Wei et al., 2025). It therefore provides a clearer and more physically controlled condition for establishing a quantitative framework of drought attenuation efficiency. Meteorological drought provides a necessary starting point for the present analysis before extending the framework to other drought types in future work.

References

Wei, Y., Huang, W., and Zhu, S.: Characterization of the propagation of two types of meteorological drought events, insufficient precipitation and excessive evaporative demand, into hydrology and agriculture, J. Hydrol., 650, 132555, doi: https://doi.org/10.1016/j.jhydrol.2024.132555, 2025.

4. While the paper contains extensive methodological detail, the presentation is overly technical and textually dense, often making it difficult to follow the main line of reasoning. Equations, parameters, and symbols are introduced in rapid succession without sufficient intuitive explanation, and transitions between conceptual discussion, mathematical derivation, and numerical experiments are often abrupt.

As a result, even readers with a strong hydrological background may struggle to understand the logical progression from the conceptual models (Section 3.4) to the construction of the DMQM and its application. The heavy use of notation (DS, DD, DI, DDP, PIP, k, t_1–t_8, toptimal) without repeated restatement of their physical meaning contributes to confusion. I strongly suggest providing a short, intuitive summary before or after key equations and reducing unnecessary algebraic detail or moving it to supplementary material and restating the physical meaning of variables (e.g., PIP, DDP, k) when they reappear in later sections.

Response: Thank you for this constructive comment. We understand that the methodological presentation in current version is dense and that the rapid introduction of equations, symbols, and terminology may hinder readers from following the conceptual progression of the DMQM framework. Below, we provide a detailed explanation of how we will address these concerns in revision.

1. We recognize the need for clearer and more intuitive transitions between conceptual reasoning, mathematical formulation, and numerical experiments. In several parts of the paper, equations appear in quick succession without sufficient explanation of their physical meaning. In the revised version, each major equation will be accompanied by a concise and intuitive explanation placed immediately before or after its presentation. These explanations will focus on clarifying why the equation is introduced, what physical reasoning it represents, and how it connects to the drought evolution process.

Furthermore, we will clarify the methodological workflow and the logic connecting the numerical experiments with the analytical formulation. As illustrated in Fig. 3, our analysis begins with a controllable hypothetical additional precipitation numerical experiment designed on the basis of the PDSI. For each MYD event, we perturb precipitation month bu month while keeping all other months unchanged. To ensure no bias, the imposed precipitation perturbation is expressed as a percentage of the multiyear mean precipitation for the corresponding month, which is formalized in $pr_{forcing}=pr_{baseline}+pr_{rate}\times\overline{pr}$, pr_{rate}=0, 5%, 10%, ..., 100% (Eq. 2). pr_{rate} denotes a dimensionless proportion (0–1) that controls the relative magnitude of the hypothetical precipitation addition, the amount of hypothetical precipitation addition is given by $pr_{rate} \times \overline{pr}$ ($\overline{pr}$ is the long-term mean monthly precipitation). For each prescribed pr_{rate} we compute the corresponding relative change in DS (DS_{rate}). Consistent with the observation described in our response to Comment 2, the DS_{rate}-pr_{rate} relationship exhibits a clear diminishing-return pattern. To formalize this behavior, we drew an analogy with systems exhibiting saturation-type adjustment and adopt a form similar to a first-order kinetic expression. After testing several functional forms, this structure provided a better performance in capturing this pattern (Table S1). Accordingly, we represent the response rate as $\frac{\partial DS_{rate}}{\partial pr_{rate}}=k\cdot(1-DS_{rate})$ (Eq. 3), where k characterizes how rapidly additional precipitation is converted into reductions in DS. The boundary and limiting conditions described in Eq. 4 and Eq. 5 then allow us to derive the analytical solution presented in Eq. 6: $DS_{rate}=1-exp(-k\cdot pr_{rate})$. Additionally, the structure of Eq. 3 further enables us to quantify the empirical sensitivity of DS to small precipitation perturbations. This sensitivity is approximated using the initial slope of the DS_{rate}-pr_{rate} curve as $k_{actual}=\frac{DS_{rate}(pr_{rate}=0.05)}{0.05}$, with 0.05 representing the step size in our numerical experiment. The $k_{theoretical}$ is obtained by fitting the analytical solution Eq. 6 to the numerically derived DS_{rate}-pr_{rate} curve. To evaluate the consistency between the empirical behavior and the analytical framework, we compute the relative error using Eq.7: $\varepsilon=\left(\frac{k_{theoretical}-k_{actual}}{k_{actual}}\right)\times 100\%$. These additions will substantially improve the transparency of the formulation and ensure that readers can clearly follow the logical progression from conceptual reasoning to numerical experimentation and analytical derivation.

2. We agree that readers may experience cognitive burden due to the heavy algebraic detail included in the current manuscript. To reduce unnecessary complexity in the main narrative, material not essential for understanding the DMQM framework will be moved to the supplementary material. This will allow the main text to focus on the conceptual logic and the key components of the framework, while still ensuring that all methodological details are available for readers who wish to examine them more deeply.

3. We acknowledge that the frequent use of symbols and acronyms such as DS, DD, DI, DDP, PIP, k, t_1–t_8, and $t_{optimal}$ can be overwhelming when their meaning is not periodically reiterated. These symbols and acronyms play central roles across multiple sections, and readers unfamiliar with the notation may lose track of their physical interpretation. To address this, brief reminders of the meaning of the most important variables will be restated when they reappear in later discussion. In addition, a consolidated table listing principal symbols, acronyms and their description will be provided so that readers can easily consult definitions at any point (Table S2).

Table S2: A list of principal symbols and acronyms

Symbols and Acronyms	Description
MYD	MultiYear Drought
PDSI	Palmer Drought Severity Index, a monthly drought index where each value represents the PDSI-defined "drought severity" for that month (the terminology is distinguished from the event-level drought severity, DS, used in this study)

DS	Drought Severity of a drought event, the cumulative deficiency of a drought index below the critical threshold throughout the event
DD	Drought Duration of a drought event, the period during which a drought index is continuously below the critical threshold during the event
DI	Drought Intensity of a drought event, the maximum deviation of the drought index below the critical threshold during the event
DDP	Drought Development Period of a drought event
PIP	DI Point of a drought event, the PDSI value corresponding to the DI
DMQM	Drought Mitigation Quantitative Model
CAFEC	Climatically Appropriate For Existing Conditions
$pr_{baseline}$	the actual precipitation in the targeted month under the baseline (no hypothetical additional precipitation is applied) scenario
$\overline{pr}$	the long-term mean monthly precipitation
pr_{rate}	a dimensionless proportion (0–1) that controls the relative magnitude of the hypothetical precipitation addition, the amount of hypothetical precipitation addition is given by $pr_{rate} \times \overline{pr}$
$pr_{forcing}$	the precipitation in the targeted month after applying hypothetical additional precipitation, $\mathrm{pr}_{\mathrm{forcing}} = \mathrm{pr}_{\mathrm{baseline}} + \mathrm{pr}_{\mathrm{rate}} \times \overline{\mathrm{pr}}$, $\mathrm{pr}_{\mathrm{rate}} = 0, 5\%, 10\%, \ldots, 100\%$
$DS_{baseline}$	the actual DS under the baseline (no hypothetical additional precipitation is applied) scenario
$DS_{forcing}$	the DS after the hypothetical additional precipitation is applied
DS_{rate}	the relative change in DS, $DS_{rate}=\frac{DS_{baseline}-DS_{forcing}}{DS_{baseline}}$
k	MYD attenuation efficiency, k characterizes how rapidly additional precipitation is converted into reductions in DS
$k_{theoretical}$	the theoretical k value, $k_{theoretical}$ is obtained by fitting the DMQM analytical solution $\mathrm{DS}_{\mathrm{rate}}=1-\exp(-\mathrm{k}\cdot\mathrm{pr}_{\mathrm{rate}})$ to the DS_{rate}-pr_{rate} curve derived from the numerical experiments
k_{actual}	the actual k value, represent the empirical sensitivity of DS to small precipitation perturbations, $k_{actual}=\frac{DS_{rate}(pr_{rate}=0.05)}{0.05}$, $pr_{rate}=0.05$ is the step size in the numerical experiments
ε	relative error, $\varepsilon=\left(\frac{k_{theoretical}-k_{actual}}{k_{actual}}\right)\times 100\%$
k_1	$k_{theoretical}$ value at the first month of a MYD event
k_2	$k_{theoretical}$ value at the second month of a MYD event
k_3	$k_{theoretical}$ value at the third month of a MYD event
k_{max}	maximum $k_{theoretical}$ within a MYD event, obtained as the largest monthly k among all months of this event
Δk_{21}	the changes in $k_{theoretical}$ between the second and the first month of a MYD event
Δk_{31}	the changes in $k_{theoretical}$ between the third and the first month of a MYD event
$t_{forcing}$	the month of hypothetical additional precipitation is applied
t_1	the first month of a MYD event
t_2	the second month of a MYD event

t_3	the third month of a MYD event
t_{PIP}	the month at PIP of a MYD event
$t_{optimal}$	the optimal timing for drought alleviation
t_a	$t_{optimal}$ in Conceptual Model 1
t_b	$t_{optimal}$ in Conceptual Model 2
t_c	$t_{optimal}$ in Optimal Solution Model

Note: To avoid ambiguity, the monthly values of the Palmer Drought Severity Index (PDSI) are referred to simply as "PDSI" in this study, whereas DS is used exclusively to denote event-level drought severity.

Overall, we greatly appreciate the suggestion to improve clarity and accessibility. The revisions described above are intended to strengthen the logical continuity between conceptual models, mathematical construction, and numerical experiments. They will also ensure that readers with hydrological expertise but without prior exposure to this modelling framework can follow the development of ideas more comfortably and gain a more intuitive understanding of the DMQM.