A Robust ~~Variational~~ Minimization-Based Framework for Cyclogeostrophic Ocean Surface Current Retrieval

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Abstract. Estimations of surface currents at submesoscales (1–50 km) are crucial for operational applications and environmental monitoring, yet accurately deriving them from satellite observations remains a challenge. While the geostrophic approximation has long been used to infer ocean surface currents from Sea Surface Height (SSH), it neglects nonlinear advection, which can become significant at submesoscales. To address this limitation, we present a robust and efficient ~~variational~~ minimization-based method for inverting the cyclogeostrophic balance equation, implemented in the open-source Python library `jaxparrow`. Unlike the traditional ~~iterative approaches~~ fixed-point approach, our method reformulates the inversion as ~~an optimization~~ a minimization problem, providing stable estimates even in regions where a cyclogeostrophic solution may not exist. Using a submesoscale-permitting model simulation and both DUACS and the high-resolution NeurOST SSH products, we demonstrate that cyclogeostrophic corrections become increasingly relevant at finer spatial scales. Validation against drifter-derived velocities shows that our approach consistently improves current estimates in energetic regions, reducing errors by up to 20 % compared to geostrophy alone in energetic regions of the global ocean. These results support the systematic inclusion of cyclogeostrophic inversion in the analysis of high-resolution SSH fields.

1 Introduction

Surface ocean currents play a critical role in a wide range of environmental and operational processes (Röhrs et al., 2023). At spatial scales from 1 to 50 km—commonly referred to as submesoscales—these currents influence the exchange of energy between the ocean and atmosphere, with important implications for climate studies (Hewitt et al., 2022). They are also essential for numerous practical applications, including offshore operations, renewable energy development (Ferreira et al., 2016), and the forecasting of object trajectories in the ocean. Accurate surface current information supports search-and-rescue missions, iceberg tracking, and the management of marine debris and oil spills (Breivik et al., 2013; Keghouche et al., 2009; Trinanes et al., 2016; De Dominicis et al., 2016). Additionally, submesoscale dynamics contribute to vertical mixing in the upper ocean, affecting biological productivity and the transport of nutrients and plankton, which are key components of marine ecosystems (Mahadevan, 2016; Lévy et al., 2018).

Satellite observations of Sea Surface Height (SSH) and Sea Surface Temperature (SST) both provide valuable insights into surface currents and fine-scale ocean dynamics. Since the 90's, satellite altimetry has provided SSH observations that are then processed into global gridded maps (Le Traon and Dibarboure, 1999) from which geostrophic velocities can be derived (Le Traon and Dibarboure, 2002). The effective resolution of these maps are estimated at nearly 200 km at mid-latitudes (Taburet et al., 2019), keeping the submesoscale spectrum invisible to us. The Surface Water and Ocean Topography mission (SWOT, Fu, 2008; Fu et al., 2012), launched in 2022, has been designed to increase the spatial resolution of earlier altimeters and reach 15 km of effective resolution in the satellite swath (Morrow et al., 2019; Nencioli et al., 2025; Wang et al., 2025). Complementary to altimetry, SST provides high-resolution snapshots of ocean surface structures, revealing submesoscale features which are not observed by conventional altimeters. Many research efforts are presently under way to derive global maps of SSH and currents with a resolution that would enable the observation of the ~~upper~~ high-wavenumber portion of the spectrum of the mesoscale dynamics by synthesizing classical altimetry with SWOT (Le Guillou et al., 2021; Ubelmann et al., 2021; Ballarotta et al., 2023; Le Guillou et al., 2023; Xiao et al., 2023) and/or SST ~~(Buongiorno Nardelli et al., 2022; Archambault et al., 2023; Fablet et al., 2024; ?; Martin et al., 2024).~~ (Buongiorno Nardelli et al., 2022; Archambault et al., 2023; Fablet et al., 2024; Le Guillou et al., 2025; Martin et al., 2024). In addition, there is a growing interest within the SWOT community in moving beyond the geostrophic approximation when exploiting the high-resolution 2D SSH fields of the SWOT swath (Archer et al., 2025; Wang et al., 2025; Zhang and Callies, 2025; Trancha

Under some dynamical conditions, accurately deriving ocean surface currents from high-resolution SSH images or maps requires using the cyclogeostrophic balance approximation rather than the usual geostrophic approximation. ~~The cyclogeostrophic balance equation mathematically describes the balance between the pressure gradient force, the Coriolis force (these two only making the geostrophic balance), and the convective force:~~ To introduce these relationships, we start from the horizontal momentum equation in a rotating frame:

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}_{cg} \cdot \nabla)\boldsymbol{u}_{cg} + f\boldsymbol{k} \wedge \boldsymbol{u}_{cg} = -g\nabla\eta + \boldsymbol{R} \tag{1}$$

where ~~$\boldsymbol{u}_{cg}$~~ $\boldsymbol{u}$ is the horizontal ~~, cyclogeostrophic~~ velocity, f the Coriolis parameter, g the gravity, η the SSH, $\boldsymbol{k}$ the vertical unit vector, and $\boldsymbol{R}$ collects frictional and unresolved processes (e.g. horizontal and vertical mixing, wind stress-driven Ekman current, and in pratice any ageostrophic contribution not captured by the balance). Bold fonts indicate vectors. The geostrophic balance results from neglecting the local acceleration, the nonlinear advective term, and the residual term:

$$f\boldsymbol{k} \wedge \boldsymbol{u}_g = -g\nabla\eta \tag{2}$$

where $\boldsymbol{u}_g$ is the geostrophic velocity. By retaining the nonlinear advective term while still neglecting the local acceleration and residual processes, one obtains the cyclogeostrophic balance:

$$(\boldsymbol{u}_{cg} \cdot \nabla)\boldsymbol{u}_{cg} + f\boldsymbol{k} \wedge \boldsymbol{u}_{cg} = -g\nabla\eta \tag{3}$$

where $\boldsymbol{u}_{cg}$ is the cyclogeostrophic velocity. This equation extends the usual geostrophic balance equation when the Rossby number Ro, defined as the ratio between the scales of the ~~convective force~~ advective term and the Coriolis ~~force~~ term, approaches

1. This "$Ro \approx 1$" regime actually characterizes the submesoscale regime (McWilliams, 2019; Taylor and Thompson, 2023). Cyclogeostrophic currents can ~~significantly~~ substantially differ from geostrophic currents in some regions such as the Mozambique channel (Penven et al., 2014)~~or~~, the Mediterranean sea (Ioannou et al., 2019), and the Antarctic Circumpolar Current

60   (Tranchant et al., 2025). A global assessment ~~has been performed by Cao et al. (2023) and leads to a similar conclusion~~by Cao et al. (2023) further indicates that important differences are also expected in the Gulf Stream, the Agulhas Current, and the Kuroshio Current.

Several methods to solve the cyclogeostrophic inverse problem have been proposed in the past literature but they all exhibit

65   drawbacks, and publicly available, well maintained implementations are missing. Penven et al. (2014) provides a review of these methods. The most widely employed, proposed by Arnason et al. (1962) and Endlich (1961), ~~is iterative.~~ solves the cyclogeostrophic balance by iteratively updating the velocity through a fixed-point relation that adds the nonlinear advective correction to the geostrophic velocity (see Eq. 7). Unfortunately, Arnason's study shows it can be unstable. This ~~has been confirmed~~ was confirmed subsequently by several authors ~~later~~ (Penven et al., 2014; Ioannou et al., 2019). In particular, the

70   method is not suitable when the ~~cyclogerostrophic~~ cyclogeostrophic equation has no solution. Further details are given in Sect. 2.

This paper proposes a new and modern numerical solution for the cyclogeostrophic inverse problem. The first novelty lies in ~~the mathematical formulation of the problem, in a variational form~~its mathematical formulation as a minimization problem. The second novelty lies in the use of the `JAX` Python library to solve the optimization problem numerically. These developments

75   make a new, open-source, and numerically efficient Python package for the cyclogeostrophic inversion, named `jaxparrow`. The ~~variational~~ minimization-based resolution corrects the shortcomings of the historical ~~iterative~~ fixed-point method and enables a quantification of the impact of cyclogeostrophic corrections as effective resolution of SSH fields increases.

The paper is structured as follows: ~~Section 2 reviews~~ Sect. 2 reviews the analytic gradient wind solution and Arnason's

80   ~~iterative~~ fixed-point method for the cyclogeostrophic inversion, describes the new ~~variational~~ minimization-based method, and its implementation with `JAX`. Section 3 details the data used and the experimental setup of our study. Section 4 presents global applications with operational SSH maps: DUACS, available through the Copernicus Marine Environment Monitoring Service (CMEMS); and NeurOST, available through the Physical Oceanography Distributed Active Archive Center (PODAAC). ~~For both products~~ Our proposed method is also compared to the fixed-point approach using pseudo-SWOT observations generated

85   from the eNATL60 simulation. Finally, for both DUACS and NeurOST products, assessments of the derived currents using drifters are included.

2   ~~Resolution of~~ Solutions to the cyclogeostrophic inversion problem

This section presents methods used to solve the cyclogeostrophic inversion problem. We first revisit the analytic gradient wind solution. We then review the historical ~~iterative~~ fixed-point approach proposed by Arnason et al. (1962), which has been

widely used despite known limitations. ~~We then~~ Finally, we introduce a novel ~~variational~~ minimization-based formulation of the inversion problem that addresses some of these shortcomings. ~~Finally,~~, and we describe our practical implementation of this ~~variational~~ minimization-based approach using modern automatic differentiation tools.

2.1 ~~State of the art: Arnason's (1962) iterative method~~ The analytic gradient wind solution

~~The~~ As discussed by Knox and Ohmann (2006), in an idealized circular and axisymmetric flow, the nonlinear term $(\boldsymbol{u}_{cg} \cdot \nabla)\boldsymbol{u}_{cg}$ simplifies to the centrifugal acceleration $-\frac{V_{gr}^2}{R}$ with V_{gr} the azimuthal component of the velocity and R the radius of curvature (which coincides with the radial distance to the vortex center in strictly axisymmetric cases). Under these assumptions, Eq. 3 becomes:

$$\frac{V_{gr}^2}{R} + fV_{gr} - fV_g = 0 \tag{4}$$

where V_g is the azimuthal geostrophic velocity, positive for cyclonic eddies and negative for anticyclonic ones. Solving this quadratic equation yields the physically relevant branch of the gradient wind solution:

$$V_{gr} = \frac{2V_g}{1 + \sqrt{1 + 4V_g/(fR)}} \tag{5}$$

Equation 5 provides useful intuition about the conditions under which the cyclogeostrophic balance admits a physical solution. For cyclonic eddies ($V_g > 0$), the term under the square root is always positive, and a real solution exists. In contrast, for anticyclonic eddies ($V_g < 0$) this term becomes negative when $|V_g/(fR)| > 0.25$, in which case no real solution exists. This situation corresponds to the occurrence of inertial instability, indicating a breakdown of the balance assumptions (Knox and Ohmann, 2006).

2.2 State of the art: Arnason's (1962) fixed-point method

While the analytic gradient wind solution is useful for understanding the existence and physical limits of cyclogeostrophic balance, it is restricted to idealized axisymmetric flows. In realistic oceanic conditions—where the flow is neither perfectly circular nor steady—numerical approaches are instead required to solve Eq. 3. A widely used strategy is the fixed-point method originally proposed by Arnason et al. (1962), which we describe below.

Taking the vector product of k with Eq. 3 ~~, denoting $\boldsymbol{u}_g$~~ and substituting the geostrophic velocity ~~, leads to~~ $\boldsymbol{u}_g$ from Eq. 2, we obtain:

$$\boldsymbol{u}_{cg} - \frac{\boldsymbol{k}}{f} \wedge (\boldsymbol{u}_{cg} \cdot \nabla)\boldsymbol{u}_{cg} = \boldsymbol{u}_g \tag{6}$$

Then the iterations proposed by Arnason et al. (1962) to get the cyclogeostrophic velocity are initialized with $\boldsymbol{u}_{cg}^{(0)} = \boldsymbol{u}_g$ and implement as:

$$\boldsymbol{u}_{cg}^{(n+1)} = \boldsymbol{u}_g + \frac{\boldsymbol{k}}{f} \wedge (\boldsymbol{u}_{cg}^{(n)} \cdot \nabla)\boldsymbol{u}_{cg}^{(n)} \tag{7}$$

This approach has traditionally been referred to as the "iterative" method. However, this terminology can be misleading, as other numerical procedures—including our minimization-based formulation (see Sect. 2.3)—are also iterative while relying on fundamentally different update mechanisms. For clarity, we therefore adopt the more precise term "fixed-point" method to describe Eq. 7.

As initially mentioned by Arnason et al. (1962), these iterations do not always converge; an *ad hoc* and imperfect stopping strategy is generally implemented to avoid their numerical divergence. A typical case of numerical divergence is when the cyclogeostrophic equation has no solution, ~~which happens in some circumstances (Ioannou et al., 2019).~~ as previously discussed in Sect. 2.1. From a fixed-point perspective, divergence also occurs whenever the initial guess $\boldsymbol{u}_{cg}^{(0)} = \boldsymbol{u}_g$ is not an attracting fixed point of the update map in Eq. 7. Knox and Ohmann (2006) provide a detailed analysis of the convergence properties of this method in the context of the idealized gradient wind balance.

To mitigate these difficulties, Penven et al. (2014) stops the iterations at any grid point i when the residual $|\boldsymbol{u}_{cg,i}^{(n+1)} - \boldsymbol{u}_{cg,i}^{(n)}|$ falls below 0.01 or starts to increase. Ioannou et al. (2019) implements this with two additional ingredients: the initial geostrophic velocity field is projected, with a cubic interpolation, on a grid 3 times finer than the initial one. This is to "improve the computation of the velocity derivatives" in Eq. 7. The second modification is in the calculation of the residual norm for each grid point, which includes now the 8 neighboring grid points. ~~From~~

Nonetheless, our own experience ~~, illustrated in Figures 2 and A, this~~ indicates that (i) the fixed-point method can fail to converge to the cyclogeostrophic solution (Fig. 1) and (ii) the local iteration-stopping strategy can ~~sometimes lead to~~ produce noisy or unrealistic velocity fields ~~. Thus there is a~~ (Fig. 2). These limitations motivate the need for an ~~alternate~~ alternative approach.

2.3 ~~Variational~~ Minimization-based formulation

We recast the cyclogeostrophic inversion problem in a ~~variational~~ minimization form, by searching for the velocity field $\boldsymbol{u}_{cg}$ that minimizes the following loss function:

$$J(\boldsymbol{u}_{cg}) = \int_{\Omega} [\Delta_{cg}(\boldsymbol{u}_{cg}(\boldsymbol{x}))]^2 d\boldsymbol{x} \tag{8}$$

where Ω is the 2D spatial domain and Δ_{cg} denotes the *cyclogeostrophic imbalance* function computed locally at each point $\boldsymbol{x} = (x, y)$ in the discretized domain:

$$\Delta_{cg}(\boldsymbol{u}_{cg}) = \left\| \boldsymbol{u}_{cg} - \frac{\boldsymbol{k}}{f} \wedge (\boldsymbol{u}_{cg} \cdot \nabla)\boldsymbol{u}_{cg} - \boldsymbol{u}_g \right\| \tag{9}$$

where $\|\cdot\|$ is the ~~L_2~~ ℓ_2 norm for a 2-component velocity vector: $\|\boldsymbol{u}(\boldsymbol{x})\| = \sqrt{u(\boldsymbol{x})^2 + v(\boldsymbol{x})^2}$, using the standard ~~notations~~ notation for the zonal and meridional velocities. In Eq. 8, we make it explicit that the loss function is the domain integral of a locally computed norm (the integration could be hidden in the norm operator) in anticipation of further discussions.

The ~~variational~~ minimization of Eq. 8 is performed using gradient descent, i.e. by taking small steps in the direction opposite to the gradient of J:

$$\boldsymbol{u}_{cg}^{(n+1)} = \boldsymbol{u}_{cg}^{(n)} - \gamma \nabla J\left(\boldsymbol{u}_{cg}^{(n)}\right) \tag{10}$$

150   where the hyperparameter γ controls the step size. The gradient $\nabla J\left(\boldsymbol{u}_{cg}^{(n)}\right)$ is computed automatically using JAX's reverse-mode automatic differentiation: JAX records the computation of J as a sequence of elementary operations with known derivatives and applies the chain rule to construct the corresponding gradient function.

The minimization-based formulation is expected to solve the numerical divergence problem of the ~~iterative~~ fixed-point method; it also provides a measure of the deviation from the cyclogeostrophic solution (when it exists). Where the cyclo-
155   geostrophic imbalance Δ_{cg} reaches 0, the solution is the cyclogeostrophic velocity. ~~And where there is no cyclogeostrophic solution, the minimum of J yields a smooth velocity field that does not exhibit the unrealistic features yielded by the iterative method~~ In regions where no exact cyclogeostrophic solution exists, the minimization-based approach—because its update strategy relies on the gradient of a globally evaluated loss involving spatial derivatives—favors a smoother and more coherent estimate of the velocity field, despite the absence of any explicit regularization term. In this sense, it is expected to avoid the
160   unrealistic features that the fixed-point method can generate, since the latter's point-wise update and stopping criterion tend to amplify noise. Interestingly, the cyclogeostrophic imbalance is a straightforward indication of where a cyclogeostrophic velocity can be found, and where it cannot. It is not possible to determine the physical nature of the velocity solution when Δ_{cg} does not reach 0. But it is still possible to quantify a deviation from the cyclogeostrophic equilibrium.

2.4 Implementation

165   Our cyclogeostrophic inversion library, jaxparrow (Bertrand et al., 2025), is implemented with JAX (Bradbury et al., 2018), a Python library developed by Google to perform two main operations on Python functions: acceleration and automatic differentiation. jaxparrow leverages both features. The automatic differentiation capability directly provides the gradient of J, which can be used for gradient-based minimization methods. For the minimization itself, jaxparrow implements Optax (DeepMind et al., 2020), a gradient processing and optimization library specifically developed for JAX.

170   jaxparrow handles gridded data, making it well-suited for estimating cyclogeostrophic currents from SSH derived from models, Level-4 products, and also 2D Level-3 products ~~as illustrated in Fig. B1. In jaxparrow, the partial derivatives are computed~~. While most altimetry products use Arakawa A-grids, where all quantities are evaluated at the grid center (T point), jaxparrow computes partial derivatives using finite differences on Arakawa C-grids~~.~~, where the SSH is defined at the grid center, the velocity components at the grid faces, and the vorticity at the grid vertices. As a result, variables must be carefully
175   interpolated when performing numerical computations. Specifically, for the kinematic diagnostics described in Sect. 3.2.1, the velocity components u and v are first interpolated to the T points prior to computing the velocity magnitude, whereas vorticity is calculated directly on the C-grid and then interpolated back to the T points.

To support further evaluation of our ~~variational~~ minimization-based method and facilitate the integration of cyclogeostrophic currents into a global operational product, our library is easily installable via pip, with its code publicly available on GitHub.

3 Data and experimental setup

This section describes the data sources and methodology used to assess cyclogeostrophic surface current reconstructions. We first present the ~~input satellite SSH products~~satellite-derived products, the model data, and the drifter dataset used for validation. We then detail the experimental setup, including the computation of derived kinematic fields and the evaluation procedure based on drifter-derived velocities.

3.1 Input and validation data

3.1.1 Operational SSH products

~~As Penven et al. (2014); Ioannou et al. (2019); Cao et al. (2023),~~ Following Penven et al. (2014), Ioannou et al. (2019), and Cao et al. (202_, we use the standard Data Unification and Altimeter Combination System ~~(?)~~ (DUACS, 2024) SSH global product. ~~In its most recent version, DUACS provides data with daily temporal resolution on a grid with a spatial resolution of 1/8°.~~ As reported by Ballarotta et al. (2019), the DUACS effective resolution (computed using the Signal to Noise Ratio method) ranges globally from 100 km at high latitudes to 800 km in the equatorial band. In its most recent version, DUACS provides data with daily temporal resolution on a grid with a spatial resolution of 1/8°.

To illustrate the relevance of cyclogeostrophic corrections as effective resolution increases, we also use the newer experimental global product NeurOST (NeurOST, 2024). NeurOST gridded data has a temporal resolution of one day and a spatial resolution of 1/10°. Martin et al. (2024) shows that by combining satellite observations of SSH and SST, NeurOST improves the effective resolution by up to 30 % compared to DUACS, particularly in the Gulf Stream region, where NeurOST achieves an effective resolution of 108 km versus 150 km for DUACS.

The present study covers the period from 2010 to 2022 (inclusive), corresponding to the availability period of both DUACS and NeurOST products.

3.1.2 eNATL60 model data

We leveraged SSH and surface currents from the eNATL60-BLB002 simulation (Brodeau et al., 2020) to illustrate the benefits of reconstructing surface currents from SSH using the cyclogeostrophic approximation rather than the geostrophic one. eNATL60 is a submesoscale-permitting North Atlantic configuration (including the Mediterranean Sea) of the NEMO ocean model, with a 1/60° horizontal resolution. We employed the tide-free version of the configuration and the daily-averaged dataset of the simulation run.

3.1.3 Global Drifter Program (GDP) dataset

We used 6-hourly interpolated surface current velocity measurements from drifters, collected in the GDP database (Lumpkin and Centurioni, 2019). The GDP database includes data from drifters of various types and shapes with differing sensitivities to wind. To ensure that the reference velocities are not influenced by direct wind forcing on the drifters, we restricted our analysis

210 to drogued SVP-type drifters. Drogue-loss detection in SVP drifters was known to be unreliable, leading to some observations being incorrectly tagged as drogued. The GDP database provides a more robust drogue presence tag, employing the procedure described by Lumpkin et al. (2013), in which drogue loss is detected based on anomalous downwind ageostrophic motion. At the global scale, over the period 2010—2022, it represents approximately 9.8 million observations from around 12,500 drifters.

3.1.4 Modeled Ekman currents

215 To remove the Ekman contribution from the drifter-derived velocities, we used the GlobCurrent product (GlobCurrent, 2024). In GlobCurrent, Ekman currents at the surface and at 15 m depth are estimated from ERA5 wind stress following the methodology of Rio et al. (2014). These estimates are provided at hourly resolution on a regular 1/4° grid.

3.2 Experimental setup

3.2.1 Derived kinematics

220 Starting from global SSH maps, we present several diagnostics to assess the impact of accurately computed cyclogeostrophic velocities. ~~Due to the use of Arakawa C-grids, variables must be carefully interpolated as needed when performing numerical computations.~~

We compute the cyclogeostrophic imbalance from Eq. 9 and use it as a local measure of deviation from cyclogeostrophy, expressed in $\mathrm{m\,s^{-1}}$. To better highlight divergences while estimating cyclogeostrophic currents, spatial deviations from
225 cyclogeostrophy are aggregated over time by taking the maximum of the ~~7-days~~ 7-day moving average, following the approach of Fig. 12 in Ioannou et al. (2019).

We derive geostrophic and cyclogeostrophic velocities from SSH using `jaxparrow`. ~~Iterative~~ Fixed-point cyclogeostrophic velocities are computed using Eq. 7, with the stopping procedure described in ~~Sec. ??~~ Sect. 2.2 (same as Penven et al., 2014; Cao et al., 2023). ~~Variational~~ Minimization-based cyclogeostrophic velocities are estimated by minimizing J ~~from~~ (Eq. 8) using
230 gradient descent (Eq. 10) with a fixed ~~learning rate~~ step size of 5×10^{-3} for 2,000 iterations, using geostrophic velocities as the initial guess.

Relative vorticity provides insight into ocean dynamics and the quality of reconstructed current velocities. It represents the spinning motion of a water parcel relative to the Earth and is defined as the curl of the velocity:

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \tag{11}$$

235 Since it requires computing spatial derivatives, it is expected to highlight noise in velocity fields. Relative vorticity maps are also computed using `jaxparrow`.

Eddy Kinetic Energy (EKE) quantifies the kinetic energy associated with the time-varying component of the flow and as such is a good indicator of the mesoscale dynamics. Following Cao et al. (2023) we compute it as:

$$\mathrm{EKE} = \frac{(u')^2 + (v')^2}{2} \tag{12}$$

240 where u' and v' are the zonal and meridional components of the velocity anomaly (i.e. deviation from the mean flow). We use the Sea Surface Height Anomaly (SSHA), rather than the full SSH, to compute geostrophic and cyclogeostrophic velocity anomalies in the same manner as for total current velocity.

3.2.2   Evaluation against ~~drifters~~total surface currents

To validate that cyclogeostrophy provides a better estimate of surface currents than geostrophy, we compute evaluation metrics
245 ~~using~~ against eNATL60 relative vorticity and drifter-derived velocities.

Pseudo-SWOT observations of the eNATL60 SSH

Because the true total sea-surface fields corresponding to satellite SSH observations are unknown, one way to evaluate the cyclogeostrophic inversion methods is the use of model data. To mimic SWOT swath observations from model output, we generate pseudo-SWOT data by re-interpolating eNATL60 SSH onto portions of the two SWOT CalVal passes that cross the
250 Balearic Sea, using the 2-km SWOT grid.

For each point of the SWOT grid, we define the inversion error for method M as:

$$\epsilon_M = (\zeta_M - \zeta)^2 \tag{13}$$

where ζ and ζ_M are relative vorticity fields computed from Eq. 11 using, respectively, the eNATL60 velocity field (interpolated onto the SWOT swath) and the velocity field obtained from the cyclogeostrophic ($M = cg$) or geostrophic ($M = g$) inversion
255 of the eNATL60 SSH field, also interpolated onto the swath. We then compute the time-averaged Root Mean Square Error (RMSE) at each grid point over August 2009:

$$\mathrm{RMSE}_M = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\epsilon_M^{(i)}} \tag{14}$$

where $N = 31$ is the number of days considered. To compare two inversion methods, we use the normalized difference between their RMSE values:

260 $$\Delta\,\mathrm{RMSE}_{M_1 - M_2} = 100\frac{\mathrm{RMSE}_{M_1} - \mathrm{RMSE}_{M_2}}{\mathrm{RMSE}_{M_1}} \tag{15}$$

This indicator measures the relative improvement (or degradation) in the fidelity of the reconstructed vorticity field when using method M_2 instead of M_1, capturing changes in both bias and variance of the inversion.

Velocities derived from drogued SVP drifters

Another way to evaluate the cyclogeostrophic inversion methods is to use drifter-derived velocities.
265 Thanks to their drogue centered at 15 m depth, SVP drifters sample the currents in the upper $\sim$10–20 m of the ocean (Lumpkin and Pazos, 2007). They provide an estimate of the total current velocity, including signatures from high-frequency

processes such as near-inertial wave, and, as illustrated by Eq. 1, these unbalanced motions are neglected in both the geostrophic and the cyclogeostrophic approximations. To mitigate the influence of these additional terms in our analysis, we follow the procedure applied by Müller et al. (2019) to 6–hourly interpolated SVP drifter data. We first remove the Ekman contribution to the drifter velocities using the 15 m Ekman current estimated in the GlobCurrent product. We then filter near-inertial signal by applying a second-order Butterworth filter with a 25–hour cutoff period to the drifter velocities.

For each drifter observation i at time t_i and position $\mathbf{X}_i$, we define the inversion error for method M as:

$$\epsilon_M^{(i)} = \|\mathbf{u}_M(t_i, \mathbf{X}_i) - \mathbf{u}_d^{(i)}\|^2 \tag{16}$$

where $\mathbf{u}_d$ is the drifter velocity vector and $\mathbf{u}_M$ is the velocity field obtained from the cyclogeostrophic ($M = cg$) or geostrophic ($M = g$) inversion, interpolated at the drifter time and position. Individual errors are binned into 1° latitude × 1° longitude boxes (Fig. ?? B1 shows the number of observations per bin). Within each bin, we compute the bias RMSE of an inversion method M using the mean inversion error:

$$\overline{\epsilon_M} = \frac{1}{N} \sum_{j=1}^{N} \epsilon_M^{(j)}$$

Eq. 14, where N is now the number of errors within the bin. We evaluate the robustness of an inversion method M by computing the standard error within each bin:

$$\hat{\sigma}_M = \sqrt{\frac{1}{N} \sum_{j=1}^{N} \left(\epsilon_M^{(j)} - \overline{\epsilon_M} \right)^2}$$

To compare error variability between (or observations) inside that bin. To compare two inversion methods spatially, we use the normalized difference between their binned standard errors, defined as : RMSE values, as defined in Eq. 15.

$$\widehat{\Delta}\sigma_{M_1 - M_2} = 100 \frac{\hat{\sigma}_{M_1} - \hat{\sigma}_{M_2}}{\hat{\sigma}_{M_1}}$$

This indicator quantifies the change in reliability of surface current estimates from SSH when using inversion method M_2 instead of M_1. RMSE values, as defined in Eq. 15.

In addition to spatial binning the spatial comparison, we also compute $\overline{\epsilon_M}$ and $\hat{\sigma}_M$ across bins defined around cyclogeostrophic EKE percentiles, as illustrated in Fig. ??. assess whether the cyclogeostrophic solution provides a better estimate than the geostrophic one as a function of the magnitude of the cyclostrophic correction. The cyclostrophic correction is defined as the difference between the cyclogeostrophic velocity (obtained using the minimization-based approach) and the geostrophic velocity:

$$\mathbf{u}_c = \mathbf{u}_{cg} - \mathbf{u}_g \tag{17}$$

For any drifter observation i, we consider the cyclogeostrophic solution to be better than the geostrophic one if $\epsilon_{cg}^{(i)} < \epsilon_g^{(i)}$. This criterion allows us to model the probability that the cyclogeostrophic solution outperforms the geostrophic one, conditionally

295 on the magnitude of the cyclostrophic correction, $\mathbb{P}(\epsilon_{cg} < \epsilon_g | \|\mathbf{u}_c\|)$, using a logistic regression. To allow for a nonlinear dependence on $\|\mathbf{u}_c\|$, we expand $x_i = \|\mathbf{u}_c^{(i)}\|$ using a natural cubic spline basis $\mathbf{s}(x_i) = (s_1(x_i), \ldots, s_K(x_i))^\top$ with $K = 4$ functions, and fit the model:

$$\text{logit}[p_i] = \beta_0 + \sum_{k=1}^{K} \beta_k s_k(x_i), \qquad \text{with} \qquad p_i = \mathbb{P}\left(\epsilon_{cg}^{(i)} < \epsilon_g^{(i)} | \|\mathbf{u}_c^{(i)}\|\right). \tag{18}$$

This provides a smooth estimate of the probability that cyclogeostrophy outperforms geostrophy as a function of the cyclostrophic
300 correction magnitude, along with 95% confidence bands computed using the delta method.

4 Application to ~~global maps of~~ SSH maps

In this section, we apply the proposed cyclogeostrophic inversion method to ~~global~~ maps of SSH. We first analyze the resulting geostrophic and cyclogeostrophic surface currents at the global scale, highlighting differences in dynamic regions. We then focus on the reconstruction skill using pseudo-SWOT swath observations over the Balearic Sea. Finally, we evaluate the reconstructed
305 currents ~~against independent drifter observations~~ globally by comparing them with independent drifter measurements from the GDP ~~to quantify improvements in accuracy~~. Unless otherwise specified, the cyclogeostrophic inversion method referred to throughout this section is the ~~variational~~ minimization-based one.

4.1 Analysis of geostrophic and cyclogeostrophic currents

Surface currents derived from SSH using the geostrophic approximation and both ~~variational and iterative~~ minimization-based
310 and fixed-point cyclogeostrophic inversion methods are here qualitatively analyzed (i) with the cyclogeostrophic imbalance from Eq. 9, (ii) by observing the velocity and relative vorticity fields, and (iii) through a comparison of EKE.

The measure of the deviation from cyclogeostrophy shows that (i) geostrophy can be a ~~coarse~~ crude approximation of cyclogeostrophy at some ~~location~~ locations in space and time and (ii) the ~~variational~~ minimization-based inversion method is
315 more accurate than the ~~iterative~~ fixed-point method to compute a cyclogeostrophic velocity field. These conclusions are drawn from the examination of Fig. 1 which presents the time-aggregated deviation from the cyclogeostrophic balance of 3 velocity fields derived from NeurOST SSH, namely the geostrophic field (top) and the cyclogeostrophic solutions from the ~~variational~~ minimization-based method (bottom left) and the ~~iterative~~ fixed-point method (bottom right). The geostrophic field exhibits large deviations from cyclogeostrophy, ~~exceeding~~ with deviations larger than 0.3 m s^{-1} at nearly 5 % of grid points, hinting that
320 the ~~advection~~ advective term should not be neglected. The solution of the ~~iterative method deviates from cyclogeostrophy by~~ fixed-point method deviates even further, with differences exceeding 0.35 ~~at over~~ m s^{-1} at more than 5 % of ~~grid points, while the variational method shows deviations below~~ points. In contrast, the minimization-based method limits deviations above 0.03 ~~at~~ m s^{-1} to fewer than 5 % of grid points. This suggests that the ~~iterative~~ fixed-point method is less reliable in converging

toward a cyclogeostrophic solution, particularly in the western boundary currents and the Agulhas current, where the ~~variational~~ minimization-based method shows that a cyclogeostrophic solution exists.
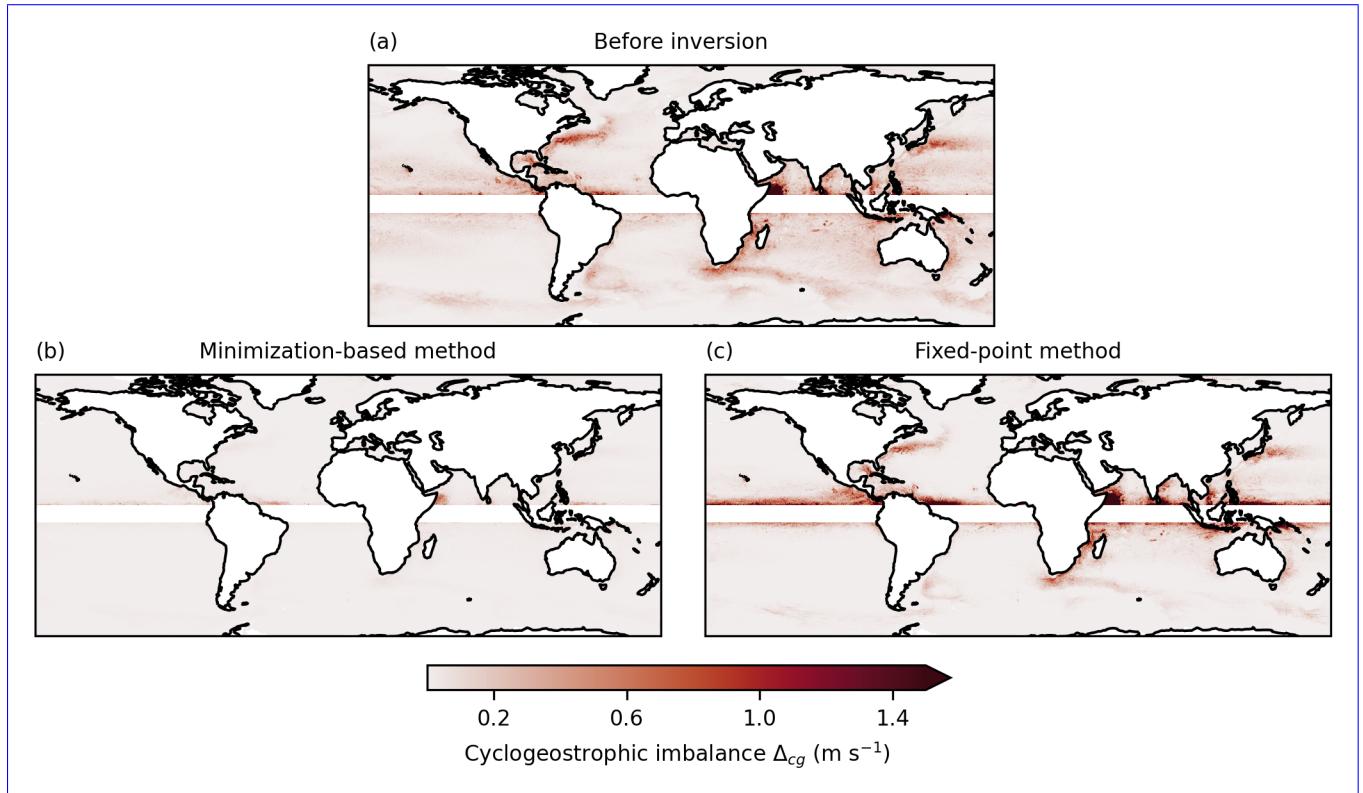
Figure 1. Maps of deviation from cyclogeostrophy, computed from Eq. 9, for the geostrophic velocity (a), the ~~variational~~ minimization-based cyclogeostrophic velocity (b), and the ~~iterative~~ fixed-point cyclogeostrophic velocity (c) derived from NeurOST SSH.

Our implementation of the proposed ~~variational~~ minimization-based method enables physically consistent estimation of cyclogeostrophic currents on a global scale, including in highly dynamic regions where cyclogeostrophic corrections substantially impact jets and eddies, and where the ~~iterative~~ fixed-point method yields unrealistic physical fields. Figure 2 presents a global snapshot of the norm of cyclogeostrophic currents derived from NeurOST SSH, along with ~~a zoom on~~ an enlargement of the Gulf Stream region where relative vorticity and differences compared to geostrophy are also displayed. In the northern ~~(southern) branches~~ meanders of the Gulf Stream jet, ~~positive (negative) cyclogeostrophic corrections~~ cyclogeostrophic corrections are positive and can reach up to $\pm$$+0.2$ m s^{-1}, while in the southern meanders they are negative, down to -0.2 m s^{-1}. Similarly, anticyclonic ~~(cyclonic) eddies exhibit~~ and cyclonic eddies exhibit respective cyclogeostrophic contributions of approximately $+0.2$ ~~(~~m s^{-1} and -0.2 ~~)~~ m s^{-1}, corresponding to ~~differences from $\pm$~~ relative increases of 10~~% to $\pm$~~–50 % in the anticyclonic case and relative decreases of 10–50 % in the cyclonic case. Finally, while the ~~variational~~ minimization-based method allows for the reconstruction of a smooth and physically coherent relative vorticity field, the ~~iterative~~ fixed-point method introduces

artifacts in the most dynamic parts of the jet and eddies. As discussed in Sections 2.2 and 2.3, these differences are likely linked to the distinct mathematical nature of the two approaches.
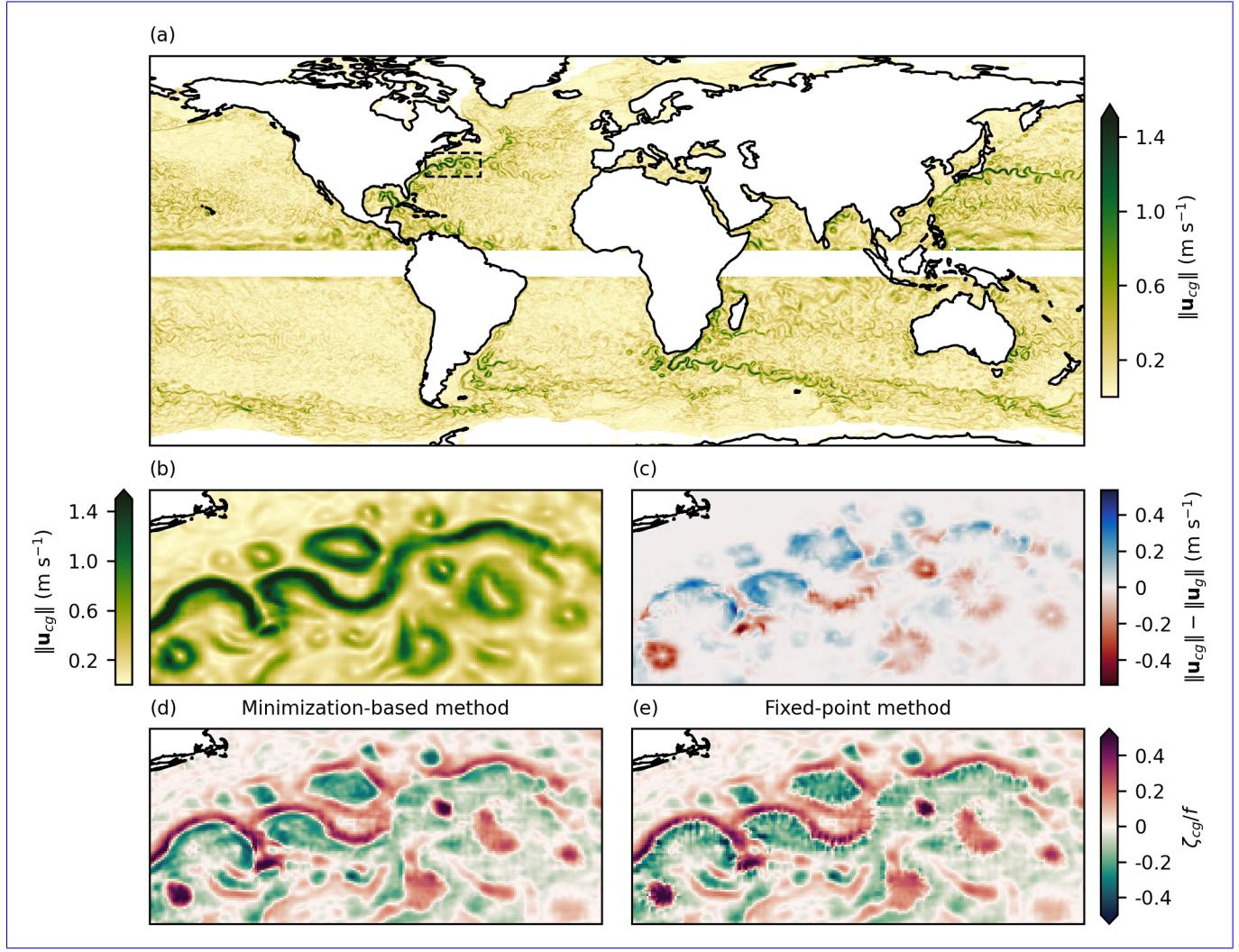
Figure 2. 16 April 2015 snapshots derived from NeurOST SSH. (a) Norm of the ~~variational~~ minimization-based cyclogeostrophic velocity. (b) Same as (a), zoomed in the Gulf Stream region. (c) Difference between the norms of ~~variational~~ minimization-based cyclogeostrophic and geostrophic velocities. (d) Relative vorticity computed from the ~~variational~~ minimization-based cyclogeostrophic velocity. (e) Same as (d), using the ~~iterative~~ fixed-point cyclogeostrophic velocity.

The EKE computed from the geostrophic and the ~~variational~~ minimization-based cyclogeostrophic velocities anomalies exhibit differences up to 20 %, essentially at low and middle latitudes. This is shown in ~~Figure~~ Fig. 3, which presents the relative difference in EKE between cyclogeostrophy and geostrophy, averaged over the whole time period. Positive differences are particularly pronounced near the equatorial band. Regions with intense dynamics such as the western boundary currents and the

Agulhas current are characterized by elongated dipole structures with both positive and negative differences. These reflect a current intensification in anticyclonic eddies detaching poleward and a damping of the current in cyclonic eddies detaching

345 equatorward, in agreement with the magnitude and sign of cyclogeostrophic corrections observed in ~~Figure~~ Fig. 2. All these observations are consistent with Cao et al. (2023) who performed a similar analysis with 1/4° DUACS maps and the historical ~~iterative~~ fixed-point method for cyclogeostrophy over the period 1993-2018. Our results suggests once more that geostrophy can be a crude approximation leading to errors up to 20 % in EKE.
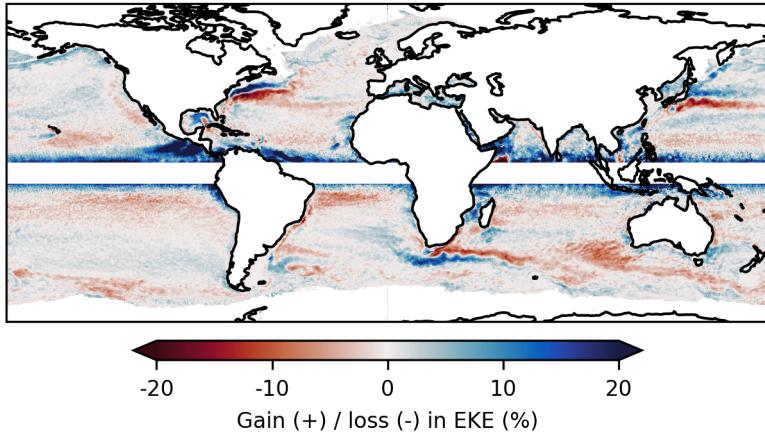
Figure 3. Relative difference in EKE between ~~variational~~ minimization-based cyclogeostrophic and geostrophic current velocity anomalies derived from NeurOST SSH.

4.2   Evaluation using pseudo-SWOT observations from eNATL60

350 Normalized relative vorticity fields obtained from geostrophy or cyclogeostrophy surface current reconstruction are compared to reference fields derived from eNATL60 total surface currents. To demonstrate the feasibility of performing the cyclogeostrophic inversion in the SWOT swath using our package `jaxparrow`, the original eNATL60 fields are first interpolated onto the 2-km grid of the SWOT swath before reconstruction.

While normalized relative vorticity fields derived from the cyclogeostrophic balance are generally in better agreement with
355 eNATL60 reference—especially near the cores of persistent anticyclonic eddies—the fixed-point method more frequently exhibits RMSE increases compared to geostrophy, particularly along eddies boundaries where the minimization-based approach continues to outperform geostrophy. This is illustrated in Fig. 4, which shows a snapshot of the reference normalized relative vorticity field (top-left), the RMSE of the minimization-based reconstruction computed over one month (top-right), and the relative change in RMSE with respect to geostrophy for both the fixed-point method (bottom-left) and the minimization-based
360 method (bottom-right). Figure A also displays the normalized relative vorticity field for the three inversion methods, together with the corresponding surface current velocity fields. Several anticyclonic submesoscale (≤50 km) eddies can be identified in the reference normalized relative vorticity field shown in panel (a). Three of these eddies—located North and South of Ibiza,

and South of Menorca—are persistent over the full month of the evaluation period (not shown). From panel (b), we observe that the RMSE of the minimization-based method exceeds 0.1 only in coastal areas, where the cyclogeostrophic assumption likely

365 breaks down. The relative difference in RMSE with respect to geostrophy in panel (d) generally indicates a better reconstruction when using the minimization-based approach, particularly in the regions of the three persistent eddies where improvements locally reach 100 %. Conversely, panel (c) shows that the fixed-point method provides slightly weaker improvements and, more notably, more frequent degradations, with pixel-like patterns similar to the artifacts seen in Fig. 2 and also noticeable in Fig. A.

370 Consistently with Archer et al. (2025); Tchonang et al. (2025); Tranchant et al. (2025), these results suggest that cyclogeostrophy should be employed when analyzing high-resolution 2D SSH fields. They also indicate that the minimization-based method may provide more reliable reconstructions than the fixed-based approach in such contexts.

4.3 Evaluation with data from the GDP

Reconstructed cyclogeostrophic and geostrophic currents are evaluated against drifter-derived velocities using ~~the binned~~
375 ~~standard error , as~~ (i) the inversion error defined in Eq. ~~??, computed (i)~~ 16 and binned within 1° latitude × 1° longitude boxes at the global scale, and (ii) ~~across EKE percentiles~~a logistic regression modeling the probability for cyclogeostrophy to outperform geostrophy as a function of the cyclostrophic correction magnitude.

When using NeurOST SSH, ~~variational~~ minimization-based cyclogeostrophic corrections improve surface current estimates,
380 particularly in energetic regions such as western boundary currents~~and the Agulhas current~~, where reconstruction ~~uncertainties~~ errors are highest. This is illustrated in Figures 5 and ~~??. Fig.~~ 6. Figure 5 presents global maps of the cyclogeostrophic ~~standard error~~ RMSE obtained from NeurOST SSH (top-left panel) and of the comparison between cyclogeostrophic and geostrophic inversion methods for NeurOST (top-right). Cyclogeostrophic ~~standard error~~ RMSE remains below 0.1 m s^{-1} across most of the ocean but increases to 0.2~~—~~0.5 m s^{-1} in energetic currents. In these regions, NeurOST-based cyclogeostrophy clearly reduces
385 standard error, with improvements of up to 10 % in the Gulf Stream and over 20 % in the Kuroshio. ~~Fig. ??~~ Figure 6 further illustrates this, showing the ~~reconstruction bias and standard error for both geostrophy (blue bars) and cyclogeostrophy (green and red bars)around EKE percentiles~~probability that cyclogeostrophy outperforms geostrophy as a function of the cyclostrophic correction magnitude. The solid lines correspond to the logistic regression fit, and the shaded envelopes indicate the 95 % confidence bands. We note that these confidence bands are estimated from the whole population of inversion errors, that is
390 why the binned empirical mean probabilities (dots)—which are computed from smaller subsets of data as the cyclostrophic corrections increases—fall outside the bands. Focusing on NeurOST-derived currents (~~solid bars~~blue), we find that ~~at the highest EKE percentiles, cyclogeostrophy reduces reconstruction uncertainty by nearly 10 % upon geostrophywhen employing the variational method, and by less than 5 % with the iterative method~~cyclogeostrophy is, on average, consistently a better estimate than geostrophy, and that this probability increases with the magnitude of the cyclostrophic correction, up to 70 % for
395 cyclostrophic corrections of 0.45 m s^{-1}.
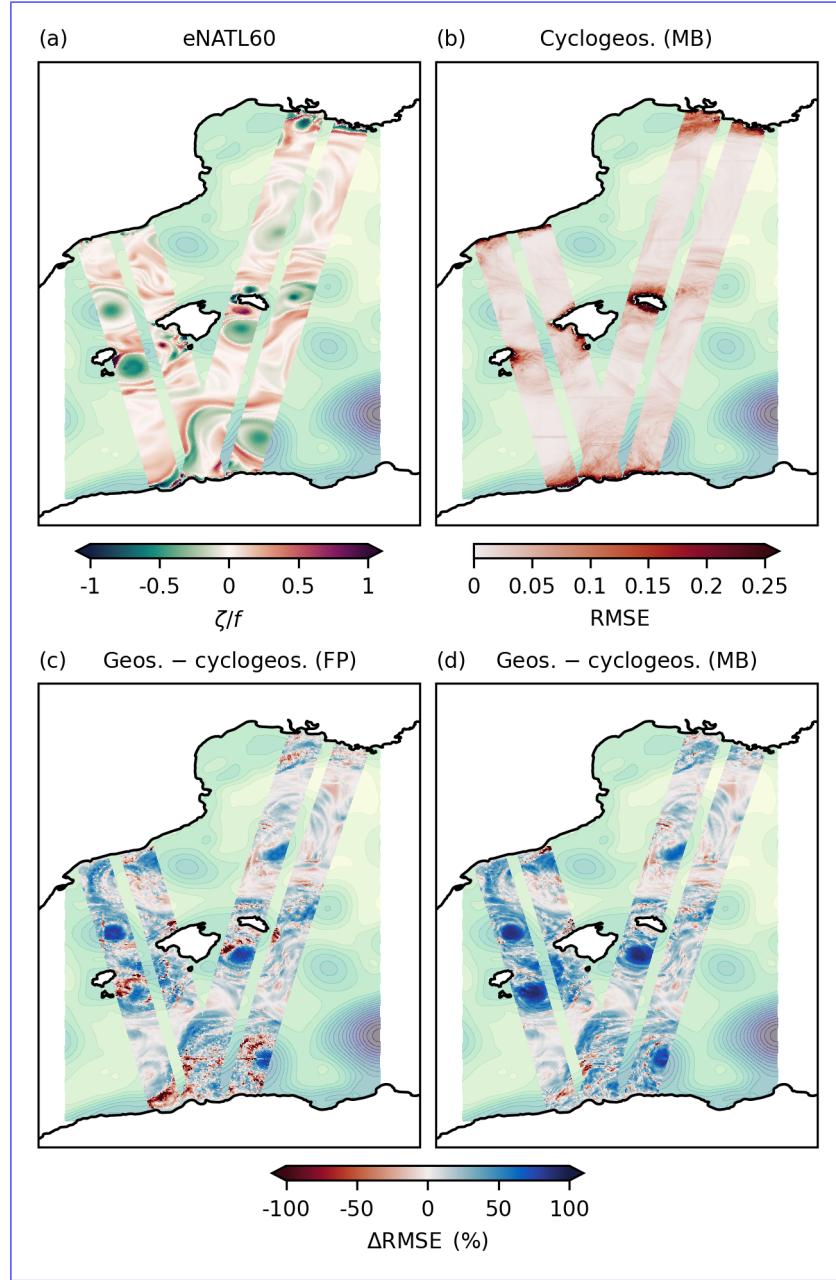
Figure 4. Performance of cyclogeostrophic inversion methods applied to eNATL60 SSH interpolated onto the SWOT swath. The background field in all four panels is the original eNATL60 SSH on 15 August 2009. (a) Normalized relative vorticity computed from eNATL60 surface currents on 15 August 2009. RMSE values of normalized relative vorticity with respect to eNATL60 in panels (b), (c) and (d) are computed over the full month of August 2009. (b) RMSE obtained when reconstructing surface currents using the minimization-based approach. (c) Relative RMSE difference between geostrophic and fixed-point cyclogeostrophic inversions. (d) Same as (c) but using the mimization-based cyclogeostrophic inversion.

In contrast, cyclogeostrophic corrections can degrade performances when applied to DUACS SSH. This is again illustrated in Figures 5 and ~~??~~6. The bottom-left panel of Fig. 5 compares cyclogeostrophic and geostrophic inversion methods based on DUACS SSH. Unlike results obtained with NeurOST, regions such as the western boundary currents ~~and the Agulhas current~~ show a degradation in performance of around 10 % when cyclogeostrophic corrections are applied. Similarly, the ~~dashed bars~~ orange line in Fig. ~~?? show the standard error across EKE percentiles for~~ 6 shows the logistic regression fit for improving the reconstruction when using cyclogeostrophy rather geostrophy for DUACS-based surface currents~~using the variational method (green), the iterative method (red), and geostrophy (blue). Across all EKE percentiles, the three methods perform similarly, with cyclogeostrophic approaches showing a slight deterioration in performance as EKE increases.~~. Cyclogeostrophy performs worse more often, on average, than geostrophy for cyclostrophic corrections smaller than $0.45 \, \mathrm{m \, s^{-1}}$. These discrepancies could stem from differences in the effective resolution of the SSH products: DUACS may insufficiently capture fine-scale structures, deteriorating the accuracy of cyclogeostrophic ~~correction~~ corrections in energetic regions.

Importantly, the combination of higher effective resolution SSH fields and cyclogeostrophic inversion yields substantial benefits over the current operational standard. As shown in Fig. 5 (bottom-right panel), applying ~~variational~~ minimization-based cyclogeostrophy to NeurOST SSH reduces reconstruction standard error by 5~~—~~ 20 % at mid-latitudes relative to DUACS geostrophy. ~~Fig. ?? confirms this result across EKE percentiles: NeurOST-based variational cyclogeostrophy (green, solid bar) yields an 8 % reduction in standard error at low EKE (0.05), and up to 15 % at higher EKE (0.35) compared to DUACS geostrophy (blue, dashed bar).~~

These results suggest that cyclogeostrophic corrections will become increasingly relevant as SSH products achieve higher effective resolution—consistent with the findings from Tranchant et al. (2025)—and could significantly benefit future operational surface current products.

5 Discussion and conclusions

We developed a new and robust method for the cyclogeostrophic inversion of surface currents by reformulating the inversion problem in a ~~variational~~ minimization-based framework, thereby overcoming the limitations of the traditional ~~iterative fixed-point~~ approach. The method is implemented as an open-source Python package, `jaxparrow`, which leverages the ~~JAX~~ JAX library for high-performance and scalable computation, enabling its application at the global scale. When applied to NeurOST SSH fields and pseudo-SWOT observations, the proposed approach yields physically consistent cyclogeostrophic current estimates, particularly in energetic regions. The relevance of the cyclogeostrophic corrections derived with our ~~variational~~ minimization-based method is supported by a global, 13-year comparison with drifter-derived velocities.

This work makes systematic application of cyclogeostrophic inversion feasible, providing a complementary tool for ~~surface current reconstruction~~ reconstructing surface currents from operational SSH products as well as from high-resolution 2D SSH observations in the SWOT swath.

Several questions were not addressed in this study. By formulating the cost functional J from Eq. 8 as a domain integral, the solution to the minimization problem depends on the entire study region. Moreover, we did not investigate the sensitivity of the
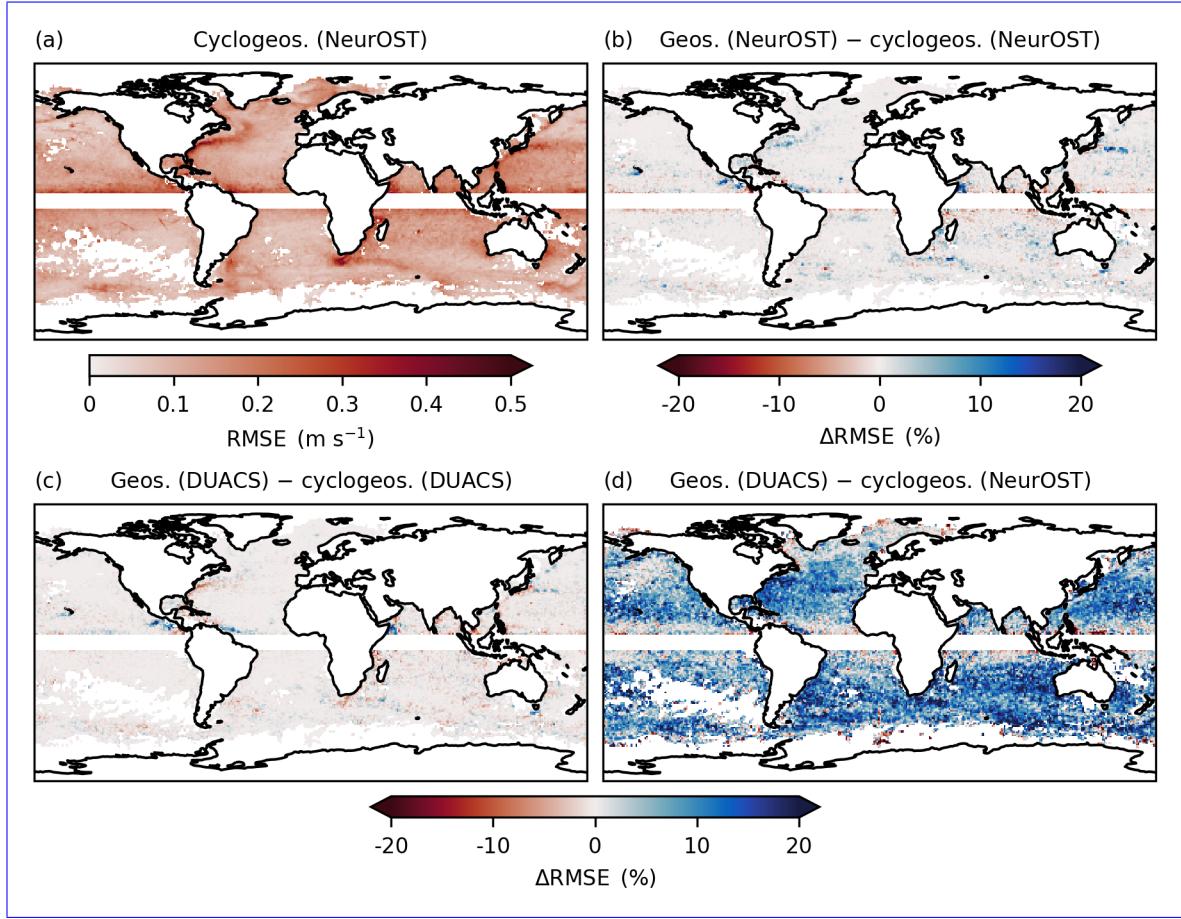
Figure 5. (a) ~~Standard errors~~ RMSE with respect to the drifters for the cyclogeostrophic velocity estimated from NeurOST SSH. (b) Relative RMSE difference of NeurOST-derived geostrophic and cyclogeostrophic velocities~~standard errors~~. (c) Same as (b) but using SSH from DUACS. (d) Same as (b) but between DUACS geostrophic velocities and NeurOST cyclogeostrophic velocities.~~See Fig. ?? for the corresponding plot of the bias instead of the standard error.~~
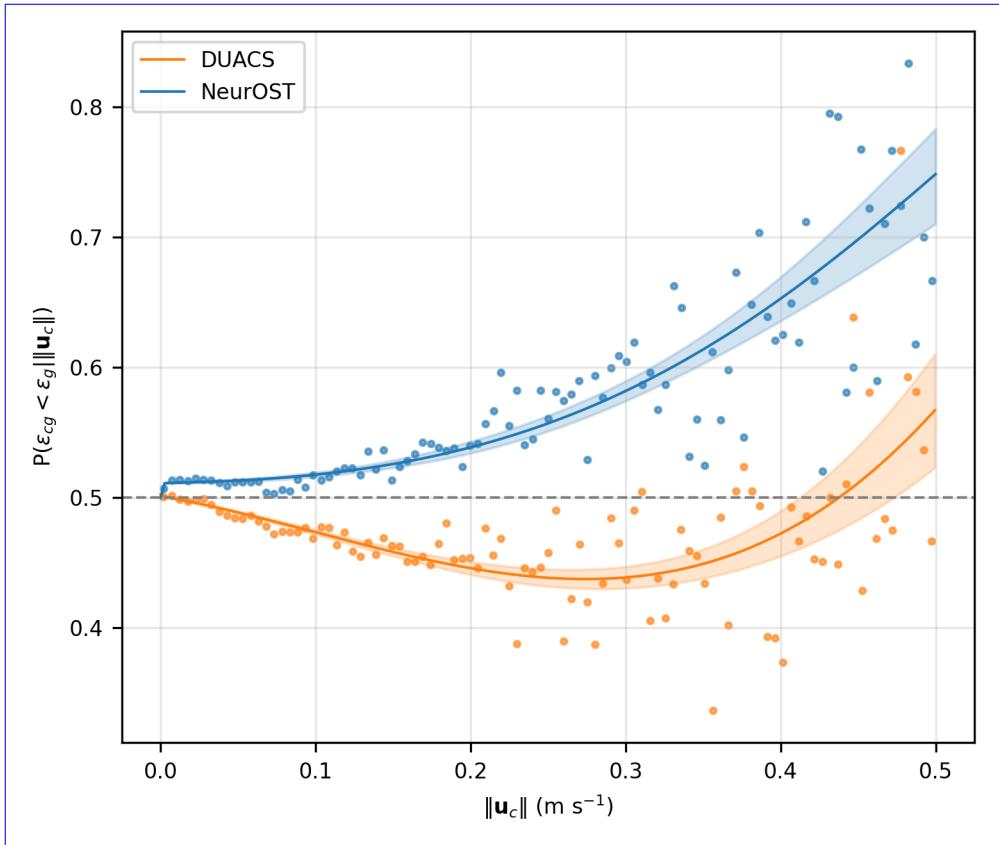
Figure 6. ~~Bias and standard error with respect~~ Probability that cyclogeostrophy improves surface current reconstruction relative to geostrophy, as a function of the ~~drifters around EKE percentiles~~cyclostrophic correction magnitude. Dots indicate empirical proportions computed per bin of cyclostrophic correction magnitude. Solid ~~error bars refer~~ lines show the logistic regression fit. Shaded envelopes denote to ~~NeurOST-derived velocities and dashed error bars to DUACS-derived velocities ; green refers to variational cyclogeostrophic velocities~~the 95 % confidence band, ~~red to iterative cyclogeostrophic velocities, and blue to geostrophic velocities~~computed using the delta method.

minimization solution to the choice of the optimizer~~and its~~ : although Eq. 10 illustrates the classical gradient descent update, the

430   `Optax` library provides many alternative optimization algorithms and corresponding hyperparameters. These points suggest potential avenues for investigation, such as partitioning the domain into sub-regions and applying different minimization strategies tailored to the energetic conditions of each area. Furthermore, ~~drifter-based validation does not fully isolate the geostrophic or cyclogeostrophic components of motion, and to our knowledge there is no well established processing (i.e. filtering)of drifter-derived velocities designed for geostrophic or cyclogeostrophic currents evaluation.~~ the iterations from Equations 7

435   and 10 are initialized using the geostrophic velocity field. An alternative—of potential interest for future work—would be to initialize from the analytical gradient wind solution (Eq. 5), relaxing the axisymmetric assumption by estimating the local radius of curvature following Meijer et al. (2022) (see their Eq. 3).

In addition to enabling the inclusion of cyclogeostrophic corrections in operational SSH and surface current products, our work opens several ~~new~~ additional opportunities. With its effective resolution reaching 15 km within the swath, the SWOT mission offers unprecedented possibilities for observing and studying the submesoscales. While several efforts are currently underway to accurately separate balanced and unbalanced signals from SWOT SSH ~~(Gao et al., 2024; Tranchant et al., 2025)~~ (Gao et al., 2024; Tranchant et al., 2025; Uchida et al., 2025), our implementation provides a practical approach for reconstructing cyclogeostrophic currents ~~and thus extending~~ from balanced SSH, thereby enabling SSH-based diagnostics to be systematically extended beyond the geostrophic approximation, ~~as illustrated in Fig. B1. Furthermore, the development of Data Assimilation (DA) systems for General Circulation Models (GCMs) or Ocean GCMs (OGCMs) typically requires building and maintaining their tangent linear and adjoint models~~. Another advantage of our minimization-based formulation is its flexibility to incorporate extra constraints or regularization terms directly into the inversion. Because the cyclogeostrophic inversion is expressed as a differentiable cost functional, the method can naturally be extended to jointly filter noisy SSH observations—~~a complex and time-consuming task. Recent progress in differentiable programming frameworks such as JAX and Julia (?) now allows for automatic differentiation of GCMs, enabling direct access to their tangent linear and adjoint models (??). As our cyclogeostrophic inversion method is fully implemented in JAX, it can be readily integrated as a modular component into such DA systems.~~ such as those from SWOT, similarly to Tranchant et al. (2025)—or to enforce consistency with ancillary surface fields, like sea surface temperature as in Le Guillou et al. (2025). While these extensions could also be embedded within larger variational or learning-based data-assimilation systems, the key advantage here is the ability to constrain the inversion itself using additional physical or observational information.

Code and data availability. The DUACS delayed-time altimeter gridded maps of sea surface height product used in this study is freely available on the CMEMS portal: https://doi.org/10.48670/moi-00148.

The NeurOST delayed-time altimeter gridded maps of sea surface height product used in this study is freely available on the PO.DAAC portal: https://doi.org/10.5067/NEURO-STV24.

The six-hourly interpolated drifters data used in this study is freely available on the NOAA portal: https://doi.org/10.25921/7ntx-z961, or via the `clouddrift` Python library: https://doi.org/10.5281/zenodo.11081647.

The SWOT L3 Expert data in its version v2_0_1 is available through the AVISO+ portal: https://doi.org/10.24400/527896/A01-2023.018.

The eNATL60-BL002 data is available on MEOM's OpeNDAP: https://ige-meom-opendap.univ-grenoble-alpes.fr/thredds/catalog/meomopendap/extract/MEOM/eNATL60/eNATL60-BLB002/1d/SSH/catalog.html.

The minimal diagnostics used in this study are available on Zenodo: https://doi.org/10.5281/zenodo.16099419. More comprehensive and larger datasets can also be found on MEOM's OpeNDAP: https://ige-meom-opendap.univ-grenoble-alpes.fr/thredds/catalog/meomopendap/extract/MEOM/cyclogeostrophy-paper/catalog.html.

The code used to run this study experiments and produce the diagnostics presented here can be found on GitHub: https://github.com/vadmbertr/cyclogeostrophy_impact_experiment.

The code of the Python library `jaxparrow` introduced in this paper is also available on GitHub: https://github.com/meom-group/jaxparrow.

Appendix A: ~~Variational cyclogeostrophic inversion from NEMO-eNATL60 SSH~~

Appendix A: Cyclogeostrophic inversion in a pseudo-SWOT swath

~~As a qualitative assessment of the benefit of the variational method for the inversion of cyclogeostrophic currents we present snapshots obtained from eNATL60-BLB002 (no tides) (Brodeau et al., 2020) simulation based on NEMO. This simulation covers the North Atlantic, plus the Mediterranean Sea, with a 1/60° horizontal resolution. We choose snapshots of the Alboran Sea as it features two large and persistent gyres subject to cyclogeostrophy.~~

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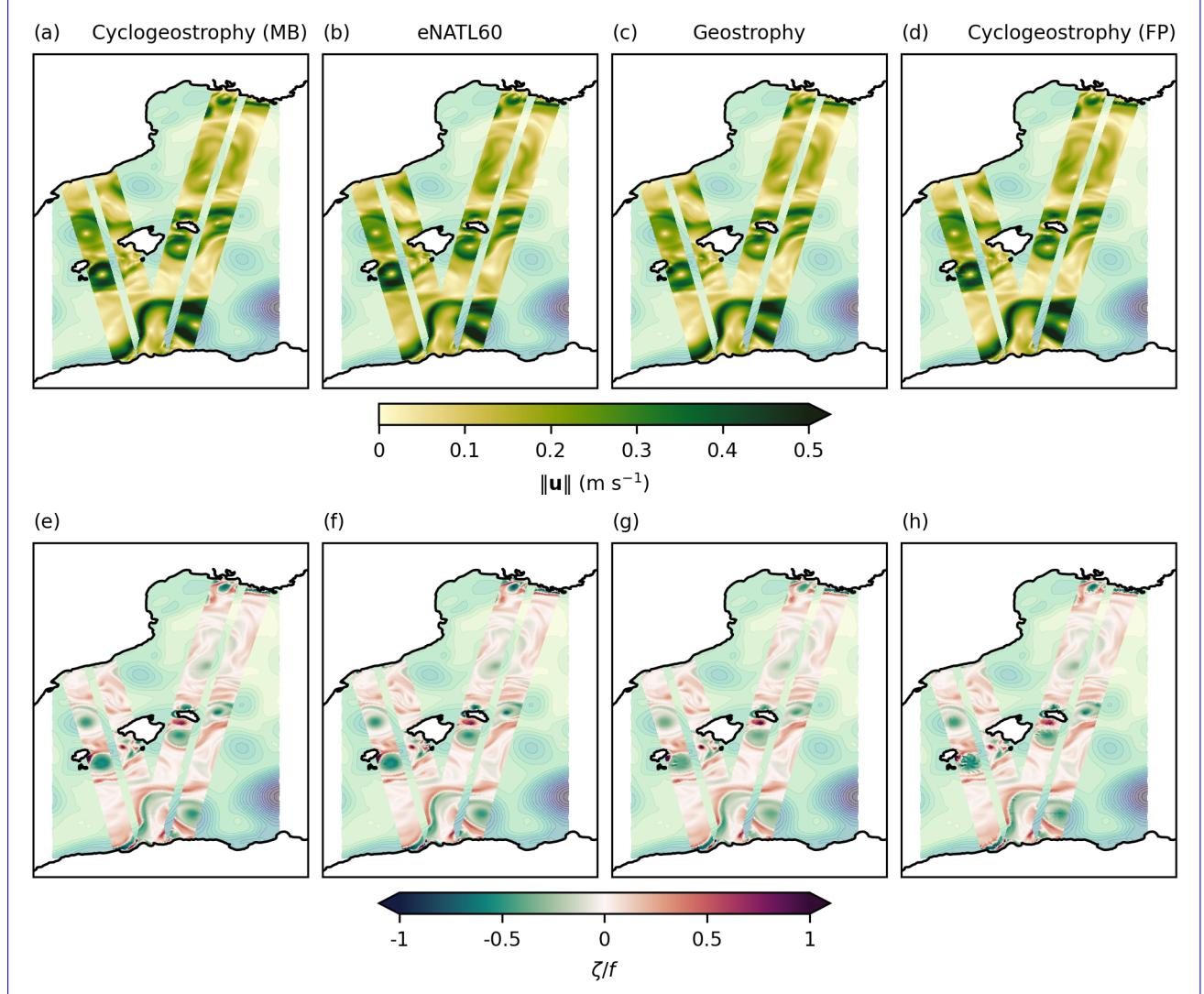
Appendix B: Variational cyclogeostrophic inversion in the SWOT swath

To illustrate the use of our implementation of the variational cyclogeostrophic inversion on irregular grids we show inversion results in the SWOT swath. We use the

8 June 2023 snapshots derived from SWOT L3 SSH. (a) SSH = MDT + SSHA_Filtered. (b) Norm of the geostrophic currents derived from the SSH. (c) Norm of the cyclogeostrophic currents derived from the SSH using the variational method. (d) Same as (c), using the iterative method. Top row: surface current magnitude. Bottom row: normalized relative vorticity. (a), (e) Cyclogeostrophic currents reconstructed with the minimization-based method. (b), (f) True eNATL60 fields interpolated onto the swath. (c), (g) Geostrophic currents reconstructed from SSH. (d), (h) Cyclogeostrophic currents reconstructed with the fixed-point method.

Appendix B: Evaluation with data from the GDP

~~This section provides additional plots supporting our evaluation of the reconstructed currents using data from the GDP.~~

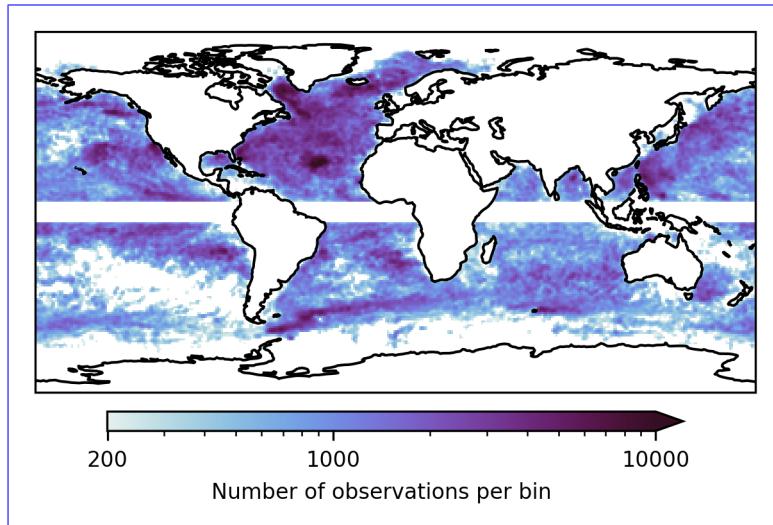



Figure B1. Number of drifter observations used for the methods' evaluation per 1° latitude × 1° longitude bin.

~~Same as Fig. 5, but using the mean instead of the standard deviation for spatial binning.~~

480 *Author contributions.* VB developed the Python package `jaxparrow`, designed and ran the experiments and analysis, and wrote the manuscript. EC proposed the minimization-based cyclogeostrophic inversion formulation. VZDA carried out the initial work on the implementation in `JAX` of the minimization-based cyclogeostrophic inversion. JLS and EC contributed to the design of the experiments, and the writing of the manuscript. JLS, AS and EC contributed to the analysis. JLS and EC acquired funding. All authors reviewed the manuscript.

Competing interests. No competing interests are present.

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