Author Responses to reviewers' comments

We thank the reviewer for carefully reading the revised manuscript and for providing a detailed and constructive comment. We agree that our previous wording did not sufficiently distinguish between the exact 3D radiative-transfer (RT) definition of albedo (α) susceptibility and the regression-based susceptibility diagnostic calculated in this study. The major changes made to the manuscript are listed below.

1. We modified the title to explicitly state that the study addresses "Regression-Based Albedo Susceptibility Estimates," rather than implying that the exact 3D RT derivative of albedo susceptibility is calculated.

2. We added text in Sect. 2.2 clarifying that, in 3D RT, the horizontally resolved albedo field depends on the height or reference surface at which the upwelling irradiance is registered. Therefore, the local α used in the regression analysis should be interpreted as a domain-top, locally registered albedo diagnostic from the forward RT calculation, not as a uniquely column-resolved TOA albedo.

3. We added explanatory text at the beginning of Sect. 3.4 distinguishing the exact, physically invariant 3D RT albedo susceptibility from the regression-based diagnostic estimated in this study. We now state explicitly that the exact quantity is a functional derivative of area-averaged albedo with respect to perturbations in cloud optical or microphysical properties, and that computing this derivative would require a linearized/adjoint 3D RT calculation or equivalent perturbation framework.

4. We revised the interpretation of Figures 11 and 12 and their captions. The green curves are no longer described as "truth" susceptibility. They are now described as LES-based

regression benchmarks computed from domain-top 3D RT albedo, (N_{d_LES}), and (LWP_{LES}) binning. We also state explicitly that these curves are diagnostic benchmarks and are not exact linearized 3D RT susceptibility derivatives.

5. We revised the abstract and conclusion to use "regression-based albedo susceptibility" terminology and to state that the reported susceptibility estimates are sensitive to the chosen vertical albedo/flux-registration convention.

Please find our point-by-point response to the referee's comment in black as well as the changes made to the revised manuscript in blue below.

Reply to Reviewer 1:

Comment:

I appreciate the effort that the authors have put in to address my extensive comments. I have one significant comment warranting major revisions.

The definition of albedo susceptibility used in the text is ambiguous as it depends entirely on the vertical reference level used to construct the spatial variation of the albedo. As touched on in my previous comments, when the altitude of the TOA albedo is a distance from the cloud top that is much larger than the horizontal domain size, the albedo will become spatially uniform. This is an immediate geometric consequence of the cosine weight in the definition of irradiance. Then, as the variance of the albedo tends to zero, the regression slope of horizontally varying albedo against (N_d) will tend to zero. Since the reference level is a geometric convention, it could be chosen in an arbitrary way to select a smaller or larger albedo variance, and hence a smaller or larger albedo susceptibility. Each convention will produce a different covariance between droplet concentration and albedo. None of them are correct. None of them are the “truth”, i.e., consistent with classical physics.

Regardless of the geometric convention used to construct the horizontal variability of albedo, the real albedo susceptibility is unchanged. The sensitivity of area-averaged TOA albedo to a particular column’s optical properties is invariant to the height of the domain top and to the geometric convention. A consistent differential form for the 3DRT preserves this property. The present study’s definition does not.

The authors have argued that the implementation of a calculation of albedo susceptibility using 3DRT is out of scope of a study that focuses on the calculation of albedo susceptibility using 3DRT. This does not make sense to me, personally. While the portions of this paper covering droplet concentration retrievals are valuable, I have yet to be convinced that this study estimates the “Impacts of three-dimensional radiative transfer effects on […] albedo susceptibility.”

In lieu of making changes to the numerical results, my request to the authors is simple; please explain everything in your article: Introduce the exact calculation of albedo susceptibility, introduce your approximation, and discuss the implications. Then, justify your interpretation of the results to the reader without skirting around the textbook definition of irradiance in 3D geometry.

Reply:

We thank the reviewer for this important clarification. We agree that our previous description of the susceptibility calculation was not sufficiently precise, especially where the regression-based 3D RT/LES susceptibility curve was described as a “truth” reference. The reviewer is correct that, in 3D geometry, the horizontally resolved albedo field obtained from flux integration is not a unique local cloud-column quantity. Its spatial variability depends on the height or reference surface at which the upwelling irradiance is registered. Therefore, a regression of locally registered albedo against (N_d) is not equivalent to the exact 3D RT susceptibility, which should be defined as the functional derivative of area-averaged albedo with respect to perturbations in column optical or microphysical properties.

To address this, we have revised the manuscript to explicitly distinguish between (i) the exact 3D RT albedo susceptibility and (ii) the regression-based susceptibility diagnostic calculated in this study. We now state that the exact susceptibility is the derivative of area-averaged albedo with respect to perturbations in the cloud optical or microphysical state, and that such a calculation would require a linearized/adjoint 3D RT calculation or an equivalent finite-difference perturbation framework. We also now state that the susceptibility values reported in this study are regression-based diagnostics computed from locally registered, domain-top albedo and should not be interpreted as exact column-resolved 3D RT derivatives of TOA albedo.

The title, abstract, Sect. 2.2, Sect. 3.4, Figure 11 and Figure 12 captions, and the conclusion have been revised accordingly. In particular, we no longer refer to the green curves as “truth” susceptibility. Instead, we call them LES-based regression benchmarks (in short as LES-based benchmarks). These benchmarks are useful for comparing retrieval-based regression estimates because they use the same forward 3D RT albedo field and LES microphysical information, while avoiding retrieval errors in (N_d) and LWP. However, we now explicitly acknowledge that these benchmarks remain dependent on the domain-top registration convention and are not exact 3D RT susceptibility derivatives.

Changes made in the revised manuscript:

1. *The title of the revised manuscript was updated from "Impacts of the Three-dimensional Radiative Effects on Cloud Droplet Number Concentration Retrieval and Albedo Susceptibility Estimates" to "Impacts of the Three-dimensional Radiative Effects on Cloud Droplet Number Concentration Retrieval and Regression-Based Albedo Susceptibility Estimates"*

2. *Regression-based albedo susceptibility was emphasized in the abstract of the revised manuscript and that the albedo is evaluated at the top of the cloud domain.*

3. *We updated Sect. 2.2 of the revised manuscript around L239–L255 to include*

   "Within a 1D RT framework, the upwelling flux and corresponding albedo can be interpreted locally because each column is radiatively independent from surrounding columns. However, within a 3D RT framework the horizontally resolved albedo field is not a uniquely local cloud-column quantity. The value of α calculated for a particular pixel depends on the height or reference at which the upwelling irradiance is registered (for our study the reference is at the top of the RT domain), because horizontal photon transport and angular integration redistribute radiative contributions from neighboring columns. Thus, different registration levels can produce different spatial variances of α and different covariances between α and local cloud properties. For example, if the irradiance field is evaluated at a level far above the cloud relative to the horizontal domain size, the locally registered albedo field can become increasingly smooth and approach a spatially uniform value, even though the area-averaged reflected flux remains physically well defined. Therefore, the local α used in the regression-based susceptibility calculations in this study should be interpreted as a domain-top, locally registered albedo diagnostic from the

forward RT calculation, not as a unique column-resolved TOA albedo or as the exact 3D RT albedo susceptibility. The physically invariant 3D susceptibility is instead the response of area-averaged albedo to perturbations in cloud optical or microphysical properties, which we discuss in Sect. 3.4."

4. *L643: The title of Sect. 3.4 in the revised manuscript was changed from "Implications for Estimating Twomey Effect from Retrievals" to "Implications for Regression-Based Albedo Susceptibility Estimates for Evaluating the Twomey Effect" Also, a paragraph distinguishing the exact 3D RT susceptibility from the regression-based diagnostic and implications of this definition utilized in the study was explained around L729 to 759 of the revised manuscript as shown below.*

"Before interpreting the susceptibility results, it is important to distinguish the exact 3D RT susceptibility from the regression-based albedo susceptibility *estimated in this study. The physically invariant 3D albedo susceptibility is the functional derivative of an area-averaged albedo with respect to perturbations in the cloud optical or microphysical properties, which describes how the area-mean albedo responds to a perturbation in a particular cloud column or group of columns. Unlike a regression slope based on locally registered albedo, this derivative is not determined by the arbitrary height chosen to register the spatially varying flux/albedo field. Computing this exact quantity in 3D requires a linearized or adjoint 3D RT calculation, or an equivalent finite-difference perturbation framework. Such a calculation requires additional modeling and computational framework and is out of scope of this present study.*

As stated in Sect. 2.2 and earlier in this section, the quantity evaluated here is an approximate, regression-based susceptibility diagnostic rather than an exact 3D RT susceptibility. Specifically, within each LWP bin, we estimate the slope of the locally registered, domain-top broadband albedo, α, with respect lnN_d *(Eq. 10). This approach is similar to regression-based observational albedo-susceptibility analyses, where TOA albedo from CERES and* N_d *inferred from passive cloud retrievals such as MODIS are used to estimate susceptibility (e.g., Painemal and Minnis, 2012; Platnick and Oreopoulos, 2008). However, under 3D RT, the locally registered α field is sensitive to the chosen reference level because horizontal photon transport and angular integration redistribute reflected radiation across neighboring columns. As a result, the spatial variance of α and its covariance with* N_d *can depend on the albedo registration convention. Therefore, the susceptibility values reported in this study should not be interpreted as exact column-resolved 3D RT derivatives of TOA albedo. Rather, they quantify how 3D RT and retrieval errors affect regression-based albedo–*N_d *susceptibility estimates under the specific domain-top registration convention used in this study. Our interpretation does not rely on the absolute value of the regression slope as a physically invariant susceptibility. Instead, we focus on how the slope changes when the same regression framework and albedo-registration convention are applied to albedo and* N_d *fields affected by different RT assumptions, spatial resolutions, and absorbing-channel choices. In this sense, the diagnostic quantifies how 3D RT effects would modify the albedo–*N_d *covariance and associated regression-inferred susceptibility under an observational-style regression framework, rather than the exact differential response of area-mean TOA albedo to a controlled perturbation in* N_d*."*

5. L848–L851: The caption of Figure 11 in the revised manuscript was updated to include "Green curves show the LES-based regression benchmark computed from domain-top 3D RT albedo, (N_{d_LES}), and (LWP_{LES}) binning; these curves are used as diagnostic benchmarks and are not exact linearized 3D RT susceptibility derivatives."

6. L873–L875: The caption of Figure 12 in the revised manuscript was updated to include "Green curves show the LES-based regression benchmark computed from domain-top 3D RT albedo, (N_{d_LES}), and (LWP_{LES}) binning; these curves are used as diagnostic benchmarks and are not exact linearized 3D RT susceptibility derivatives."

7. In the summary and conclusion of the revised manuscript around L1017–L1019 we added the sentence below to emphasize that the exact 3D RT susceptibility is different from our regression-based albedo susceptibility estimates used in our study.

   "We emphasize that these susceptibility estimates are not exact 3D RT derivatives of area-averaged TOA albedo. They are regression-based diagnostics computed from locally registered domain-top albedo and are therefore sensitive to the chosen flux-registration convention."