

# Response to the Reviewer 2: A multi-physics Eulerian framework for long-term contrail evolution

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We sincerely thank the reviewer for dedicating their valuable time to reviewing our manuscript and for their careful and insightful comments, which we believe were very helpful in enhancing the quality of the paper. Before responding one to one to the reviewer's comments, we would like to summarize the modifications/additions included in the revised version of the manuscript:

1. We have revised the paper's structure, in accordance with both Reviewers' 1 and 2 comments. Together with some re-arrangements, a concept figure has been added. This intends to strength the focus of the paper in our "multi-physics Eulerian framework for long-term contrail evolution", which we herein coin as MuliCon. We also hope it helps identifying the main building blocks of the model.
2. We have included a direct comparison with the CoCiP contrail model (following both Reviewers' recommendations), expanding also the discussions of the results and, thus, we believe now we have found a better balance between the MultiCon model development and the application of the model to obtain results (as suggested by Reviewer 1).
3. We have added the wind-shear effect into the MultiCon modeling framework using a tracer (characteristic) equation. This also responds to a suggestion from Reviewer 2.
4. Following Reviewer's 2 recommendation, the monodispersity assumption has been made prominent in the introduction, the modelling assumptions, and the problem statement. We have also discussed extensions to accommodate the polydispersity effect in our MultiCon model.
5. We have provided additional insight on the habit dynamics, including further justifications and explanations of the validity of the habit model. This was one of the main criticism of Reviewer 2.
6. We have derived a representative Partial Differential Equation (PDE) for ice supersaturation in detail and explained precisely how it is implemented in our simulations, including the initialization schemes employed, addressing an important comment from Reviewer 2.
7. We have highlighted the computational time required for single-plume analysis (the analysis conducted in this paper) and discussed its implications for potential future work targeting large-scale aircraft/contrail simulations.
8. We have included minor changes in accordance with both reviewers' comments.

Now, we proceed answering one by one the comments raised by the Reviewer. For the sake of clarity, the following criteria has been adopted throughout this response document:

- Reviewer's comments
  - Authors' responses
  - New words added to the manuscript after revision
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We would like to begin by expressing our sincere gratitude for the time you have dedicated to reviewing our paper, and especially for your insightful comments, which have greatly helped us improve our work. Please find our specific responses to your comments below:

**Comment 1:** First of all, I would like to note that the manuscript is difficult to read because it addresses many aspects that I have not encountered in the contrail literature. I greatly appreciate that concepts, phenomena etc. from other research fields are being incorporated into the field of contrail research. Although these ideas may be common in other disciplines, it is unclear whether all of them are of primary importance for contrail evolution. You must clearly demonstrate that your novel model components are essential for contrail evolution.

In my view, the text is verbose and contains numerous technical details that should be relocated to the appendix or to a supplement. It is difficult to discern which aspects of the model and its assumptions are truly significant. Other essential information—such as the assumptions regarding a monodisperse size distribution—is buried deep within the text. The manuscript requires a much clearer structure and hierarchy regarding where which type of information is provided. Because many technical details appear in the main body, it is challenging to find the most essential information.

**Response:** In the revised version of the paper, we have considerably polished the structure to better emphasize the novel aspects of the present model. These aspects are now explicitly highlighted in the Introduction. The monodisperse assumption has been made clear in the introduction, Section 2 which provides modeling assumptions in the present research and also reflecting directly in the title of Section. 3.

Changes made in the **Introduction:** Clarifications on the scope of the present work:

In this research, we present a unified Eulerian framework for contrail evolution that rigorously couples macro- and microphysical processes within a single computational domain. Macro-scale dynamics are described by moment equations derived from the Population Balance Equation (PBE), which include a nonlinear diffusion term representing the (possible) diffusion-blocking mechanisms and also includes polydispersity (mass space) by construction. Microphysics are represented by Eulerian field equations obtained by translating the Lagrangian growth kinetics of individual ice crystals into spatially and temporally resolved moment fields. These microphysical fields are further coupled to a habit-dynamics field equation that solves shape-evolution, enabling a continuous representation of ice-crystal geometry throughout the contrail life cycle. Within this framework, we distinguish the particle-scale terminal velocity (function of crystal mass and projected area-known as Stokes formula) from the ensemble-scale bulk settling velocity. By performing an analysis of the multi-phase flow equations under high turbulent mixing, we derive a first-order Burgers'-type equation that accounts for the collective bulk settling velocity. In addition, we demonstrate that, under mild assumptions, the governing equations exhibit separability, making the model particularly well-suited for large-scale simulations with a favorable balance between accuracy and computational cost.

Changes made in **Introduction**: Clarifications on the generality of the Eulerian framework, which inherently has the potential to be applied under monodisperse or polydisperse assumption:

Notably, although the presented Eulerian framework is general, the corresponding micro- and macro-scale field equations are derived primarily under the assumption of monodispersity for simulation purposes. Extension to polydispersity is doable in a seamless way. However, because the main focus of the present work is to introduce the framework itself and to study the effects of habit dynamics and the ensemble-scale bulk settling-velocity closures, the simulations reported here employ the minimal (monodisperse) representation of the Eulerian framework, following a mathematical separation ansatz.

Changes made in **Section 2**: Further clarifications on the monodisperse assumption made in this work:

Finally, each computational cell is characterized by an average ice-particle mass, number concentration, mass concentration, and shape index, thereby assuming a locally monodisperse particle distribution through the use of the Kronecker delta function in the PBE. Nevertheless, the grid dependency of the plume in the sensitive vertical dimension has been assessed, and a fine mesh resolution was selected at which the field equations were convergent.

Changes made in **Section 3**: Revised title, directly reflecting the monodispersity assumption in this work:

Multi-Physics Monodisperse Equations for Long-Term Contrail Evolution

Changes made in **Conclusion**: on the polydispersity extension:

The vertical solver can be extended to treat particle mass explicitly and represent polydispersity; with low-parameter closures, the additional computational cost remains modest. Moreover, microphysical fidelity may be increased by augmenting the growth and habit-dynamics terms to include habit-specific ventilation corrections, and morphological evolution (e.g., hollowing or branching) which directly modifies deposition density.

Regarding the structure of the paper, we initially considered transferring the entire appendix to a separate supplementary document, which remains a possible option. However, because the appendix material is closely and sequentially connected to the main text, we were concerned that separating it would require readers to move back and forth between two documents, potentially disrupting the flow and readability of the paper.

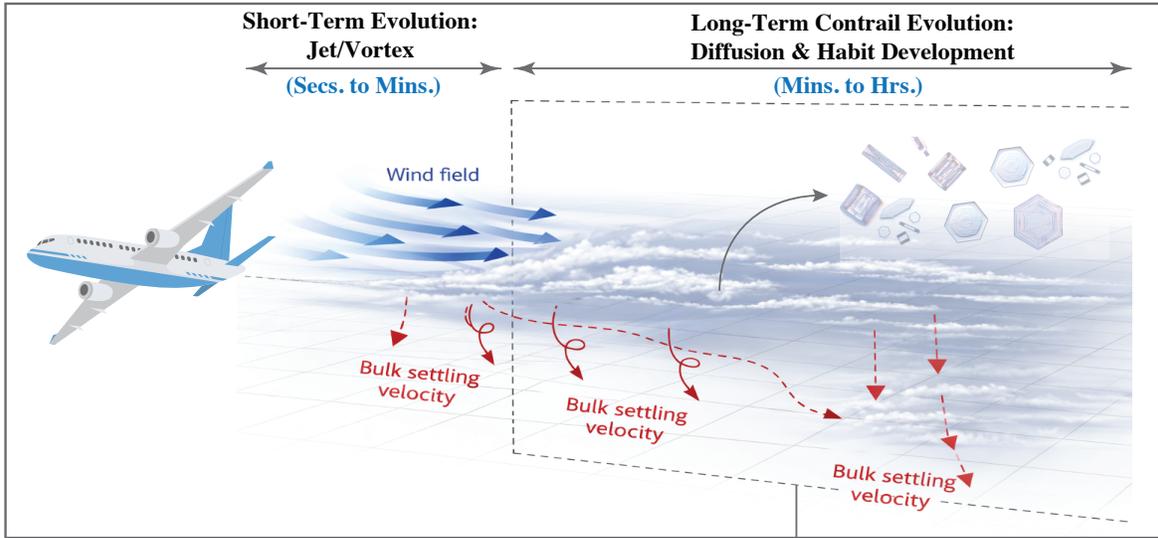
Regarding the relevance of the newly introduced aspects to the contrail domain, we have rigorously justified their applicability by citing key references beginning in the Introduction and throughout the relevant sections of the manuscript.

Finally, in **Section 2**, after presenting the general framework, we have highlighted the modeling assumptions clearly in a subsection and clarified the structure of the paper through additional text and a schematic diagram. This part now reads:

Fig. 2 elucidates the flow of this study focusing on the primary building blocks of the Eulerian framework to resolve the long-term evolution of contrails. We begin by deriving the monodisperse equation from the PBE in **Sec. 3**, with detailed steps provided in **Appendix B**. The formulation further requires the derivation of key components, including the bulk settling velocity (presented in **Sec. 4** and detailed in **Appendices A, C, D**, growth and sublimation microphysics (detailed in **Appendix G**, and habit dynamics (detailed in **Appendix F**). These elements constitute indeed the primary building blocks.

The need for considering secondary building blocks associated with our model is highlighted in **Sec. 5 and 6.4** with detailed derivations/explanations presented in **Appendix H, I J**.

## Contrail Evolution



## Modelling Framework for Long-Term Contrail Evolution

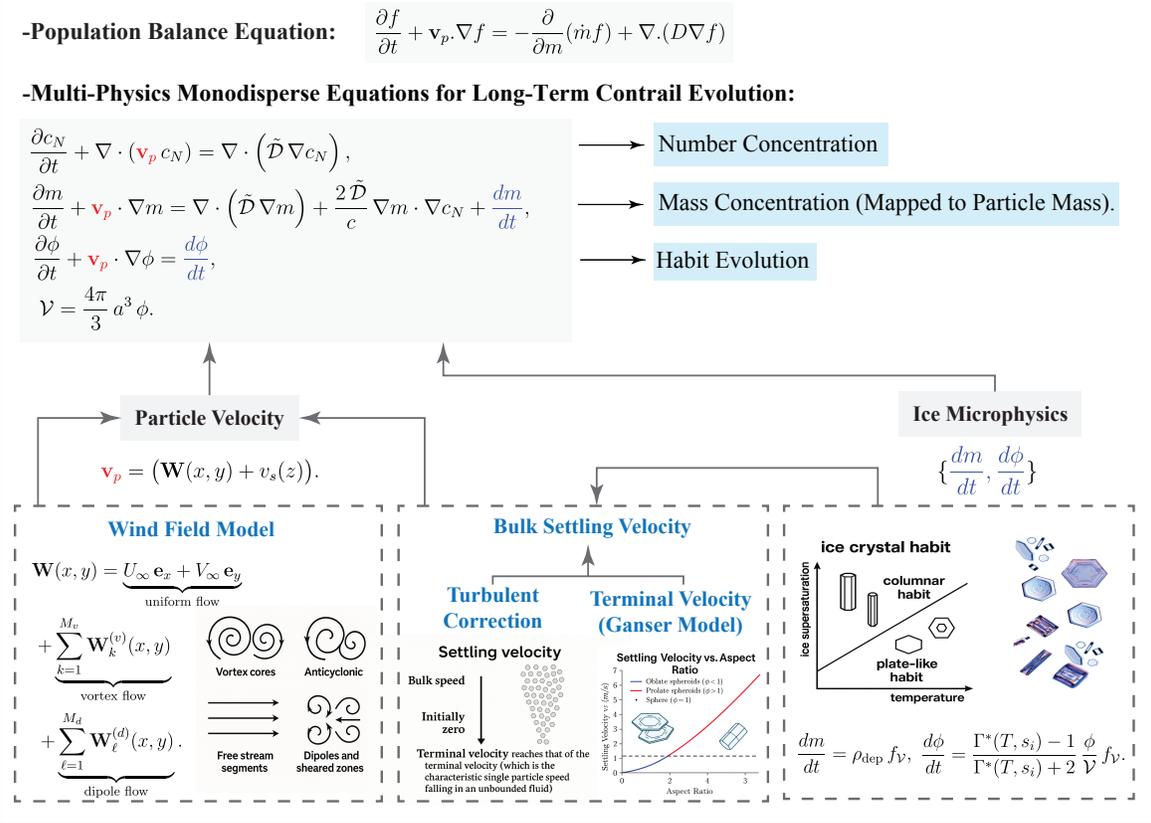


Figure 1: Schematic diagram for the flow of the study focusing on the primary building blocks

**Comment 2:** A monodisperse size distribution is a very strong assumption. Earlier LES modelling studies (Unterstrasser und Gierens 2010, Lewellen 2014, Lewellen, Meza und Huebsch 2014) have shown that only the largest ice crystals fall out from the contrail to form a fall streak, while the smaller ice crystals remain at the original altitude or fall only very slowly. I doubt that the claimed superiority of the numerical implementation matters as long as a monodisperse size distribution is assumed.

Allowing for a size distribution with pre-defined shapes would be one step forward. This can, however, lead to spurious sedimentation effects when ice mass and ice number vertical fluxes become decoupled. The bin approach (i.e. spectral microphysics) used by Lewellen (see citations above) or the Lagrangian approaches (Naiman, Lele und Jacobson 2011, Paoli, H elie und Poinso 2004, Unterstrasser, Gierens und S olch, et al. 2017, Nybelen und Paoli 2009) are conceptually far more advanced than the present scheme. Back in 2006, (Huebsch und Lewellen 2006) demonstrated already that a monodisperse assumption is inadequate.

**Response:** We would like to thank the reviewer for their thoughtful observations and for providing such valuable references. In response, we emphasize that although the simulations presented in this study are based on a monodisperse assumption, the underlying Eulerian framework is general and can be extended to a polydisperse representation with only modest additional computational cost. We refer the reviewer to the **Conclusion** section, where this point is further emphasized as a direction for future work. The details of the polydispersity version of the MultiCon model will be presented in a separate paper.

Moreover, we emphasize that the present model is not intended to outperform size-resolved LES approaches. Rather, the objective of this study is to introduce a new framework for long-term contrail evolution, comparable in scope to existing models such as CoCiP and APCEMM. The primary goal is to incorporate key missing micro- and macrophysical processes (well-justified in the manuscript) while maintaining low computational cost and scalability for large-scale simulations. Notably, even LES-based contrail studies generally lack explicit coupling to ice crystal habit dynamics, and effects such as loitering or preferential sweeping may not emerge automatically and typically require careful/correct treatment of particle–turbulence scale interactions.

In addition, we have included a dedicated section comparing our model with CoCiP and examining how the new physical processes introduced here modify in-cloud structure, producing changes in the spatio-temporal distributions of ice water content (IWC) and ice number concentration. That section also presents a comparison of center-of-mass settling behavior, highlighting differences attributable to the newly introduced microphysical processes embedded in a consistent Eulerian framework employed in our model. The associated figures are presented below.

Moreover, the accompanying .gif files show animations for the single-plume evolution by MultiCon over 9 hours simulation Examples of Single-Plume Evolution by MultiCon. The details associated with the evolution of the plume are given in **Sec. 6.4** of the revised version.

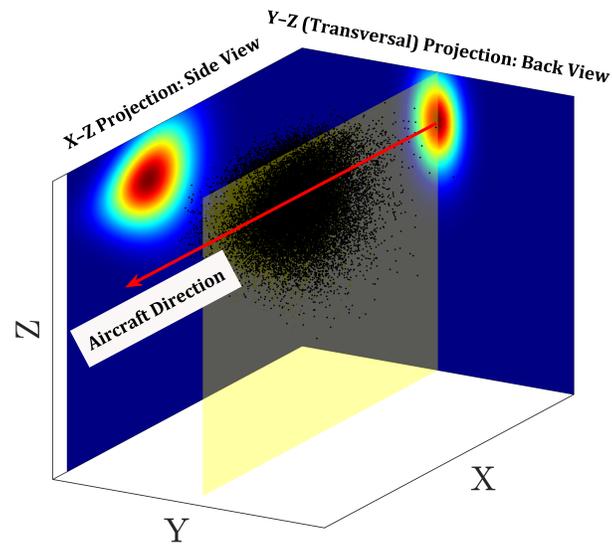
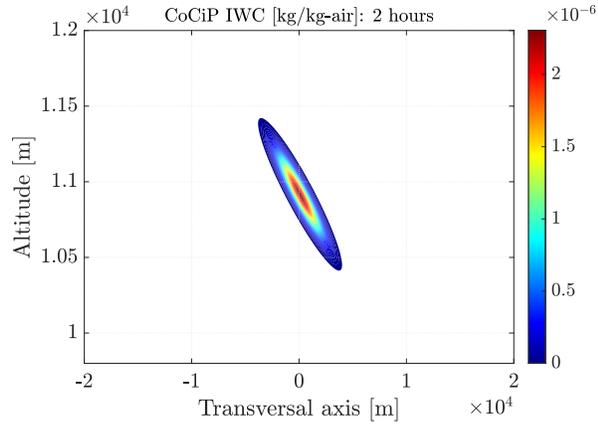
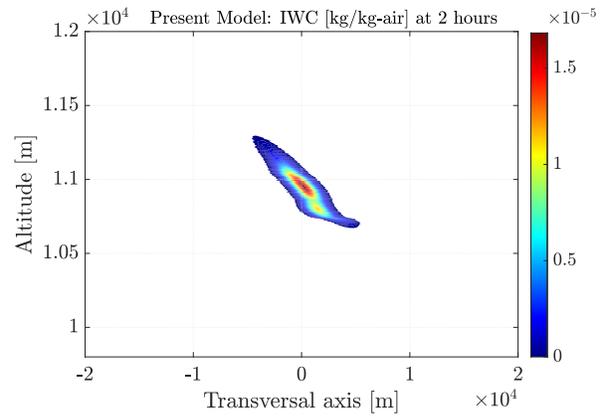


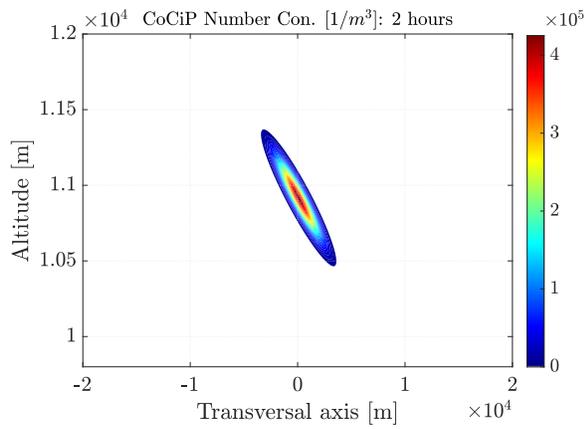
Figure 2: Schismatic for the plume projections used for the visualization purpose



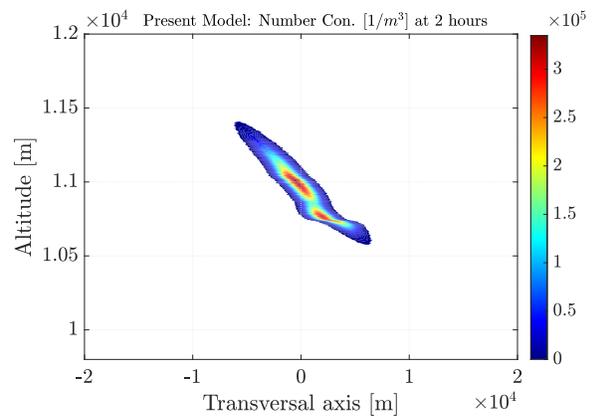
(a) CoCiP IWC: 2 hours



(b) IWC obtained from the present model: 2 hours



(c) CoCiP number concentration: 2 hours



(d) Present model number concentration: 2 hours

Figure 3: Comparison with CoCiP

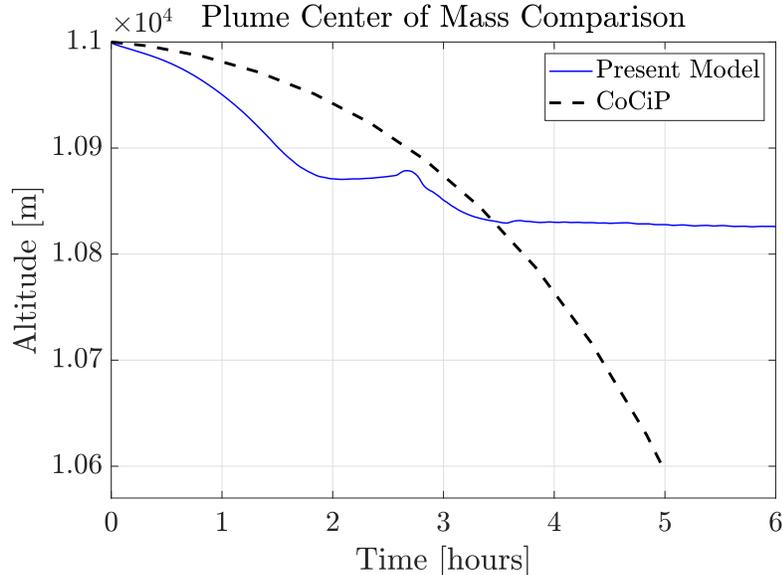


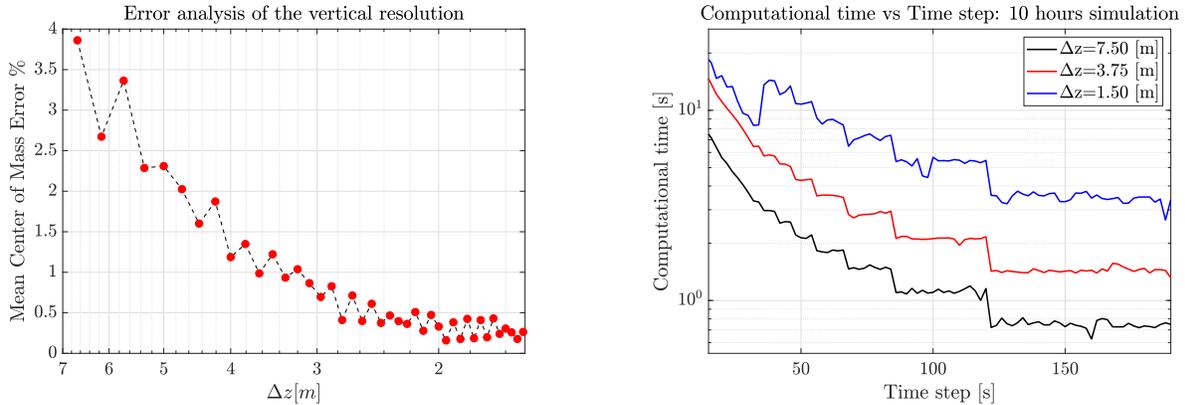
Figure 4: Center of mass comparison between CoCiP and the present model

We would also like to emphasize that one of the key advantages of the present Eulerian framework is its flexibility and remarkable computational speed (see Fig. 5a, and Fig. 5b and the accompanying text). In addition to accommodating polydispersity, the framework can incorporate additional habit modules, such as ventilation effects and hollowing/branching morphologies, without significant structural modification. At the same time, the computational cost remains sufficiently low to make the approach promising for large-scale simulations involving multiple aircraft over extended geographical domains. In particular, tube-based methods could be employed to extend the single-plume analysis presented in this paper to multi-plume or multi-aircraft scenarios at reasonable spatial resolution.

Finally, compared with the microphysical and macrophysical treatments in current long-term, large-scale contrail models such as CoCiP and APCEMM, the present model enables the inclusion or parameterization of more detailed physical processes and dimensions that remain underexplored in contrail modeling, while ensuring that the computational cost remains sufficiently low (roughly below 1 hour to simulate 1,000 flights over the entire European airspace at a reasonable spatial resolution).

Nevertheless, further discussion of tube-based solvers is beyond the scope of the present work; however (as we presented in **the ECATS conference Jan. 2026**), we would like to refer to the following link as a demonstration of ongoing activities on this topic ([An Example of Continuous Aircraft-Plume Evolution by MultiCon](#))

Figure 5b shows the computational time of the vertical solver over a 10-hour simulation as a function of the solver time step, for several vertical resolutions. When the present Eulerian framework is deployed in large-scale analyses involving many plumes (e.g., using tube-based methods), the vertical solver remains the dominant contributor to overall computational cost. Although this study does not target large-scale applications, the timings in Fig. 5b indicate that the model is promising for such scenarios. As can be seen, for moderate vertical resolution and solver time step, the computational time of the vertical solver can fall below one second.



(a) Vertical-resolution convergence of the 10-hour mean center-of-mass position

(b) Computational time of the vertical solver for a 10-hour simulation as a function of time step

Figure 5: Vertical-resolution convergence and computational cost analysis.

**Comment 3:** I do not understand what the habit percentages shown in Fig.6 are based on? Why should there be differences in young contrails based on background supersaturation?

**Response:** As indicated in the paper, the initial time  $t = 0$  corresponds to the onset of the long-term diffusion regime, occurring typically a few minutes after early-stage contrail formation and the wake-vortex regime. In other words, since the early-stage contrail regimes are not directly parameterized in this study, as they are outside the scope of this work, a synthetic initialization is adopted to represent the plume characteristics after passing through these early phases. This point has now been clarified in the revised manuscript, and the relevant part in Section 6.3 reads:

As noted earlier in this section, the initial time ( $t = 0$ ) corresponds to the onset of the long-term diffusion regime, which typically occurs on a timescale of several minutes after contrail formation.

Besides, to compute the habit percentage, we simply check, at each time, which cells have  $\phi > 1$  (columns) and which have  $\phi < 1$  (plates), considering only non-empty cells (number concentration above a threshold). Since time zero corresponds to the onset of long-term propagation, we already

assume that habit classification does not apply to young contrails. This assumption is also supported by the fact that habits typically emerge only once the ice crystal radius exceeds roughly  $2.5 \mu\text{m}$ , which is not the case for very young contrails.

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**Comment 4:** Why do the habit percentages change over time? Do you brute-force prescribe a habit depending on several atmospheric parameters? How is this backed up by observations?

**Response:** We do not brute-force the habit dynamics; rather, they naturally emerge from the habit dynamics model considered in this study. In this revised version, we have added further discussion justifying the model choice for contrail applications. Accordingly, Appendix F has been carefully revised to address this issue. Specifically, in the present work, the IGF is formulated in a way to be consistent with state-of-the-art habit-dynamics models describing transitions between plate-like and columnar crystals, while remaining sufficiently simple to allow future extensions and refinement as controlled in-cloud observational data become available. The relevant parts now reads as follows:

It should be emphasized that the current habit-dynamics framework does not represent bullet-rosette geometries, internal hollowing/branching morphologies, or ventilation effect.

To elaborate, incorporation of hollowing and branching together with habit-dependent ventilation is conceptually straightforward and can be implemented using well-documented microphysical relations [16]; doing so does not invalidate the terminal-velocity formulations for generalized spheroidal particles (for example, the Ganser-type parameterizations employed in this study [5]) because those relationships remain applicable to the modified bulk shape and density descriptions.

By contrast, explicit treatment of bullet-rosette crystals poses practical difficulties. These habits typically develop after prolonged exposure to large ice supersaturations and therefore are likely less relevant to contrails. Specifically, in expanding contrails, the local relative humidity often relaxes to ice saturation level for a considerable period, and the subsequent recovery toward ambient humidity can be slow. Such a thermodynamic history does not generally provide the sustained, extreme supersaturation episodes necessary for bullet-rosette formation. Moreover, in-situ studies of contrail and contrail-cirrus microphysics usually report that plate- and column-like morphologies are observed more frequently than bullet-rosettes. Finally, parameterizations of the aerodynamic properties of complex rosette aggregates—especially broadly applicable terminal-velocity formulations—are scarce and poorly constrained, which further complicates their reliable inclusion in a habit-dynamics model.

For these reasons — and in order to introduce a clear, state-of-the-art baseline for habit dynamics in contrail microphysics — we omit bullet-rosettes, hollowing/branching, and ventilation effects in the present work and reserve them for future extension and validation.

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**Comment 5:** The observations may only provide an overall picture of habit classification (for example to include the bulk habit effect in radiative transfer applications as in Yang, 2010). However, I do not believe that those empirical relations should be used to predict the habit of individual contrail-cirrus lifecycles. Prescribing a habit transition for each contrail and for all ice crystals in a contrail is probably not meaningful.

**Response:** Our previous response already addresses this comment. We refer the respected reviewer to Appendix F, where we have clarified the habit dynamics model, its relevance to the contrail context, and addressed the possible sources of confusion.

To further elaborate, **once the habit dynamics are coupled to the other equations, the crystal shape factor  $\phi$  naturally evolves over time according to the local temperature, ice supersaturation, and the mass/shape history through its differential equation.** In contrast to previous works that assign equal settling velocities to equal-mass particles, crystals of the same mass can now exhibit different settling depending on their shape. The current model successfully predicts the faster settling of columns compared to plate-like crystals, even when they have equal mass, leaving the core of the plume dominated by plate-like crystals. Nonetheless, our current comparison remains qualitative, as controlled in-cloud observations are not yet available. Finally, it is worth noting that an advantage of the habit dynamics model is that it not only follows state-of-the-art habit formulations but also accommodates uncertainties associated with the IGF through tunable parameters, which can be refined if detailed observations become available. The relevant parts in **Section 6.3** of the revised version read:

In particular, our simulations indicate that the dominant crystal habits are plate-like ice crystals, with columnar crystals gradually migrating toward the edges of the contrail layer over time. Although higher ice supersaturation typically provides more favorable conditions for the formation of needle-like, bullet rosette and columnar crystals, the coupling between settling velocity and crystal growth suggests that these crystal types are advected out of the ice-supersaturated layer and begin to sublime sooner than plate-like crystals.

Specifically, we conducted a simulation at  $-61^\circ\text{C}$  (similar to the observations reported in Yang, 2010) at the reference altitude with an initial layer of ice supersaturation peaking locally at 17%, and 27% i.e.,  $s_{i,\text{peak}}(z, 0) \approx 17\%, 27\%$ .

Because the reports in Yang, 2010 do not include information on local ice supersaturation, our comparison is necessarily qualitative. The lack of supersaturation data in Yang, 2010 prevents a clean quantitative comparison and model parameter tuning. Nevertheless, since the reported plume age is around 40 min, the simulation with a 17% ice supersaturation peak appears quantitatively more consistent with Yang, 2010.

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**Comment 6:** A change in habit percentage within a time period (e.g. during one time step) implies that a certain amount of ice crystals change from columnar to plate-like shape? Does this habit change override the non-isotropic growth of ice crystals described in Appendix E?

**Response:** Let us initially remind that  $\phi > 1$  corresponds to columnar and  $\phi < 1$  to plate-like crystals. Therefore, for our qualitative comparison presented in **Section 6.3**, as already mentioned in response to **Comment 3**, even  $1 + \epsilon$ ,  $\epsilon \rightarrow 0^+$  is categorized algorithmically as columns and  $\epsilon \rightarrow 0^-$  as plates. **However, in general, during habit transition, the capacitance, thermal conductivity and vapor diffusivity change according to the well-established formulas cited in this research. Furthermore, if hollowing/branching is to be considered,  $\rho_{\text{dep}}$  will also be affected (not considered in this paper).**

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**Comment 7:** Your habit formulations probably imply an increase in the aspect ratios of the ice crystals. Concerning, the difference between the two habit models, is it mostly an effect of more elongated ice crystal in your habit model? What other properties of your habit definitions (in addition to the aspect ratio) are relevant for your simulated contrail properties? How do your aspect ratios relate to other empirical studies defining habit types (Erfani und Mitchell 2016, Finlon, et al. 2019, Heymsfield, Lewis, et al. 2002)?

Appendix E describes a non-isotropic growth of ice crystals. I cannot figure out how this relates to the habit definitions, which you seem to prescribe in a brute-force manner. You apply the Chen and Lamb and Nelson and Baker parametrizations, which are poorly constrained in the temperature range, where most contrails form (around 220K). Extrapolating formulas that are valid only above 243 K to lower temperatures is, in my opinion, unreliable. Temperatures of 243 K and higher belong to the mixed-phase cloud regime, whereas contrails generally form in the pure-ice cloud regime. Moreover, ice crystal habits can differ between natural cirrus and contrail cirrus. Moreover, you state that discrepancies are largest below 233K. Contrails form typically only for temperatures below 225K. I am not convinced that the papers cited in Appendix E are suited to describe the situation in contrails. In the end, you apply corrections that are not well-justified. The statement that riming is omitted because of sparse data reveals a misinterpretation: riming requires supercooled droplets that splash onto ice crystals, but such droplets do not exist at contrail-forming temperatures.

In summary, given the many uncertainties in your approach, I am unsure whether including habits in the way you do makes the model better. To be clear, I do not argue that treating ice crystals as spheres is reasonable. Several contrail LES models described in the literature incorporate

ice crystal habits with increasing aspect ratios, but do not include any additional habit aspects that are extrapolated from a different temperature regime.

**Response:** Initially, we would like to refer to the added texts in the revised version which may address concerns about the habit model and its relevance to the contrail area:

The available mathematical frameworks for habit dynamics are those by Chen and Lamb [4] (based on the aspect-ratio hypothesis, defining  $\frac{dc}{da} = \frac{\alpha_c}{\alpha_a} \frac{c}{a} := \Gamma \phi$ ) and Nelson and Baker [13] (based on the facet hypothesis, defining  $\frac{dc}{da} = \frac{\alpha_c}{\alpha_a} := \Gamma$ ), where  $\alpha_a$  and  $\alpha_c$  are the deposition coefficients of the basal and prism axes, respectively,  $a$  is half the basal-plane maximum width (equatorial radius),  $c$  is half the prism-plane height (transverse radius), and  $\Gamma$  is known as the Inherent Growth Factor (IGF). It is straightforward to show that the above dynamics are equivalent to [7]:

$$\frac{d\phi}{d\mathcal{V}} = \frac{\Gamma^* - 1}{\Gamma^* + 2} \frac{\phi}{\mathcal{V}},$$

where  $\Gamma^* = \Gamma$  in the Chen and Lamb model, and  $\Gamma^* = \frac{\Gamma}{\phi}$  in the Nelson and Baker model.

Here, we should also highlight that the mathematical derivation of the above differential equation is general and does not include assumptions regarding temperature ranges or mixed-phase cloud regimes (where tiny supercooled water droplets and ice crystals coexist) [7]. Therefore, the framework is also valid for contrail crystals, provided that the IGF factor is properly estimated or tuned. **Diagnostic approaches to habit classification have also been proposed in the literature (e.g., [10]). However, a limitation of such diagnostic approaches is that temporal variations caused by local temperature and supersaturation changes may not be captured by static relations [16].** Another important point is that the well-known Schmidt–Appleman criterion sets the threshold for contrail formation, and the temperature required for contrail formation typically lies between approximately  $-35$  and  $-40^\circ\text{C}$  [12], which coincides with the ice-cloud regime. However, once contrail crystals are formed, they may be advected into a mixed-phase regime, altering the vapor-transfer pathways and further complicating habit microphysics. However, this possibility is not considered in the present work, based on the assumption that temperature variations over the course of the contrail lifetime may not be significant.

Reviewing the literature on habit dynamics, predicting the IGF factor remains an active area of research, and in general, the available data are sparse for  $T < -30^\circ\text{C}$ . **However, extrapolations and fits to the limited available data have already been pursued in the literature (see, for example, [7] (considering supercooling temperatures up to  $T=-70$ ) and [9]). To date, however, most IGF estimations rely on partially empirical or ad hoc closure assumptions.**

In the present work, the IGF is formulated in a way to be consistent with the above state-of-the-art habit-dynamics models describing transitions between plate-like and columnar crystals, while remaining sufficiently simple to allow future extensions and refinement as controlled in-cloud observational data become available.

We use Nelson and Baker model which directly accounts for dislocation growth and step nucleation theories, characterized by supersaturation immediately above the surface (i.e., surface supersaturation  $s_{\text{surf}}$ ), a temperature-dependent characteristic supersaturation describing the supersaturation-dependence of surface-kinetic mediated growth (i.e.,  $s_{\text{char}}$ ), and the parameter  $N$  that describes the surface growth mode. An approximation for  $\alpha$  that captures both dislocation growth and step nucleation theories was suggested by Nelson and Baker [13]:

$$\alpha(T, s_i) = \alpha_s \left( \frac{s_{\text{surf}}}{s_{\text{char}}} \right)^N \tanh \left( \frac{s_{\text{char}}}{s_{\text{surf}}} \right)^N \quad (1)$$

where  $\alpha_s$  is the sticking probability/adsorption efficiency and is thought to be near unity [9, 11]. In addition,  $N = 1$  is consistent with the dislocation growth whereas  $N \geq 10$  is amenable to step nucleation.

Following [9], we compute  $s_{\text{surf}}$  as:

$$s_{\text{surf}} \approx s_{\text{diff}}^{1-\beta} s_{\text{char}}^\beta \quad (2)$$

with:

$$s_{\text{diff},a} = \frac{s_i}{1 + L_a}, \quad L_a = \frac{a c \bar{v}_v}{4D_v C_\Delta(c, a)}, \quad s_{\text{diff},c} = \frac{s_i}{1 + L_c}, \quad L_c = \frac{a^2 \bar{v}_v}{4D_v C_\Delta(c, a)}. \quad (3)$$

Notably, the Nelson and Baker formulation is expected to perform well at contrail-range temperatures and moderate supersaturation, but it falls short at low supersaturations [7, 9]. In this respect, it is suggested to account for residual errors by applying additional approximate parameterization techniques, more recently proposed in [9], to correct the surface supersaturation. However, the validity of such approximations has not yet been verified against experimental measurements (interested readers are referred to [6–9, 11]).

From habit diagrams (e.g., [1–3]), at temperatures colder than  $-40^{\circ}\text{C}$  (contrail-range temperatures), there is a marked shift toward columnar behavior, except at low to moderate ice supersaturation (below  $\approx 10\%$ ), where thick and irregular plates are observed. At moderate ice supersaturation (10%–25%), long solid columns and polycrystals with columnar and plate-like components are observed. Above approximately 25% ice supersaturation, bullet rosettes, long columns, and column-containing polycrystals are observed, with the frequency of bullet rosettes and columns increasing as ice supersaturation increases.

Therefore, for simplicity, consistency with the Nelson and Baker model, and correction of the low-supersaturation limit directly following the habit diagrams—while also explicitly addressing the frequently observed plate-like crystals in persistent contrails—we propose a small modification that corrects only the low-supersaturation shortcoming of the Nelson and Baker model by allowing plate-like crystals to reach a minimum of  $\Gamma = 0.2$ , corresponding to thick-plate habits.

Therefore, the applied correction only modify the low-supersaturation shortcoming of the Nelson-Baker model (please refer to the **Appendix** for more details).

We also point out that extrapolating the available data to cover a wider range, down to  $-70^{\circ}\text{C}$ , does not change the structure of the characteristic supersaturation, as long as the monotonic trend documented in the literature is preserved. However, the choice of extrapolation function primarily affects the upper-bound extent of the columnar crystals.

Further tuning of the model in the contrail regime should address a more general parameterization of low supersaturation and different types of monotonic extrapolation functions for the characteristic supersaturations. Such additional tuning or modification would mainly control the minimum and maximum bounds of  $\phi$ .

**Comment 8:** How does your formulation of terminal velocity compare with other empirical fall-speed parameterisations (Mitchell 1996, Böhm 1989, McCorquodale und Westbrook 2021, Heymsfield und Westbrook 2010)? I appreciate that you question the universal application of terminal velocity as the fall speed and that your model extends beyond this assumption. However, I have strong doubts that sedimentation is adequately resolved in models that assume a monodisperse size distribution in each grid cell. Does contrail layering in your simulations resemble that of Fig. 4 in (Lewellen, Meza und Huebsch 2014)?

**Response:** First, we would like to thank the respected reviewer once more, for their time, careful reading of our manuscript, and valuable insights and references.

With regard to the terminal velocity, we have carefully reviewed a broad range of studies available in the literature, including some of the valuable works you mentioned, such as Zastawny (2012), Ganser (1993), Datta and Srivastava (1999), and Heymsfield and Westbrook (2010), etc. Among these, Ganser (1993) provides formulations detailed enough for both columnar and plate-like particles (covering all ranges associated with ice crystal habits), explicitly incorporating the Newton

and Stokes shape factors, Reynolds-number dependence, and random particle orientation, while also being validated against experiments over a wide parameter range. Although we have not implemented every available parameterization for settling velocities of irregular and non-spherical particles, we are confident that the present approach adequately represents plate and columnar ice crystals for the purposes of this study. For more detailed analyses however, particularly concerning irregular habits such as bullet-rosette crystals, we plan to adopt more advanced formulations available in the literature.

Regarding the monodispersity–polydispersity issue, we agree with the reviewer that assuming monodispersity alone may not be sufficient to fully capture sedimentation phenomena, and that its validity should ultimately be assessed through careful comparison with a polydisperse description. However, as already emphasized in **Response to Comment 1**, incorporating polydispersity is reserved as a future extension of MultiCon.

With regard to the question whether contrail layering in our simulations resemble that of Fig. 4 in the valuable work by Lewellen, Meza und Huebsch 2014 (already cited in the revised version), indeed similarities can be observed for the overall behavior of number and mass concentrations presented in Fig. 7, 8 and 9 of our paper. However, the extent of these similarities, and possible deviations require a separate work considering the polydisperse version of MultiCon compared with LES models.

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**Comment 9:** Relative humidity rises and water vapor becomes available when air masses ascend and cool adiabatically (or via other diabatic mechanisms). This is the typical driver of continued cloud growth. I am unaware of studies in which water vapor is replenished by vigorous vertical diffusion; in stratified flows vertical exchanges are usually weak. Could you provide evidence for the formula presented in line 458? Moreover, your formulation appears to suggest that the value of  $\alpha$  was selected to achieve a balanced state, yet the two processes need not be in equilibrium (and I doubt that vertical replenishment plays an important role).

**Response:** In the revised version, we have derived the PDE associated with  $s_i$  in Appendix I, and clarified the simplified form used in the present work. The additional text also responds to the concern w.r.t. the diffusion term:

...To represent this effect in our synthetic simulations we employ the following auxiliary equation which is considered as the first-order approximation for the PDE associated with the ice-supersaturation balance:  $\frac{\partial s_i}{\partial t} = -\alpha \dot{m}(z, t) + \frac{s_{i,\text{env}} - s_i}{\tau_{\text{entrain}}}$ , where  $\alpha = \frac{c_N}{q_s \rho_{\text{air}}}$  represents the supersaturation depletion rate by ice growth. Moreover,  $s_{i,\text{env}}$  denotes the supersaturation of the environment and  $\tau_{\text{entrain}}$  is the effective recovery/relaxation timescale (the derivation of  $\alpha$  and  $\tau_{\text{entrain}}$  is provided in Appendix I). We further note that, to improve numerical stability (avoiding stiffness), a small diffusion term may be introduced; this addition has a negligible impact on the results.

## Appendix I

Ice supersaturation is defined as  $s_i(\mathbf{x}, t) \equiv \frac{q_v}{q_{vs}(T, p)} - 1$ , so  $q_v = q_{vs}(T, p)(1 + s_i)$ , where  $q_v$  is the water-vapour mixing ratio,  $q_{vs}(T, p) = \frac{\varepsilon e_{si}(T)}{p - e_{si}(T)}$  the saturation mixing ratio over ice (with  $\varepsilon = R_d/R_v$ ),  $c_N$  the ice number concentration,  $\dot{m}$  the single-particle mass growth rate, and the volumetric deposition rate  $S_{\text{dep}} = c_N \dot{m}$ .

For compactness below we write  $q_s \equiv q_{vs}(T, p)$  (so  $q_v = q_s(1 + s_i)$ ). We start from the vapour conservation equation with advection, an explicit deposition sink, and a general entrainment (mixing) operator  $\mathcal{E}[q_v]$  which represents the net tendency of ambient vapour to be mixed into the plume:

$$\frac{Dq_v}{Dt} = -\frac{S_{\text{dep}}}{\rho_{\text{air}}} + \mathcal{E}[q_v]. \quad (4)$$

Taking the material derivative of  $q_v = q_s(1 + s_i)$  gives

$$\frac{Dq_v}{Dt} = q_s \frac{Ds_i}{Dt} + (1 + s_i) \frac{Dq_s}{Dt}. \quad (5)$$

Equating the material form of (4) with (5) and solving for  $Ds_i/Dt$  yields the exact Eulerian evolution equation:

$$\frac{Ds_i}{Dt} = -\frac{1}{q_s} \frac{S_{\text{dep}}}{\rho_{\text{air}}} - \frac{1 + s_i}{q_s} \frac{Dq_s}{Dt} + \frac{1}{q_s} \mathcal{E}[q_v]. \quad (6)$$

A commonly used closure for the entrainment operator is the bulk relaxation form:

$$\mathcal{E}[q_v] = \frac{q_{\text{env}} - q_v}{\tau_{\text{entrain}}}, \quad (7)$$

where  $q_{\text{env}}(\mathbf{x}, t)$  is the ambient vapour mixing ratio sampled at the plume/element location and  $\tau_{\text{entrain}}$  is the entrainment timescale. Substituting (7) into (6) and using  $q_v = q_s(1 + s_i)$  gives

$$\frac{Ds_i}{Dt} = -\frac{1}{q_s} \frac{S_{\text{dep}}}{\rho_{\text{air}}} - \frac{1 + s_i}{q_s} \frac{Dq_s}{Dt} + \frac{1}{q_s} \frac{q_{\text{env}} - q_s(1 + s_i)}{\tau_{\text{entrain}}}. \quad (8)$$

Noting that

$$\frac{q_{\text{env}} - q_s(1 + s_i)}{q_s} = \frac{q_{\text{env}}}{q_s} - 1 - s_i \equiv s_{\text{env}} - s_i, \quad s_{\text{env}} \equiv \frac{q_{\text{env}}}{q_s} - 1,$$

the entrainment contribution reduces to:

$$\frac{Ds_i}{Dt} = -\frac{1}{q_s} \frac{S_{\text{dep}}}{\rho_{\text{air}}} - \frac{1 + s_i}{q_s} \frac{Dq_s}{Dt} + \frac{s_{\text{env}} - s_i}{\tau_{\text{entrain}}}. \quad (9)$$

To obtain  $\tau_{\text{entrain}}$ , we consider a Lagrangian plume element of mass  $M$  that entrains  $\Delta M$  during  $\Delta t$ . The post-mix vapour is [14]:

$$q_v(t + \Delta t) = \frac{M q_v(t) + \Delta M q_{\text{env}}}{M + \Delta M} \Rightarrow \Delta q_v = \frac{\Delta M}{M + \Delta M} (q_{\text{env}} - q_v). \quad (10)$$

Dividing by  $\Delta t$  and taking  $\Delta t \rightarrow 0$  (to first order  $\Delta M/(M + \Delta M) \approx \Delta M/M$ ) yields:

$$\frac{dq_v}{dt} = \frac{1}{M} \frac{dM}{dt} (q_{\text{env}} - q_v). \quad (11)$$

Comparing with (7) shows that the appropriate entrainment timescale is  $\tau_{\text{entrain}} = \frac{M}{dM/dt}$ .

We also note that  $\tau_{\text{entrain}}$  can be simply parameterized using Gaussian or cylindrical plume element segment hypothesis. However, in the present research, we have used the minimal representation of Eq. (9) by removing the mixing term and also assuming constant  $q_s$ . Moreover, in simulations without explicit entrainment, the ice number concentration  $c_N$  was prescribed to values representative of aged contrails ( $10^5 - 10^6 \text{ m}^{-3}$ ).

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**Comment 10:** Initializing a contrail as a three-dimensional Gaussian plume is, in my view, inappropriate. For your type of application, a uniform contrail along flight direction would be more reasonable. In the cross-section perpendicular to the flight direction, a 2D Gaussian plume may be used, yet keeping in mind that this is already a strong simplification. Currently, your Figs. 7 and 8 show a particular slice according to “We present the static contours at different times in the x–z plane along the track, i.e., along the center of the plume where maximum concentration is expected”. This sentence is ambiguous and raises more questions than it answers. (unfortunately, such sentences occur throughout the manuscript and it is out of scope to mention all of them). What does the word “static” imply here? Why do the authors use the word “expect” instead of stating that the maximum concentration is determined? Do you pick the same slice for all times or does it jump from location to another? Is the displayed slice close to where the initial Gaussian plume has its maximum? It does not seem very robust to search for the maximum IWC as it can depend on unimportant turbulent fluctuations. It can be beneficial to pick the slice by evaluating the total ice mass over the x-z plane. If the contrail is initialized uniformly along the flight direc-

tion, the results can be averaged over that direction, eliminating the need to choose a particular slice.

**Response:** We have carefully checked the manuscript, trying to detect and correct any ambiguous sentence. Owing to your comments, we believe that the quality of our paper has now improved substantially. Moreover, as for the initialization of the plume, we agree that there are several ways, each deserving careful attention, to evaluate which type of geometrical initialization can best represent the plume for the onset of the diffusion regime. Fortunately, the 3D Eulerian framework presented in this work allows detailed assessment of this issue. In the present work however, because the focus has been given to study the new elements introduced in the paper, the type of initialization for geometry is preliminary and for the environmental conditions synthetic. For the purpose of comparison with CoCiP however, we have used the same type of geometrical initialization, extruding the 2D ellipses in flight direction. Please refer to Fig. 3, and the detailed new Section 6.4 in our paper. The modified part in the Result section now reads as follows:

To illustrate the different visualization planes associated with the plume, a schematic showing the appearance of plume projections in different planes is presented in Fig. 2.

In **Sec. 6.1**, we present the horizontal evolution of the contrail plume (the  $x$ - $y$  plane solution/horizontal solver). Specifically, in this subsection, the purpose is to highlight the effect of the diffusion-blocking mechanism on contrail spreading. In this respect, the nonlinear diffusion-blocking coefficient is considered only in this subsection and is deactivated in the subsequent subsections.

In **Sec. 6.2**, we present the vertical evolution of the contrail plume, an error analysis of the vertical resolution, and a computational time analysis. We also compare the results with the spherical model (by deactivating the habit module inside the solver), and examine the evolution of IWC, number concentration, bulk settling velocity, crystal radius, and the habit shape function.

In **Sec. 6.3**, we contrast the crystal habit percentages (where the initial time corresponds to the beginning of the long-term diffusion regime) obtained from the present model with available reports. In general, the results appear consistent; however, a detailed comparison and further fine-tuning of the model parameters require access to controlled experimental observations, which are lacking in the literature.

In **Sec. 6.4**, we compare the present model with CoCiP by analyzing  $y$ - $z$  transverse projections of ice water content (IWC) and number concentration. In addition, we examine the evolution of the plume's center-of-mass settling and provide a detailed theoretical interpretation of the results obtained from the present model.

For the initial plume geometry associated with **Sec. 6.2** and **Sec. 6.3**, we set  $N_{\text{lin}} = \frac{EI_N \dot{m}_f}{v_{ac}} \approx \frac{4 \times 10^{14} \times 1.3}{240} \approx 2.17 \times 10^{12} \text{ m}^{-1}$  [12], which is close to the value reported in [15]. The survival/activation fraction is set to  $s = 0.35$  [15]. In addition, for  $\sigma_x$  we use the full width at half maximum to define  $L \equiv 2\sigma_x \sqrt{2 \ln 2}$ , taking  $L \approx 50 \text{ km}$ . The transverse variances are given by,  $\sigma_{y,z}^2 = \sigma_{0,y,z}^2 + 2D_{y,z} \frac{L}{v_{ac}}$ , with synthetic values for the initial cross-sectional Gaussian standard deviations,  $\sigma_{0,z} = 100$  and  $\sigma_{0,y} = 200$  (roughly reflecting the plume geometry after passing through the early phases of contrail formation). The initial ice-supersaturation layer is taken to be approximately 1.3 km thick; however, supersaturation within the layer is not uniform; it follows a Gaussian profile that peaks near the flight level (i.e., the reference altitude), decays smoothly to near zero above the layer, and decreases below the layer to about  $-0.08$ . Moreover, the mean wind is zero, and the synthetic background turbulence follows the standard von Kármán wind turbulence closure in Appendix A, with  $C = 2$  and integral scale ( $L = 500$ ). It should be highlighted that the considered setup is intentionally synthetic, as the purpose of these subsections is not to reproduce a specific atmospheric case but to provide a controlled qualitative comparison between the habit dynamic model and the spherical model (**Sec. 6.2**) under identical conditions. This setup further allows us to isolate the overall trend of crystal shape transition emerging naturally from the implemented habit dynamics and to discuss how this trend might be comparable to the limited observational evidence (**Sec. 6.3**).

Finally, we note that in **Sec. 6.4**, the initial geometry and environmental conditions are set to those of CoCiP for the purpose of inter-model comparison, as will be explained therein.

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**Comment 11:** Figures 12 and 13 show the contrail top rising by several hundred meters—a behavior that appears unphysical. While turbulence should not be so strong, sedimentation acts in opposite direction and radiative heating is absent in your simulations. What’s the physical process behind this? Can you rule out numerical artifacts such as excessive numerical diffusion in your scheme?

**Response:** Thanks for your sharp observation. In this respect, the observed apparent large upward movement is primarily due to a relatively broad vertical initialization for the onset of the long-term diffusion regime in our synthetic simulations. Although the vertical diffusion remains within the range typically considered in contrail models, we believe it is near the upper bound ( $D_z = 0.5$ ) in our study. By adjusting both the initialization and the diffusion coefficient, this upward movement can be readily controlled.

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**Comment 12:** Your separable ansatz seems to not work for scenarios with non-negligible vertical wind shear and slanted contrails. How severe is this restriction? Is it possible to simulate scenarios with vertical wind shear with your model?

**Response:** In the revised version, we have derived the mathematics to account for wind shear in our separation framework. However, the effect of wind shear has been simulated only in Section 6.4 where we compare the model with CoCiP. The new Appendix J reads as follows:

Vertical wind shear produces height-dependent horizontal advection that tilts and laterally displaces the contrail cross-section. Under the present separation ansatz the simplest consistent treatment is a kinematic remapping: a mass-conserving, levelwise horizontal translation of tracers that reproduces geometric distortion but does not feedback on local thermodynamic or microphysical state. Full two-way coupling requires abandoning the separation ansatz and solving the 3-D coupled system.

Nevertheless, in this research, the effect of vertical wind shear is incorporated by linearizing the horizontal wind field about a reference height  $z_{\text{ref}}$ :

$$u(z) = u(z_{\text{ref}}) + S_x (z - z_{\text{ref}}), \quad v(z) = v(z_{\text{ref}}) + S_y (z - z_{\text{ref}}), \quad (12)$$

where  $u$  and  $v$  denote the streamwise ( $x$ ) and cross-stream ( $y$ ) wind components, respectively, and  $S_x = \partial u / \partial z$  and  $S_y = \partial v / \partial z$  are the corresponding vertical shear rates.

Separating the mean advection from the shear-induced contribution, the differential horizontal velocities experienced by a parcel at height  $z(t)$  are

$$\delta u(z, t) = S_x (z(t) - z_{\text{ref}}), \quad \delta v(z, t) = S_y (z(t) - z_{\text{ref}}). \quad (13)$$

The cumulative lateral displacement due to shear over the time interval  $[t_0, t_1]$  is then given by

$$\Delta x = S_x \int_{t_0}^{t_1} (z(t) - z_{\text{ref}}) dt, \quad \Delta y = S_y \int_{t_0}^{t_1} (z(t) - z_{\text{ref}}) dt. \quad (14)$$

In practice, these integrals are evaluated numerically using a discrete representation of the vertical trajectory and a trapezoidal quadrature.

Vertical parcel motion is modeled using the following equation,

$$\frac{dz}{dt} = -v_s(z, t). \quad (15)$$

The resulting discrete vertical positions  $z(t_n)$  are used to evaluate the shear-induced displacements  $\Delta x$  and  $\Delta y$ .

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**Comment 13:** Line 118: this sentence is not clear. What does vertical loss refer to, ice mass loss by sedimentation or a decrease in the contrail altitude? Small ambient updrafts are certainly not included in models to counteract sedimentation effects. They are the prerequisite of most cloud

formation processes.

**Response:** By vertical loss we meant a decrease in the contrail altitude. However, the corresponding ambiguous sentence has been removed in this version.

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**Comment 14:** Line 170: number concentrations certainly drop due to dilution. But even the total ice crystal number is not conserved, as sedimentation and in-situ losses occur (Lewellen 2012, Lewellen 2014, Unterstrasser, Gierens und Sölch, et al. 2017).

**Response:** We agree to the reviewer’s observation. However, it should be pointed out that although total ice crystal number is not conserved due to processes like aggregation, crystals competition, etc., the percentage of the crystal number loss is uncertain and there is clear disagreement between models like CoCiP and APCEMM on this matter. Nonetheless, in this paper we have assumed the conservation of crystal numbers (stated in Section 2). However, in our polydisperse version of MultiCon, we will also adopt a suitable parameterization technique for number loss.

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**Comment 20:** I do not understand what is depicted in Fig. 4. Why does each panel contain three contrails? Why do your simulations start with contrails as broad as 30km? I cannot image which of your implemented physical processes lead to such a pronounced wavy pattern at later times?

**Response:** This figures illustrate only the effect of the (possible) diffusion-blocking mechanism. The background represents the advection field of normalized wind, including synoptic-scale vortices and dipoles generated by the wind-field model. In the Results section, we explicitly describe the purpose of each subsection. Moreover, the apparent wavy pattern is associated with the wind profile. In realistic atmospheric scenarios, when solving the ADE with a spatio-temporal wind field  $w(x, y, t)$ , we have observed even more pronounced distortions. This behavior implies that transport governed by a spatio-temporal advection field cannot be adequately represented by pointwise Lagrangian advection combined with independently parameterized diffusion, highlighting the intrinsically coupled nature of advection–diffusion dynamics. With regard to this comment, the modified text reads as follows:

In **Sec. 6.1**, we present the horizontal evolution of the contrail plume (the  $x$ – $y$  plane solution/horizontal solver). Specifically, in this subsection, the purpose is to highlight the effect of the diffusion-blocking mechanism on contrail spreading. In this respect, the nonlinear diffusion-blocking coefficient is considered only in this subsection and is deactivated in the subsequent subsections.

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**Comment 21:** I have never heard about diffusion blocking, loitering and preferential sweeping in the context of contrail evolution. These phenomena seem to be connected (at least partly) with the used numerical scheme. Unfortunately, I was not able to figure out whether these issues are also relevant to other contrail models.

**Response:** We believe that the processes described above are generally relevant to contrail modeling. As discussed in the manuscript, loitering and sweeping mechanisms are well documented in fluid dynamics, particularly through DNS modeling. The diffusion-blocking mechanism has also been investigated in the context of ADEs. However, the extent to which the latter applies to contrail spreading still requires confirmation through comparisons with ground-based observational data.

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**Comment 22:** What’s “necliated” at several occurrences?

**Response:** In this revised version, the manuscript has been carefully proofread and typographical errors have been corrected.

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**Comment 23:** In Section 6.3, no information on the horizontal resolution is given, only vertical resolution is mentioned.

**Response:** The horizontal resolution for the displayed figures is now provided in this revision.

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**Comment 24:** Line 492: What’s a “lack of water vapour budget”?

**Response:** By water vapour budget we were referring to the entrainment term. Nevertheless, this word has been removed in this version.

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**Comment 24:** Picking up my point, that information is not provided at ideal location: Appendix F presents the ice mass growth equation. Information about vertical profiles of thermodynamic conditions should be given elsewhere.

**Response:** In this version, the paper has been carefully restructured, and the figures have been positioned appropriately.

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