

1 **Threshold Atmospheric Electric Fields for Initiating Relativistic Runaway Electron  
2 Avalanches: Theoretical Estimates and CORSIKA Simulations**

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10 **Abstract**

12 We examine the threshold atmospheric electric field ( $E_{th}$ ) required to initiate a runaway  
13 avalanche in Earth's atmosphere. We compare the traditional, thirty-year-old theoretical  
14 threshold with its recently updated value and with the threshold derived from CORSIKA-  
15 simulated avalanches ( $E_z$ ). The altitude dependence of these thresholds is analyzed,  
16 considering how changes in air density affect avalanche development. This study is vital for  
17 understanding high-energy atmospheric phenomena in both the lower and upper atmosphere,  
18 including thunderstorm ground enhancements (TGEs) and gamma glows, and for refining  
19 atmospheric electric field (AEF) models based on particle flux measurements.

21 **Short Summary**

23 Thunderstorms can accelerate particles in the atmosphere, producing bursts of radiation at  
24 the ground. We investigated how strong the electric field inside a cloud must be to initiate  
25 such events. Using advanced computer simulations and comparing with measurements  
26 from mountain stations, we found that the fields must be stronger than earlier theory  
27 suggested. Our results improve understanding of storm electricity and its role in natural  
28 radiation.

30 **Highlights**

- 31 • Introduces a refined framework for determining the threshold atmospheric electric fields  
32 ( $E_{th}$ ) required to initiate relativistic runaway electron avalanches (RREAs) and  
33 thunderstorm ground enhancements (TGEs).
- 34 • Compares classical ( $E_{th} \approx 2.80 \text{ kV/cm} \times n$ ) and updated ( $E_{th} \approx 2.67 \text{ kV/cm} \times n$ ) theoretical  
35 thresholds with altitude-dependent thresholds derived from CORSIKA simulations.
- 36 • Demonstrates that realistic avalanche development requires fields of 15–22%  
37 stronger than theoretical values, depending on altitude and air density.
- 38 • Provides a reproducible simulation methodology for integrating experimental  
39 particle-flux measurements into atmospheric electricity models across multiple  
40 research stations.

42 **Introduction**

44 Free electrons are abundant in the troposphere. The altitude at which their flux reaches

45 its highest point, called the Regener–Pfotzer maximum (Regener, 1933). It depends on  
46 the geomagnetic cutoff rigidity ( $R_c$ ), the type of particles being measured, and  
47 the phase and strength of the solar cycle. Recent observations, supported by PARMA4.0  
48 calculations (Sato, 2016), show that at middle to low latitudes ( $R_c = 3\text{--}8$  GV), the highest  
49 flux of charged particles occurs at altitudes around 12–14 km (see Figure 3 in Ambrozova et  
50 al., 2023).

51 Atmospheric electric fields (AEFs) generated by thunderstorms transfer energy to free  
52 electrons, accelerate them, and, under certain conditions, induce electron-photon avalanches.  
53 In 1992, Gurevich, Milikh, and Roussel-Dupré identified the conditions necessary for  
54 extensive multiplication of electrons from an energetic seed electron injected into a strong  
55 AEF region (Gurevich et al., 1992). This process is known as the Relativistic Runaway  
56 Electron Avalanche (RREA; Babich et al., 2001; Alexeenko et al., 2002). A numerical  
57 approach for solving the relativistic Boltzmann equation for runaway electron beams  
58 (Symbalisty et al., 1998) aids in estimating the threshold AEF (Babich et al., 2001; Dwyer et  
59 al., 2003) required to trigger RREA. As demonstrated by GEANT4 and CORSIKA  
60 simulations (Chilingarian et al., 2012, 2022), the RREA process is a threshold phenomenon,  
61 with avalanches initiating when the atmospheric AEF exceeds a certain threshold that depends  
62 on air density. The AEF must also be sufficiently extended to support the growth of  
63 avalanches. At standard temperature and pressure in dry air at sea level,  $E_{th} \approx 2.80$  kV/cm  $\times$  n,  
64 where air density n is relative to the International Standard Atmosphere (ISA) sea-level value  
65 (see the recent update of the threshold energy  $E_{th} \approx 2.67$  kV/cm  $\times$  n in Dwyer and Rassoul,  
66 2024). This threshold field is slightly higher than the breakeven field, which corresponds to  
67 the electron energy at which minimum ionization occurs. If electrons traveled exactly along  
68 AEF lines, it would define the threshold for runaway electron propagation and the start of  
69 avalanche formation. However, the paths of electrons deviate due to Coulomb scattering with  
70 atomic nuclei and Møller scattering with atomic electrons, causing deviations from the near-  
71 vertical AEF. Additionally, secondary electrons produced by Møller scattering are not  
72 generated along the field line; therefore, AEFs must exceed the theoretical RREA threshold  
73  $E_{th}$  by approximately 10–20% for electrons to run away and trigger an avalanche.

74

## 75 1. CORSIKA simulations of RREAs reaching the high-altitude stations

76

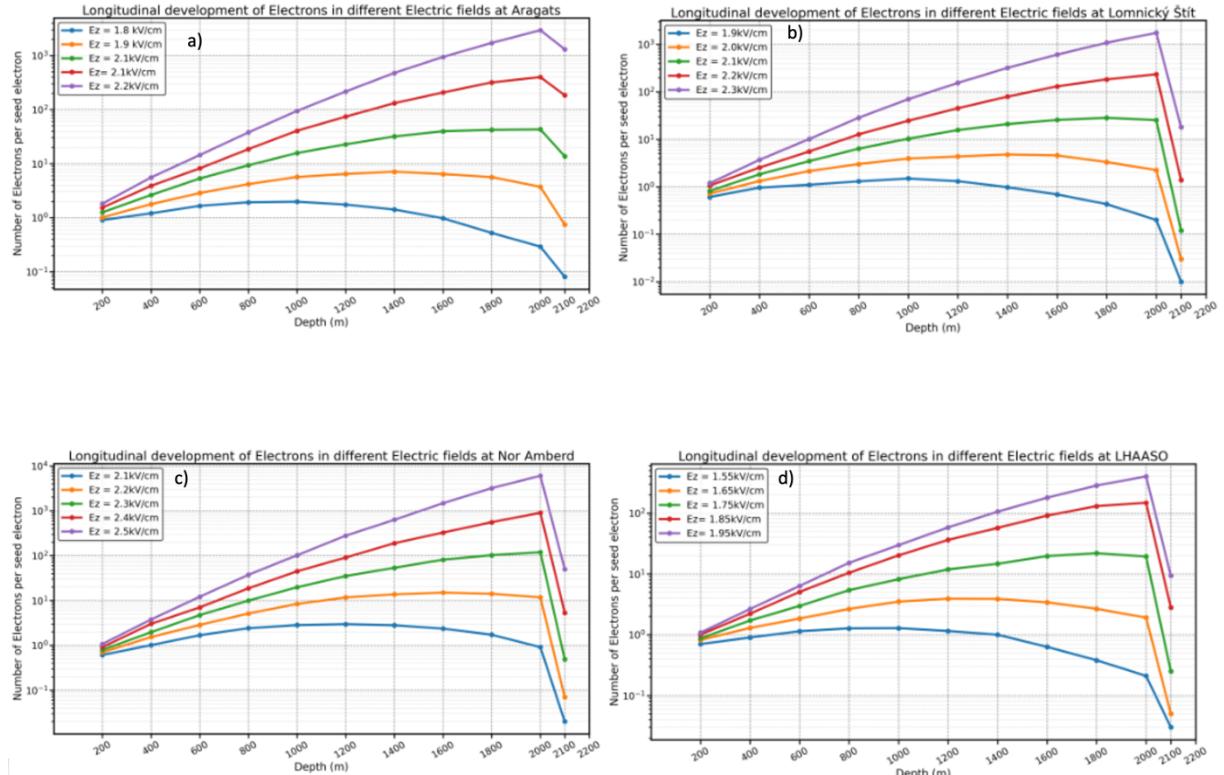
77 To understand how avalanches develop in an electrified atmosphere and to compare the  
78 new and updated  $E_{th}$  with the particle-intensity abrupt growth, we used the CORSIKA code  
79 (Heck et al., 1998), version 7.7500, which accounts for the effect of AEFs on particle  
80 transport (Buitink et al., 2009). The growth of RREA increases the cloud's  
81 electrical conductivity. Numerous studies (Marshall et al., 1995; Stolzenburg et al., 2007)  
82 have indicated that lightning flashes tend to occur when the applied electric field exceeds the  
83 RREA threshold by roughly 20–30%.

84 RREA simulation codes do not include a lightning initiation mechanism. Therefore, one  
85 can artificially raise the AEF strength beyond a realistic value to produce billions of  
86 avalanche particles; however, this approach lacks physical justification. As a result, we  
87 do not test AEFs stronger than 2.5 kV/cm at altitudes of 3–6 km. The RREA simulation  
88 was performed for vertical seed electrons with a uniform AEF that exceeded the  $E_{th}$  by a

89 few tens of percent. An introduced fixed uniform AEF shifts the surplus to  $E_{th}$  at different  
90 heights by different percentages, corresponding to air density. The chosen seed electron  
91 energy spectrum was based on the EXPACS Excel-based program (Sato 2015, Sato 2016),  
92 following a power-law with an index of 1.173 for energies from 1 to 300 MeV. During TGE  
93 events on Aragats, the typical distance to the cloud base is estimated to be  
94 25–200 m (see Figure 17 in Chilingarian et al., 2020); therefore, in our simulations,  
95 particle propagation continued in dense air for an additional 25, 50, 100, and  
96 200 meters before detection.

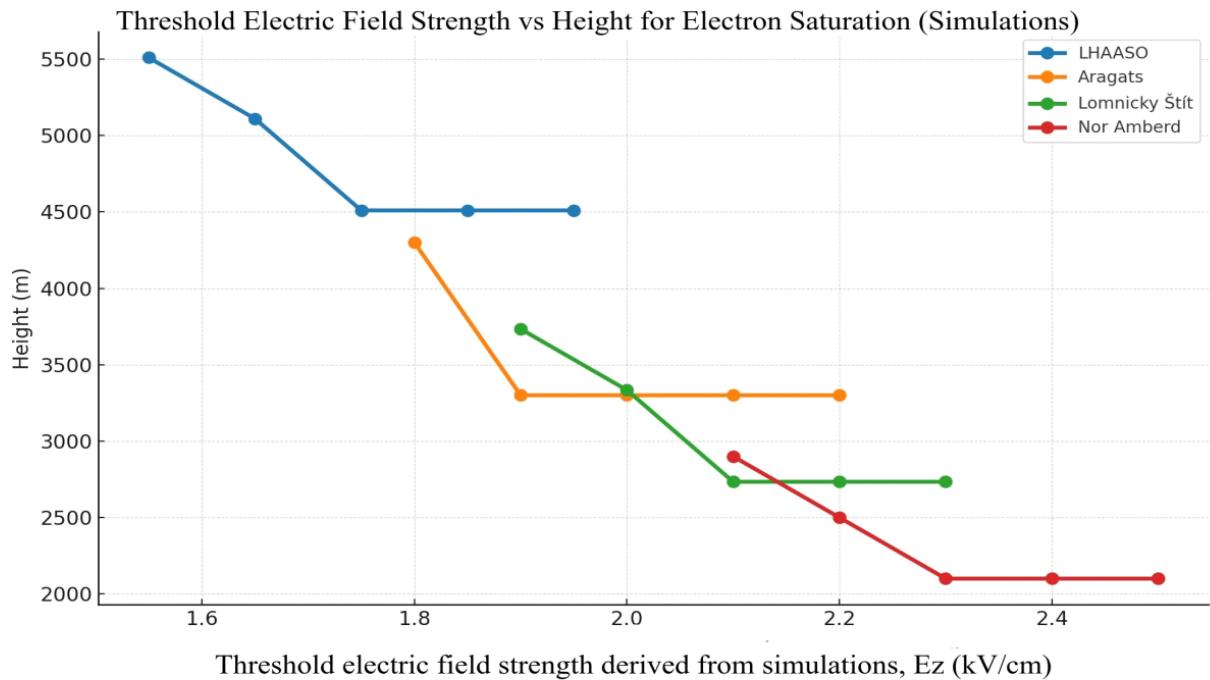
97 The simulations included 1,000 to 10,000 events for AEF strengths from 1.55 to 2.5 kV/cm.  
98 Electron and gamma-ray propagation were tracked until their energies dropped to 0.05 MeV.  
99 Each simulation event corresponds to the propagation of a single seed electron; multiple  
100 events were used to obtain statistically stable averages to reliably estimate  
101 the resulting threshold electric fields. The CORSIKA code  
102 models the development of RREA by calculating the number of electrons and gamma  
103 rays at different stages of the cascade development at 200-meter intervals. At all stations, the  
104 atmospheric electric field was implemented as a vertically uniform layer with a thickness of  
105 2000 m above the observation levels.

106 Besides the Aragats and Nor Amberd research stations on the slopes of Mt. Aragats in  
107 Armenia, we also conducted simulations for the Slovakian and Chinese research stations at  
108 Lomnický Štít (Chum et al., 2020) and the Tibetan plateau. LHAASO (Large High Altitude  
109 Air Shower Observatory, Aharonian et al., 2023) is situated at 4410 meters above sea level. It  
110 provides an ideal platform for studying atmospheric particle acceleration, owing to its thin  
111 atmosphere and the high likelihood of runaway electron avalanche formation. For LHAASO,  
112 we present CORSIKA simulation results showing increases in electron and photon fluxes  
113 under AEF strengths ranging from 1.55 to 1.9 kV/cm. The number of electrons and photons  
114 was recorded at altitudes ranging from 6510 meters to 4510 meters. Lomnický Štít is located  
115 at an altitude of 2630 meters in Slovakia. CORSIKA simulations were performed for various  
116 vertical AEFs ranging from 1.9 to 2.3 kV/cm. The number of electrons and photons was  
117 recorded at altitudes ranging from 4734 meters to 2734 meters. Significant increases in flux  
118 were observed with stronger-than-threshold fields, confirming the development of robust  
119 RREA. Saturation trends in the growth of electrons and photons suggest that the threshold  
120 field,  $E_{th}$ , at Lomnický Štít is approximately 2.3 kV/cm. These results support earlier findings  
121 from Aragats and Nor Amberd and emphasize the altitude dependence of  $E_{th}$ . Due to the  
122 thinner air at LHAASO, the TGEs occurred at a much lower value of 1.7 kV/cm. In Figures  
123 1a-1d, we display the development of RREA at different atmospheric depths and for various  
124 physically justified strengths of the AEF. The curves are scaled for a single seed electron for  
125 easier comparison with experimentally measured intensities.  
126 For large values of AEFs, the number of avalanche particles rose exponentially. For lower  
127 values of AEF, we observe saturation of the particle flux when AEF becomes lower than the  
128 threshold electric field (dependent on air density); the RREA process attenuates before  
129 reaching the observation level (see the yellow and blue curves in Figures 1a-d).



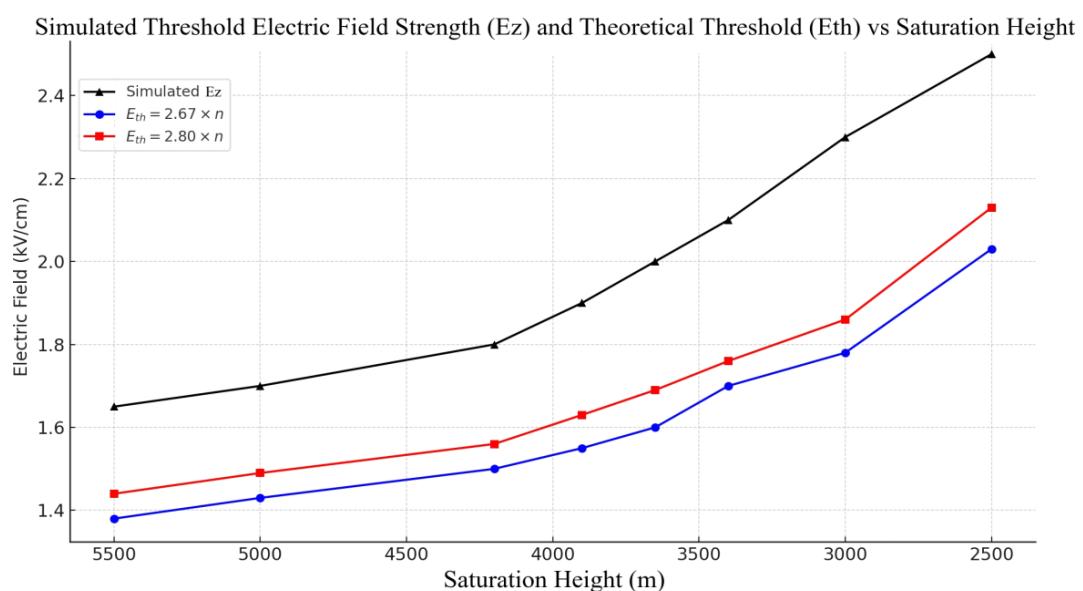
130  
131 *Figure 1. Longitudinal development of relativistic runaway electron avalanches (RREA) at*  
132 *four high-altitude observation sites: (a) Aragats, (b) Lomnický Štit, (c) Nor Amberd, and (d)*  
133 *LHAASO. The number of electrons is normalized to a single seed electron and shown as a*  
134 *function of depth within the electric field region. For each site, simulations were performed*  
135 *for several electric field strengths, as indicated in the legends. Avalanche development is*  
136 *sampled every 200 m across the field region. After exiting the electric field, electron*  
137 *propagation is followed for an additional 100 m in free space before reaching the detector.*

138  
139 Figure 1 illustrates the dependence of electron multiplication on electric field strength and  
140 highlights the altitude-dependent conditions required for sustained avalanche development.  
141 We estimate the “simulated” thresholds,  $E_z$  values, at the heights where the number of  
142 avalanche particles stops rising, as shown in Figure 2.  
143



144  
145 *Figure 2. Simulated threshold electric field strength, Ez, versus altitude for several high-*  
146 *altitudes stations. The threshold is defined as the electric field strength at which the growth of*  
147 *avalanche electrons saturates.*

148  
149 In Figure 3 and Table 1, we compare the “simulated” threshold Ez with the  
150 theoretical ones. Simulations derive higher values than theoretical estimates,  
151 especially for high  $E_{th}$  values (low altitudes) at all four research stations.  
152 Theoretical threshold fields are computed as  $2.67 \text{ kV/cm} \times n$  and  $2.80 \text{ kV/cm} \times n$ .  
153 The percentage of enhancement indicates how much the applied field exceeds the  
154 theoretical thresholds. Strong AEFs, where the cascade did not attenuate, were not  
155 included in the table.



157 *Figure 3. Simulated threshold electric field strength,  $E_z$ , and theoretical threshold electric*  
 158 *fields,  $E_{th}$ , as a function of saturation (rise stopping) altitude.*

159  
 160 *Table 1. Excess of  $E_z$  over  $E_{th}$ . Stopping altitudes and theoretical threshold field comparisons for*  
 161 *heights 2500- 5550 m.*

| Input $E_z$<br>(kV/cm) | Enhancement<br>Stopsath(m) | n<br>(relative<br>density) | 2.67×n<br>(kV/cm) | 2.80×n<br>(kV/cm) | Rel.<br>Excess.<br>(%)<br>(2.80<br>kV/cm) | Rel.<br>Excess.<br>(%)<br>(2.67<br>kV/cm) | Station                     |
|------------------------|----------------------------|----------------------------|-------------------|-------------------|---|---|-----------------------------|
| 1.55                   | 5510                       | 0.465                      | 1.24              | 1.30              | 19.0                                      | 24.8                                      | LHAASO<br>(4400m)           |
| 1.65                   | 5110                       | 0.492                      | 1.31              | 1.38              | 19.8                                      | 25.7                                      | LHAASO<br>(4400m)           |
| 1.8                    | 4200.0                     | 0.558                      | 1.49              | 1.56              | 15.2                                      | 20.8                                      | Aragats<br>(3200m)          |
| 1.9                    | 3900.0                     | 0.582                      | 1.55              | 1.63              | 16.6                                      | 22.3                                      | Aragats<br>(3200m)          |
| 1.9                    | 3734                       | 0.595                      | 1.59              | 1.67              | 14.0                                      | 19.5                                      | Lomnický<br>Štít(2630<br>m) |
| 2.0                    | 3334                       | 0.629                      | 1.68              | 1.76              | 13.5                                      | 19.0                                      | Lomnický<br>Štít(2630<br>m) |
| 2.1                    | 2700.0                     | 0.687                      | 1.84              | 1.92              | 9.1                                       | 14.4                                      | Nor<br>Amberd<br>(2000m)    |
| 2.2                    | 2500.0                     | 0.707                      | 1.89              | 1.98              | 11.2                                      | 16.6                                      | Nor<br>Amberd<br>(2000m)    |

162  
 163 **3. Discussion and conclusion**

164  
 165 Both the classical threshold field ( $E_{th} \approx 2.80 \text{ kV/cm} \times n$ ) and its updated  
 166 version ( $E_{th} \approx 2.67 \text{ kV/cm} \times n$ ) are derived under idealized assumptions; the difference  
 167 between them results from refinements in modeling particle energy loss processes. The  
 168 earlier estimate of  $2.80 \text{ kV/cm} \times n$  was based on basic energy balance considerations using  
 169 older ionization loss models and assumed monoenergetic electrons. This threshold  
 170 is slightly above the breakeven field, where energy gain equals average energy loss.  
 171 The updated  $2.67 \text{ kV/cm} \times n$  value, introduced by Dwyer and Rassoul (2024),  
 172 incorporates more accurate relativistic Boltzmann solutions, improved ionization  
 173 and bremsstrahlung cross-sections, and a probabilistic treatment of runaway thresholds  
 174 across realistic energy spectra. While both thresholds assume idealized, field-aligned  
 175 electron motion in a uniform medium, the updated value is physically more consistent.  
 176 It predicts a slightly lower field strength needed for initial runaway.

177 However, CORSIKA simulations show that this refined threshold is insufficient for sustained  
178 avalanche growth under real atmospheric conditions due to scattering and finite path  
179 effects. Moreover, it deviates more from the simulated value than the “classical”,  
180 30-year-old estimate. Multiple physical processes inhibit ideal runaway propagation.  
181 Coulomb scattering with atmospheric nuclei and Møller scattering with electrons  
182 cause substantial angular deflection and energy redistribution. Secondary electrons are  
183 not generated strictly along the field direction, and many lose energy before gaining  
184 sufficient momentum to continue avalanche growth. As a result, electrons must be  
185 accelerated in fields stronger than the threshold to overcome these losses and maintain  
186 avalanche conditions. CORSIKA simulations, which incorporate all major interaction  
187 mechanisms—including Coulomb and Møller scattering, bremsstrahlung losses, finite  
188 propagation distances, and realistic secondary cosmic ray spectra—show that avalanches fully  
189 develop only when the applied field exceeds the theoretical threshold by a measurable margin.  
190 For the updated  $2.67 \text{ kV/cm} \times n$  value, we observe a required excess of approximately 20–22%  
191 at the Aragats station ( $\sim 3200$ – $4200$  m a.s.l.), whereas for the classical  $2.80 \text{ kV/cm} \times n$   
192 threshold, the excess is typically 15–17%. Interestingly, this required excess decreases with  
193 increasing air density, as observed in the Nor Amberd simulations. At lower altitudes ( $\sim 2500$ –  
194  $2700$  m a.s.l.), the difference between the applied and threshold fields is reduced: only 14–16%  
195 above  $2.67 \text{ kV/cm} \times n$ , and about 9–11% above  $2.80 \text{ kV/cm} \times n$ .

196 This trend can be explained as follows:

197 In denser air, the chances of energy-loss interactions increase, but so does the likelihood  
198 of electron multiplication through ionization and bremsstrahlung over shorter distances.  
199 The avalanche can develop more quickly because seed electrons encounter more target  
200 atoms in a given path length. As a result, the necessary “headroom” above the threshold  
201 field for sustained multiplication is smaller. Simply put, the efficiency of avalanche  
202 formation improves in denser air, even though the absolute threshold field is higher. This  
203 results in a smaller relative excess being required above the theoretical threshold.  
204 Therefore, although the threshold field scales linearly with air density, the required  
205 enhancement factor does not. It decreases with increasing density due to a balance  
206 between energy loss and multiplication processes, all of which are faithfully captured in  
207 the CORSIKA simulation framework. This emphasizes the importance of altitude-dependent  
208 analysis in interpreting Thunderstorm Ground Enhancements (TGEs) and  
209 suggests that scaling laws based solely on density may overlook subtler effects arising  
210 from atmospheric structure and shower development dynamics. Among these effects is the local  
211 temperature profile, which can modify air density and, consequently, slightly affect the effective  
212 threshold field. A more detailed treatment incorporating measured or modeled temperature  
213 profiles could further refine threshold estimates for individual events; however, such event-  
214 specific modeling is beyond the scope of the present work.

215

## 216 **Code and data availability**

217

218 All materials under the authors’ control that are required to reproduce the results presented  
219 in this manuscript are publicly available in a Zenodo repository:

220 <https://doi.org/10.5281/zenodo.17986152>

221

222 The Zenodo archive is organized as follows:

223

224 code/

225 This directory contains auxiliary materials and user-level post-processing codes  
226 (e.g., easread.f) required to ensure the reproducibility of the simulations. These codes  
227 were used to analyze the output data of the CORSIKA simulations and to derive the  
228 numerical results presented in the manuscript.

229

230 inputs/  
231 This directory contains all CORSIKA input files used in the simulations for each observation  
232 site, including the complete input cards and definitions of the thunderstorm electric-field  
233 configurations (el.input, elfield.c), observation levels, energy cutoffs, and all relevant  
234 simulation parameters.

235

236 data/  
237 This directory contains the CORSIKA simulation output files (DAT files) corresponding to  
238 the electric-field configurations and observation sites analyzed in the manuscript. The data  
239 are organized by station and electric-field strength.

240

241 tables/  
242 This directory contains the final numerical tables used in the manuscript, including threshold  
243 electric-field values, stopping altitudes, relative air densities, and percentage excesses over  
244 theoretical thresholds.

245

246 figures/  
247 This directory contains all figures included in the manuscript, generated directly from the  
248 simulation output and the processed numerical data.

249

250 In addition, the repository root contains the official technical documentation of the CORSIKA  
251 simulation framework (CORSIKA\_GUIDE7.7550.pdf) and a README file describing the  
252 structure and contents of the archive.

253

254 The CORSIKA simulation framework is a licensed third-party Monte Carlo code developed  
255 and maintained by the Karlsruhe Institute of Technology (KIT). The exact CORSIKA version  
256 used in this work is specified in the manuscript and is available for scientific use directly  
257 from the official KIT distribution portal. All user-provided inputs, configurations, auxiliary  
258 codes, and simulation outputs required for reproducibility are provided in the Zenodo archive.

259

260 Together, these materials ensure full reproducibility of the simulations and results presented in  
261 this study for any user with legitimate access to the CORSIKA framework.

262

263 **Author contribution**  
264 AC and MZ designed the simulation experiments with the CORSIKA code, and LH  
265 performed the simulations. AC prepared the manuscript with contributions from all co-  
266 authors

267

268 **Acknowledgment**

269 The authors acknowledge the support of the Science Committee of the Republic of Armenia  
270 (Research Project No. 21AG~1C012)  
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