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Chaotic fluctuations in Greenland outlet glaciers limit predictability of a future ice sheet collapse

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Abstract. The future evolution of the Greenland ice sheet (GrIS) depends on the intensity and the speed of climate change. By applying different rates of temperature change in a state-of-the-art comprehensive ice-sheet model coupled to a regional energy-moisture balance atmospheric model, oscillations in the total ice-sheet volume are found under warming magnitudes between 1.0 and 1.3 K above present-day temperatures. These are located in the northwestern drainage basin of the GrIS and are due to two ice streams which alternate between fast and slow basal velocities, manifesting in a build-up/surge variability. These ice streams interact due to their spatial proximity, resulting in irregular periodicity. The ice streams appear in a region where tipping of the entire GrIS begins, leading the oscillations to affect the tipping behaviour. These oscillations directly impact the time it takes before the ice sheet collapses at a given external forcing magnitude by hundreds of thousands of years for an ensemble of rates of forcing and initial conditions. These long tipping times are proposed to be due to chaotic transients. Our results suggest that ice-stream oscillations are a potential source of internal chaotic variability in ice sheets that affect tipping behaviour, thereby complicating prospects of anticipating such a tipping.

1 Introduction

The Greenland ice sheet (GrIS) is one of the principal tipping elements in Earth's climate system (Armstrong McKay et al., 2022), meaning it could experience a massive and potentially irreversible change when an external forcing parameter, specifically the global mean temperature, increases beyond a critical threshold known as a 'tipping point' (Robinson et al., 2012). This phenomenon is also termed 'bifurcation-induced tipping' (b-tipping). This tipping involves the large-scale loss of ice mass (or 'collapse' of the ice sheet) through melting and has a straightforward impact on the rest of the Earth by raising the global sea level (Gregory et al., 2004; Rignot et al., 2011).

Tipping of the GrIS can occur due to the presence of two strong positive feedbacks: First, a decrease in ice-sheet thickness means that the temperature of the ice surface increases, promoting further melting, known as the melt-elevation feedback. The second is the decrease of the surface albedo as a result of the retreat of the ice sheet increasing the amount of solar

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energy absorbed, known as the ice-albedo feedback. In steady state, where the ice sheet is in mass balance, the dynamics must, however, be dominated by negative feedbacks: As atmospheric temperatures increase, so does the precipitation and the surface mass balance, leading to a thickening of the ice sheet. Furthermore, the melt-elevation feedback is reduced by the effect of glacial isostatic adjustment: when ice thins, the resulting bedrock uplift partially compensates the reduced surface elevation and thus the surface melt. This effect is, however, mainly relevant on long time scales (Wake et al., 2016; Zeitz et al., 2022). The atmospheric temperature is considered the critical parameter. As it increases, the negative feedbacks weaken and at the bifurcation point, the negative feedback no longer control the state and the positive feedbacks will force it into the alternative state.

While the critical temperature for tipping of the GrIS has been assessed in different studies (Gutiérrez-González et al.; Höning et al., 2023; Robinson et al., 2012; Zeitz et al., 2022), the effect of the rate of change of the forcing on the tipping behaviour has not yet been investigated. The feedbacks that determine the stability of the GrIS are influenced strongly by the surface air temperature of Greenland, which is increasing at a rapid pace (Intergovernmental Panel On Climate Change (IPCC), 2023). In non-autonomous systems with multiple dynamic time scales, high rates of forcing compared to the time scale for restoring to the equilibrium state could lead to a tipping of the system at a forcing less than the bifurcation point, a phenomenon known as 'rate-induced tipping' (r-tipping) (Ashwin et al., 2012; Feudel, 2023).

The bulk of the GrIS evolves over time according to slow shear flow, but areas of fast flowing ice, such as topographically confined outlet glaciers or large ice streams, represent a source of variability with a faster dynamic timescale. Ice streams in particular are relevant, as they are characterized by sliding of ice at the base due to a till that is saturated with water. A fast warming rate can lead to an easier saturation of the base because it cannot be compensated by the drainage rate, activating ice streams and increasing mass loss. The discharge of ice through ice streams contributes a large amount to the total ice-sheet mass loss despite their relatively small spatial extent (The IMBIE Team, 2020; Van Den Broeke et al., 2009). Furthermore, ice streams have accelerated due to increased atmospheric and oceanic forcing, contributing up to 50% of the GrIS mass loss in the last decades (Holland et al., 2008; Howat et al., 2008; Khan et al., 2014; Krabill et al., 2004; Larocca et al., 2023; Luthcke et al., 2006; Rignot and Kanagaratnam, 2006; Rignot et al., 2011; Trusel et al., 2018).

Rate-induced tipping of a component of the Earth system has been investigated previously in comprehensive models of the Atlantic Meridional Overturning Circulation (AMOC) (Lohmann and Ditlevsen, 2021) and the west Antarctic ice sheet (Swierczek-Jereczek et al., 2025). The rate of forcing is also important when considering the ability to prevent a transition after overshooting a tipping point by imposing a subsequent cooling (Bochow et al., 2023). In this study, we use a state-of-the-art ice-sheet model coupled to a regional atmospheric energy-moisture balance model to investigate the GrIS response under different magnitudes and rates of warming in order to determine whether r-tipping of the GrIS is possible.

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2 Methods

2.1 Model description

The model used in this study is the three-dimensional thermomechanical ice-sheet model Yelmo (Robinson et al., 2020) coupled with the regional energy-moisture balance climate model REMBO (Robinson et al., 2010). This model setup is similar to that of Robinson et al. (2012) but with a newer ice-sheet model. The model domain covers the entirety of Greenland at a horizontal resolution of 16 km. The surface mass balance (SMB) is determined by the temperature and precipitation calculated by REMBO. The SMB can be separated into a positive contribution from the precipitation and a negative contribution resulting from surface melt. The latter is calculated using an insolation-temperature melt method (ITM), whereby insolation and albedo are explicitly taken into account. The isostatic adjustment of the bedrock under the ice sheet uses the elastic lithosphere-relaxing asthenosphere (ELRA) model (Meur and Huybrechts, 1996) with a relaxation timescale of 3000 years.

To compute the ice dynamics, Yelmo uses the depth-integrated viscosity approximation (Goldberg, 2011; Robinson et al., 2022). The basal frictional stress $\boldsymbol{\tau}_b = (\tau_{b,x}, \tau_{b,y})$ is modelled using a regularized Coulomb friction law (Schoof, 2005; Joughin et al., 2019),

$$\boldsymbol{\tau}_b = -c_b \left(\frac{|\mathbf{u}_b|}{|\mathbf{u}_b| + u_0} \right)^q \frac{\mathbf{u}_b}{|\mathbf{u}_b|}, \tag{1}$$

where $\mathbf{u_b} = (u_b, v_b)$ is the basal velocity vector, $u_0 = 100$ m a^{-1} and $q = 0.2$ are empirical parameters derived from laboratory experiments (Zoet and Iverson, 2020), and c_b is a field defining the bed-friction coefficient. The threshold speed u_0 allows the basal stress to saturate at large velocities, where it becomes independent of basal velocity (Blasco et al., 2024; Zoet and Iverson, 2020). The coefficient c_b depends linearly on the effective pressure N at the base of the ice,

$$c_b = \lambda N. \tag{2}$$

The factor λ represents the till strength of the bedrock and depends on the elevation above or below sea level. This factor leads to increased sliding velocity by reducing basal friction at lower elevations, owing to the till in these areas being composed primarily of sediments that are softer and more easily deformable. A similar elevation-dependent till strength is also seen in Albrecht et al. (2020) and Martin et al. (2011).

The effective pressure $\hat{N}$ differs from the overburden pressure depending on the basal water content, following the parameterization of Bueler and van Pelt (2015):

$$\hat{N} = N_0 \left(\frac{\delta P_0}{N_0} \right)^s 10^{\frac{e_0}{C_c}(1-s)}. \tag{3}$$

The effective pressure decreases with increasing till saturation $s = H_w / H_{w,\max}$ to a minimum value of δP_0 at $s = 1$. The parameter P_0 is the overburden pressure, with $\delta = 0.02$ being an empirical minimum fraction. This means that the effective pressure at the base is equal to 2% of the overburden pressure when the till is saturated, and any excess water is considered to be drained from the system (Bueler and van Pelt, 2015). The parameter $N_0 = 1000$ Pa is a reference pressure at a reference

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void ratio $e_0 = 0.69$, and $C_c = 0.12$ is the coefficient of compressibility of the till. Finally, since the overburden pressure is an upper limit for the effective pressure, the minimum of the two is taken,

$$N = \min\left\{P_0, \hat{N}\right\}. \tag{4}$$

The basal water layer thickness H_w changes with the basal mass balance $\dot{b}_g$ and is removed via a constant drainage rate C_d,

$$\frac{\partial H_w}{\partial t} = -\frac{\rho_i}{\rho_w}\dot{b}_g - C_d, \tag{5}$$

where ρ_i and ρ_w are the ice and water densities, respectively, and $\dot{b}_g$ is the basal mass balance,

$$\dot{b}_g = -\frac{1}{\rho_i L}\left(Q_b + k\frac{\partial T}{\partial z}\bigg|_b + Q_{\mathrm{r}}\right). \tag{6}$$

The function $\dot{b}_g$ therefore converts the heat flux at the base to a melt rate via the latent heat of ice fusion, L. The heat flux consists of the heat generated by friction due to sliding of the ice along the base Q_b as well as the heat conducted into the column of ice above given by the coefficient of conduction of ice k and the temperature gradient of the ice at the base $\frac{\partial T}{\partial z}\big|_b$. In addition to this, there is a geothermal heat flux boundary condition Q_{geo} imposed 2 km below the surface and the heat diffuses vertically through the bedrock. The term Q_{r} is then the flow of heat into the ice at the interface of bedrock and ice.

2.2 Experimental setup and boundary conditions

The model is first run to equilibrium for 400 ka to arrive at an initial state (Fig. 1) corresponding to the present-day GrIS. The ice-sheet thickness and bedrock topography are initialized using data from version 5 of the BedMachine mapping of Greenland (Morlighem et al., 2017). The surface boundary conditions of Yelmo are determined by REMBO, which uses the ERA-40 climatology (Uppala et al., 2005) as its boundary conditions. The states of the GrIS at 200 ka and 300 ka are also saved as initial states, representing a small perturbation to the initial state at 400 ka. This is the starting point from which the model is forced in subsequent simulations. The use of an ensemble of multiple initial states is not common for ice-sheet simulations, but is implemented to test the robustness of the results and a possible dependence on initial conditions.

By definition, r-tipping occurs at a forcing temperature below the critical b-tipping value, so the latter must first be estimated. To estimate the bifurcation point of the GrIS, the sea-level summer air temperature and ocean temperature at the boundaries of REMBO are gradually increased, as in Robinson et al. (2012). This regional summer temperature anomaly is approximately equivalent to a global mean temperature increase. This forcing is applied using an adaptive quasi-equilibrium forcing (AQEF) function starting at each of the three initial states. This AQEF interactively adapts the forcing in order to maintain the ice sheet in a quasi-steady state by ensuring that the rate of ice-volume loss stays below a given threshold, equal to 2 gigatons per year averaged over 100 years. The forcing increase is done in an adaptive way such that the longer it takes the model to equilibrate, the less the forcing is subsequently increased. In this way, the forcing parameter does not increase while the tipping is occurring.

Once the tipping point has been estimated, it is used to determine a range of parameters for the subsequent ramping experiments that are used to assess the possibility of r-tipping. In these ramping experiments, the forcing is increased at a linear rate

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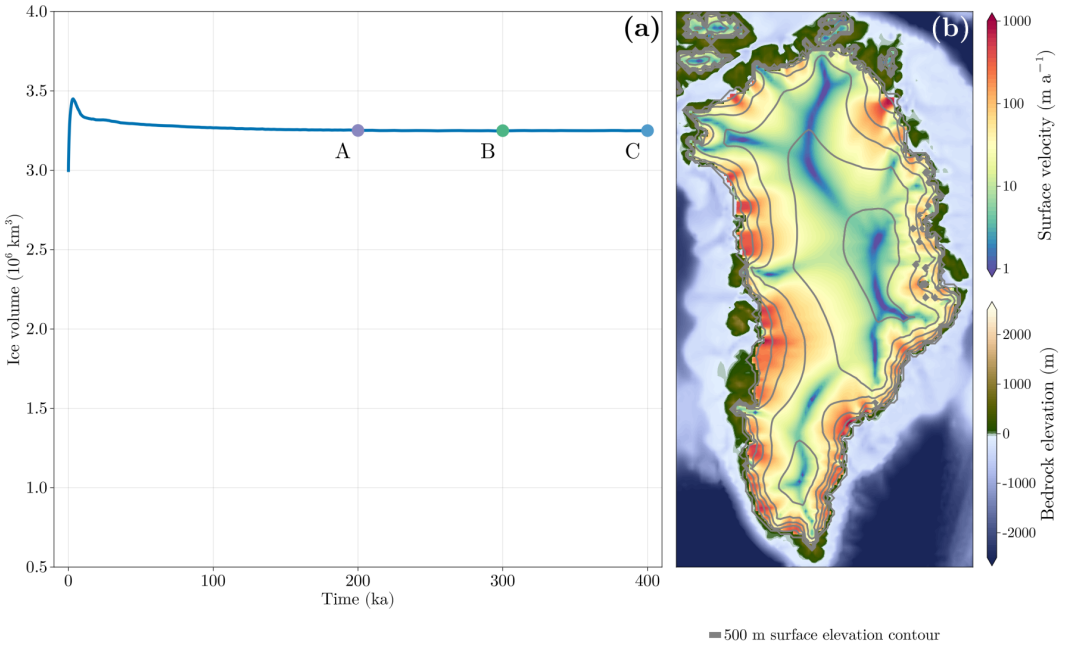

Figure 1. (a): Ice volume time series of the equilibrium simulation for the three initial states A to C. (b): Ice sheet extent of the initial state C at the end of the 400 ka spin-up run.

to some value less than the tipping point. Thereafter, the forcing is kept constant and the ice sheet is allowed to equilibrate over 400 ka. An ensemble of simulations is generated by applying different rates of increase and maximal forcing values, allowing the effect of the rate and magnitude of warming to be evaluated. These ramping experiments are also performed starting at each of the three initial states to investigate the effect of different initial conditions.

3 Results

3.1 Tipping of the ice sheet

Figure 2 shows the ice volume as a function of the forcing parameter, the regional summer temperature anomaly, for the three initial states. There is clear tipping behaviour for a forcing of +1.275 K, which is essentially independent of the choice of initial state. This tipping resembles what is expected for a saddle-node bifurcation.

The retreat of the GrIS during the tipping event is seen in Fig. 3. Spatially, the retreat begins in the north. Due to the similarities in the model setups between this study and that of Robinson et al. (2012), the spatial tipping pattern is equivalent. The pattern of ice sheet loss is also similar to that of Zeitz et al. (2022) (their Fig. 3a) albeit without the regrowth of the ice sheet.

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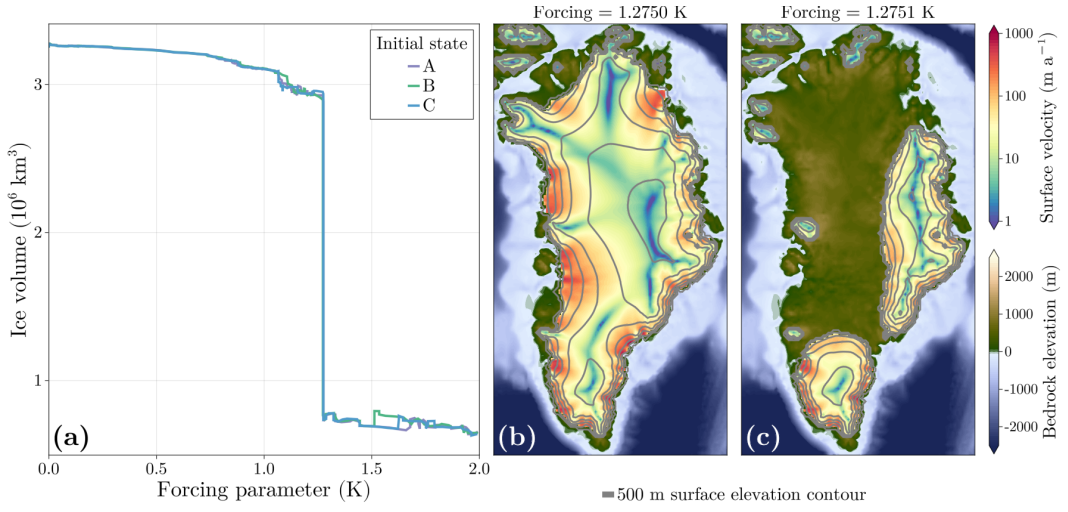

Figure 2. (a): Equilibrium ice volume of the GrIS as a function of the applied regional summer temperature anomaly in parameter space to estimate the tipping value for initial states A to C. (b): Ice-sheet before tipping. (c): Ice-sheet after tipping.

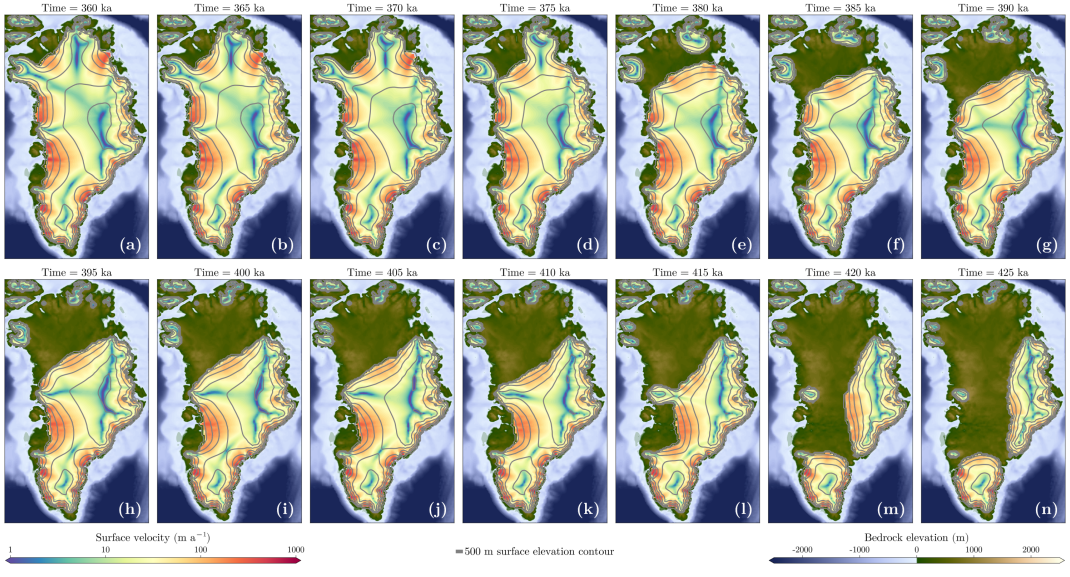

Figure 3. Ice extent and velocities during the tipping event at a fixed forcing value of +1.275 K.

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3.2 Ramping experiments

Since the tipping point is located at +1.275 K, the maximal forcing of the ramping experiments are chosen as +1.00, 1.05, 1.10, 1.15, 1.20, and 1.25 K. A value of +1.30 K past the tipping point is also included to confirm that tipping does indeed always occur when forced past +1.275 K. The forcing increases linearly at a rate of 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4}, or 10^{-5} K a^{-1}, up to one of the seven maximal forcing values noted previously. The time series of the forcing is shown in Fig. 4. As each of these 120 ramping experiments is initialized from one of three states A, B and C, the entire ensemble consists of $7 \times 5 \times 3 = 105$ members.

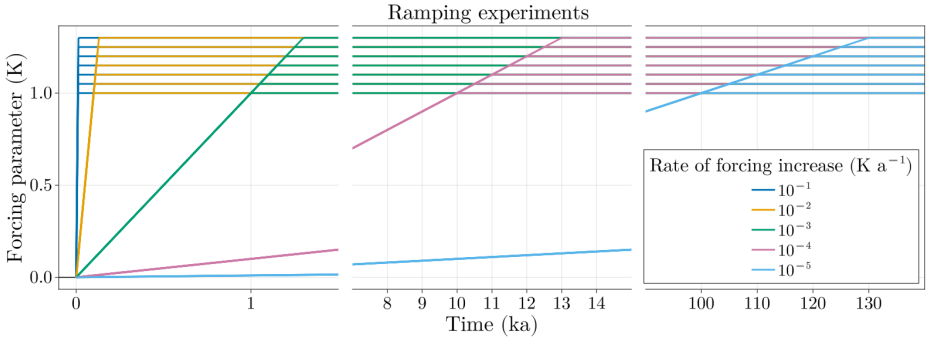

Figure 4. Time series of the forcing amplitude during the ramping experiments.

The simulations in Fig. 5 suggests rate-induced tipping. Tipping is observed for simulations forced to values lower than the critical value for b-tipping of +1.275 K, occurring between +1.10 and +1.25 K. Tipping also occurs for all of the simulations forced past the tipping point, that is, those forced to +1.30 K. What is absent, however, is some critical rate below which tipping does not occur, implying that r-tipping of the GrIS can occur even for very slow rates of forcing.

One prominent feature in this ensemble of simulations is the non-monotonicity in the tipping times. There is some agreement in tipping times among a given initial condition, as well as a dependence of the mean tipping time on the maximal forcing, being shorter for larger values. However, there is no clear relationship between the tipping time and the rate of forcing. Even for the same initial conditions, sometimes longer tipping times are achieved at lower forcing rates. For instance, for initial condition C at a forcing level of +1.25 K (panel (m) of Fig. 5), the trajectory forced at a rate of 10^{-2} K a^{-1} tips around 300 ka, where the trajectory forced at slower rate of 10^{-3} K a^{-1} tips earlier, around 100 ka.

In addition, tipping times for a maximum forcing of 1.30 K can be longer than those for 1.25 K. For example, for initial condition B and the slowest rate of 10^{-5} K a^{-1}, the tipping occurs around 250 ka for a forcing of +1.25 K and 400 ka for +1.30 K (panels (m) and (n) of Fig. 5, respectively). Finally, even for the same magnitude of forcing and rate of forcing, the tipping times are not consistent over initial conditions. For example, in panel (l) of Fig. 5, the simulation forced at a rate of 10^{-3} K a^{-1} tip at 300, 350 and 250 ka for initial conditions A, B and C respectively. Overall, sensitive dependence on initial condition, indicative of chaos, appears to be affecting the outcome of the tipping.

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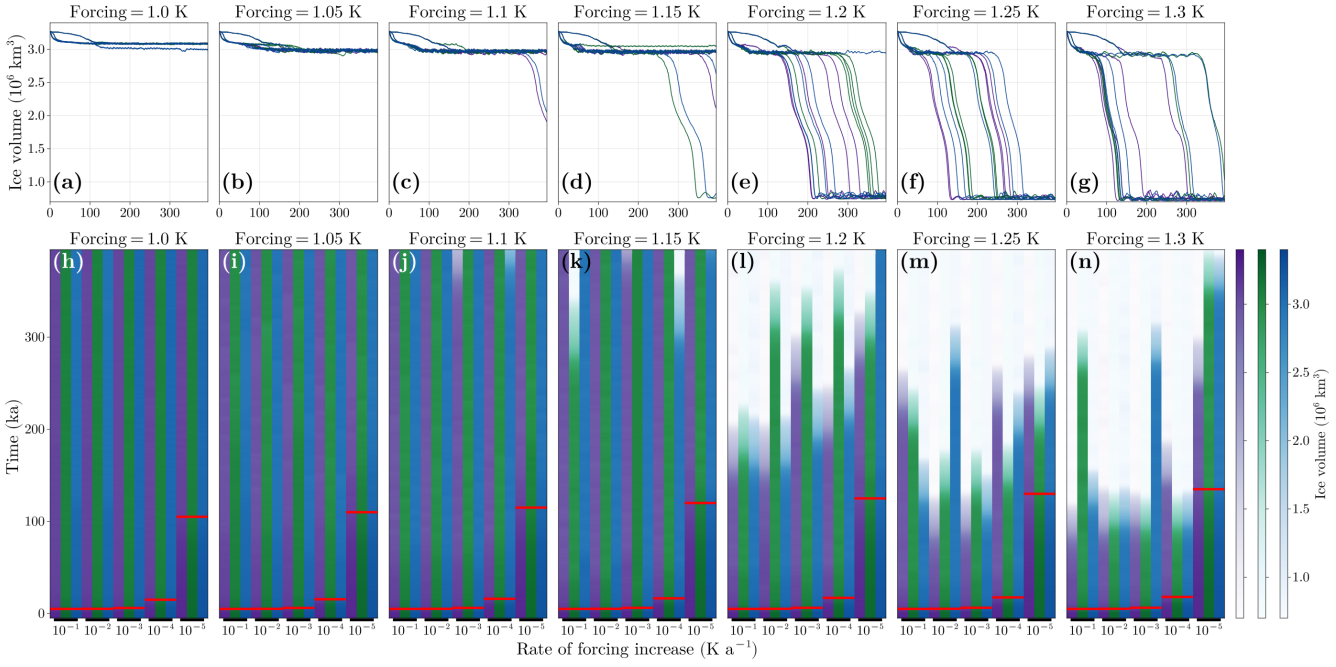

Figure 5. (a-g): Time series of the ice-sheet volume for all simulations at a given maximal forcing. (h-n): Time series of the ice volume for all of the simulations grouped by maximum forcing value. The initial state of each simulation is indicated by the different colours: purple for A, green for B and blue for C. For a given maximal forcing, the simulations are ordered along the bottom axis in decreasing rate of forcing, with the red line indicating the time where the forcing reaches its maximum.

While investigating the cause of these random tipping times, irregular oscillations in the ice sheet volume before tipping were observed. These oscillations can be isolated to a single region of the ice sheet: the northwest drainage basin, where the ice-sheet also begins its retreat during a tipping event. It is hypothesized that the random tipping times are linked to these irregular oscillations. An example of the oscillations for each maximal forcing is shown in Fig. 6. For all these simulations, there is an initial loss of mass during the first 50 - 80 ka of simulation time. Thereafter, for a maximal forcing of +1.00 K, there is a relatively small amount of variability around a steady ice volume of about 3.09 $\times 10^6$ km^3. For maximal forcing values between +1.05 and +1.15 K, the mean ice volume is lower, between 2.9 and 3.0 $\times 10^6$ km^3. The variability is also larger in both amplitude and period. Finally, the simulations for maximal forcing of +1.20 to +1.30 K in Fig. 6 retreat to a much smaller ice volume, representing a collapsed GrIS as seen in Fig. 2.c.

3.3 Spatial and temporal behaviour of the oscillations

Two simulations at a forcing of +1.00 1.05 K are compared, as this represents the onset of the large-amplitude variability seen in Fig. 6. Further examination of the oscillations show they are a result of two ice streams that alternate between periods of

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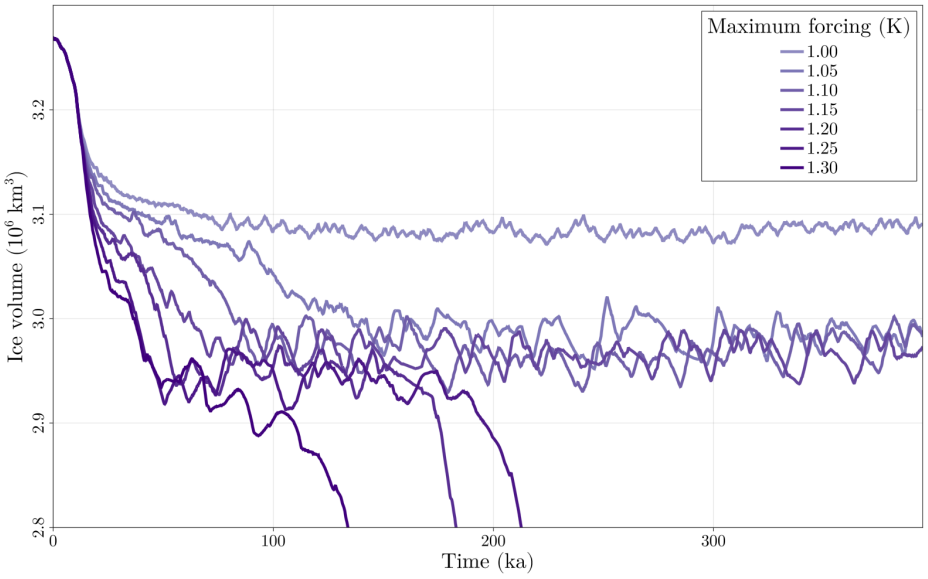

Figure 6. Time series of ice volume for simulations from Fig. 5 for initial condition A, rate of warming 10^{-4} K a^{-1}, and a range of maximal forcing values.

stagnant and rapid basal sliding. These are the Humboldt and Petermann glaciers, which lie in close proximity to each other on the northwestern ice-sheet edge. These glaciers are both regions of fast-flowing ice (Carr et al., 2015; Ehrenfeucht et al., 2023; Hillebrand et al., 2022) and behave as ice streams in the model simulations. In the model, ice streams can be identified by grounded ice with a significant basal velocity.

The first difference to be seen is the extent of the ice sheet in this region. For a forcing of +1.00 K, the ice-sheet margin is such that the Humboldt ice stream (HIS) is marine-terminating, with the Petermann ice stream (PIS) covering the Petermann fjord. The ice sheet in this case is termed 'unretreated'. In the simulation with a forcing of +1.05 K, the ice sheet extent is much reduced. From the mean ice-thickness profiles, the ice margin is almost 100 km further inland (panels a, c, d and f of Fig. 7). The HIS is no longer connected to the ocean, although the PIS terminates at the Petermann fjord. We refer to this as the 'retreated' configuration, with the two separate ice streams being designated the retreated HIS and retreated PIS.

The temporal behaviour of the two ice sheet extents is compared by taking the spatial mean in two grid boxes, one containing the PIS/retreated PIS and the other the HIS/retreated HIS, as seen in panels (a) and (j) of Fig. 8. For the unretreated case, the basal velocities in the two different ice streams behave quite differently. The PIS is in a state of steady flow of around 100 m a^{-1}, facilitated by a constant basal water layer thickness. The HIS, in contrast, alternates between near zero basal movement and sliding velocities of 100 m a^{-1}. The periods of stagnation are due to an increase of basal friction because of a reduction in the water content of the till due to freezing or drainage. While the basal velocities are minimal, the ice thickness increases, establishing a 'build up' phase. The subsequent loss of mass due to rapid ice streaming is a 'surge' phase. The period of these

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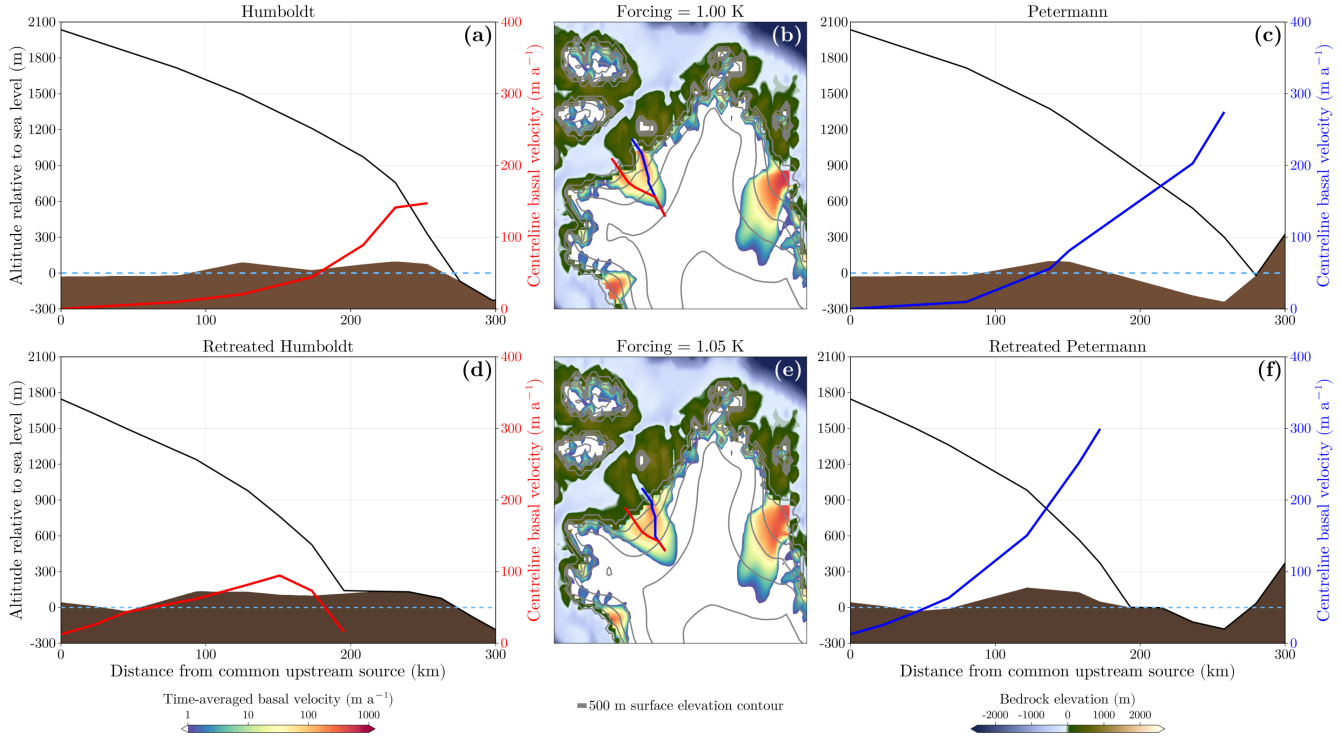

Figure 7. (a), (c): Centreline ice-thickness profile (black) and basal velocities along the centreline for the HIS and PIS. (d), (f): same as (a), (c) but for the retreated HIS and retreated PIS. (b), (e): time-averaged basal velocity field. The lines perpendicular to the grey height contours are the ice-stream centrelines for Humboldt (red) and Petermann (blue).

oscillations varies between approximately 5 and 9 ka. This steady cycle of mass gain during the build-up phase and loss during the surge results in an oscillation in mean ice thickness between 100 and 200 metres. The PIS also shows a smaller alternation in ice thickness with a similar temporal pattern while maintaining constant ice-stream flow, suggesting the thickness variations are influenced by the oscillations of the nearby HIS.

In the retreated configuration, the oscillations of ice thickness and basal velocity have larger amplitude and periodicity, as mentioned previously. While the exact location of the retreated PIS and retreated HIS differs slightly in all the simulations showing oscillations, their existence is robust among the ensemble. In contrast to the PIS, the retreated PIS is no longer in a steady-flow state and instead displays the same build-up/surge variability as the HIS and retreated HIS.

Two broad patterns for the oscillations of the retreated ice extent emerge. The first is one of a long, asymmetric build-up and surge event. An example is seen around 225 to 260 ka in panels c, e, g and i of Fig. 8. While the basal velocity in retreated PIS switches abruptly between the build-up and surge phases, the increase of the basal velocity in the retreated HIS is gradual. This slows the mass loss until a point where the basal water layer thickness rapidly decreases in both ice streams when the ice thickness reaches its minimum. The second pattern is that of short oscillations with a period of around 8 ka. These are seen

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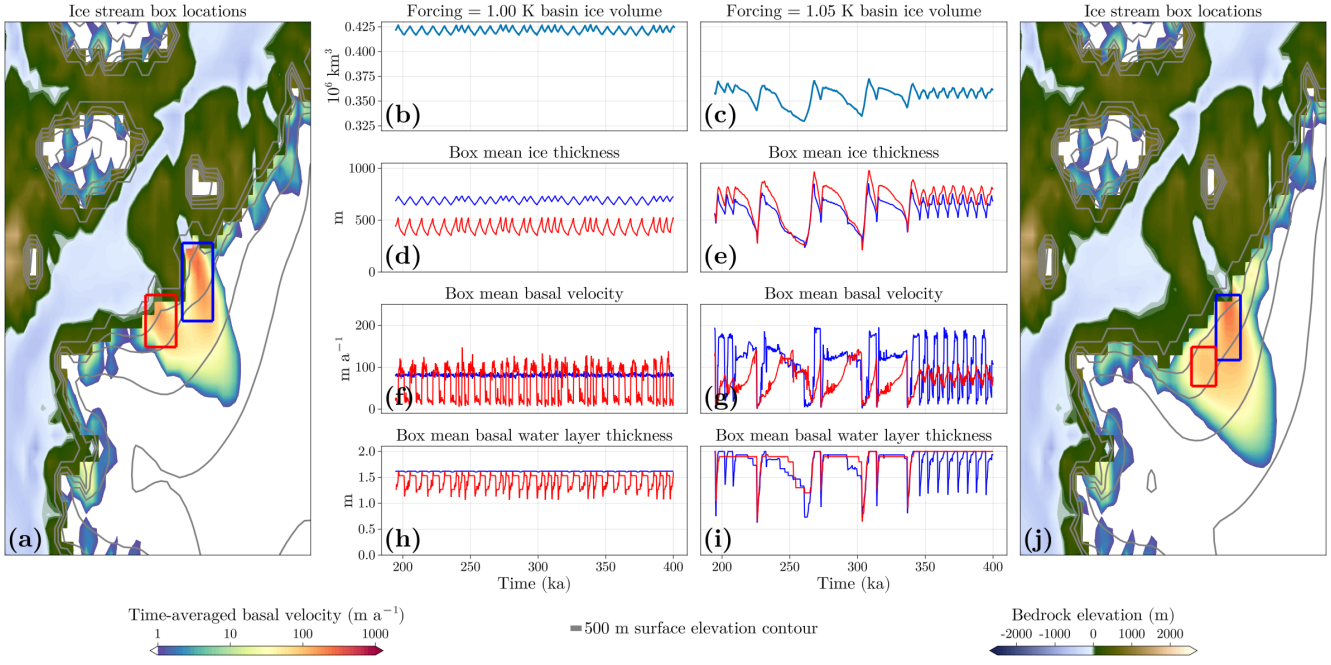

Figure 8. (a), (j): Location of the PIS/retreated PIS (blue) and HIS/retreated HIS (red) ice-stream boxes superimposed over the temporal average of the basal velocity during the oscillations of the unretreated (a) and retreated (j) configurations. (b-i): Time series of basin ice volume, mean ice thickness, basal velocity, and basal water content in the ice-stream boxes in the unretreated (d, b, f, h) and retreated (c, e, g, i) configurations.

in the last 60 ka of the time series in the panels c, e, g and i of Fig. 8. The basal velocity in retreated PIS abruptly switches between maximal and minimal, with corresponding drops in basal water-layer thickness when the velocity is near zero. In retreated HIS, the till remains saturated with water. However, the basal velocity and thereby the flow is not steady. It increases during the surge and decreases during the buildup of the nearby retreated PIS. This indicates an influence of the retreated PIS on the retreated HIS. The magnitude of ice-thickness change during this pattern is not as large as for build-up and surge variability, being around tens of metres per year in the spatial mean.

Thus the distinguishing factor between the two patterns is that during the short oscillations, retreated PIS is in a build-up/surge mode and retreated HIS has a constant till saturation associated with steady flow but experiences small oscillations due to the retreated PIS. On the other hand, both retreated HIS and retreated PIS are in the build-up/surge mode during the longer-period events. Additional to these modes, in some simulations with these oscillations there is occasionally a period where both retreated HIS and retreated PIS are in a steady flow state with intermediate mean basal velocities of around 100 m a^{-1}, with an associated minimum in the ice volume which may last tens of thousands of years.

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3.4 Removal of the oscillations

To asses whether the lack of predictability of the tipping is due to ice-stream oscillations or otherwise due to other factors, we change the parameterization of the ice streams to eliminate their ability to oscillate and investigate the consequences. As described previously, the ice-stream oscillations are surges in the basal velocity of the ice stream due to the thermomechanical coupling at the base of the ice sheet. To remove the oscillations but maintain the ice stream, the basal velocities in the region of interest need to be lower but non-zero. For lower basal velocities, the mass lost due to streaming is closer to the accumulation rate, bringing the ice stream to a steady flow state (Robel et al., 2013). To achieve this, the value of δ in Eq. 3 is increased. This raises the minimal effective pressure (Bueler and van Pelt, 2015) and thereby increases the basal frictional stress.

Simulations with the original value of δ = 0.02 as well as increasing values of δ to 0.04 and 0.10 are shown in Fig. 9. As δ is increased, we note an increase in the oscillatory period until the oscillations disappear completely. As this value affects ice streams across the entire ice sheet, the tipping points for each of these parameter values is different. Specifically, the tipping point increases as δ increases, as the ice sheet is losing less mass due to lower ice stream velocities.

For $\delta = 0.02$, all (no) simulations with the strongest (weakest) forcing tip, but for intermediate maximum forcing values the tipping times do not decrease monotonically with the forcing rates. For a large enough value of δ=0.1, the ice-stream oscillations disappear and the tipping time for a maximal forcing of +2.10 K occurs at approximately the same time for each rate. That is, the dependency on the forcing rates disappears and the tipping is much more predictable. This suggests that the ice-stream oscillations introduce a delay in the tipping.

4 Discussion

4.1 Ice-stream oscillations

Ice streams not only represent a mechanism of rapid mass loss in ice sheets, but have also been shown to be a source of internal periodic variability in models. Periodic behaviour of ice masses can be seen in glaciers that are confined to some topographical valley and/or have a basal slope, see for example Budd (1975); Kamb et al. (1985); Clarke (1987). Their reduced spatial extent also means their periodic behaviour is on a much shorter time scale of decades to centuries and thus directly observable. Models show that ice sheets and ice streams, similar to valley glaciers, can also exhibit oscillatory behaviour under the right conditions.

Studies of oscillatory behaviour in ice sheets include parameterized models (Oerlemans, 1983; Fowler and Johnson, 1996; Payne, 1995; Robel et al., 2013) and comprehensive ice-sheet models with both idealized geometries (Calov et al., 2010; Van Pelt and Oerlemans, 2012; Feldmann and Levermann, 2017) and realistic topographies (Papa et al., 2006; Roberts et al., 2016; Schannwell et al., 2023). Additionally, some studies include a coupling to additional components of the climate system (Calov et al., 2002; Ziemen et al., 2019). Similarities between the oscillations seen in this paper and those observed in paleoclimate modelling studies are discussed in Appendix A.

Common to these studies is the use of a single initial state for a given combination of parameters rather than an ensemble. The use of an ensemble of states for investigating the evolution of an ice-sheet model is not too common, appearing in the

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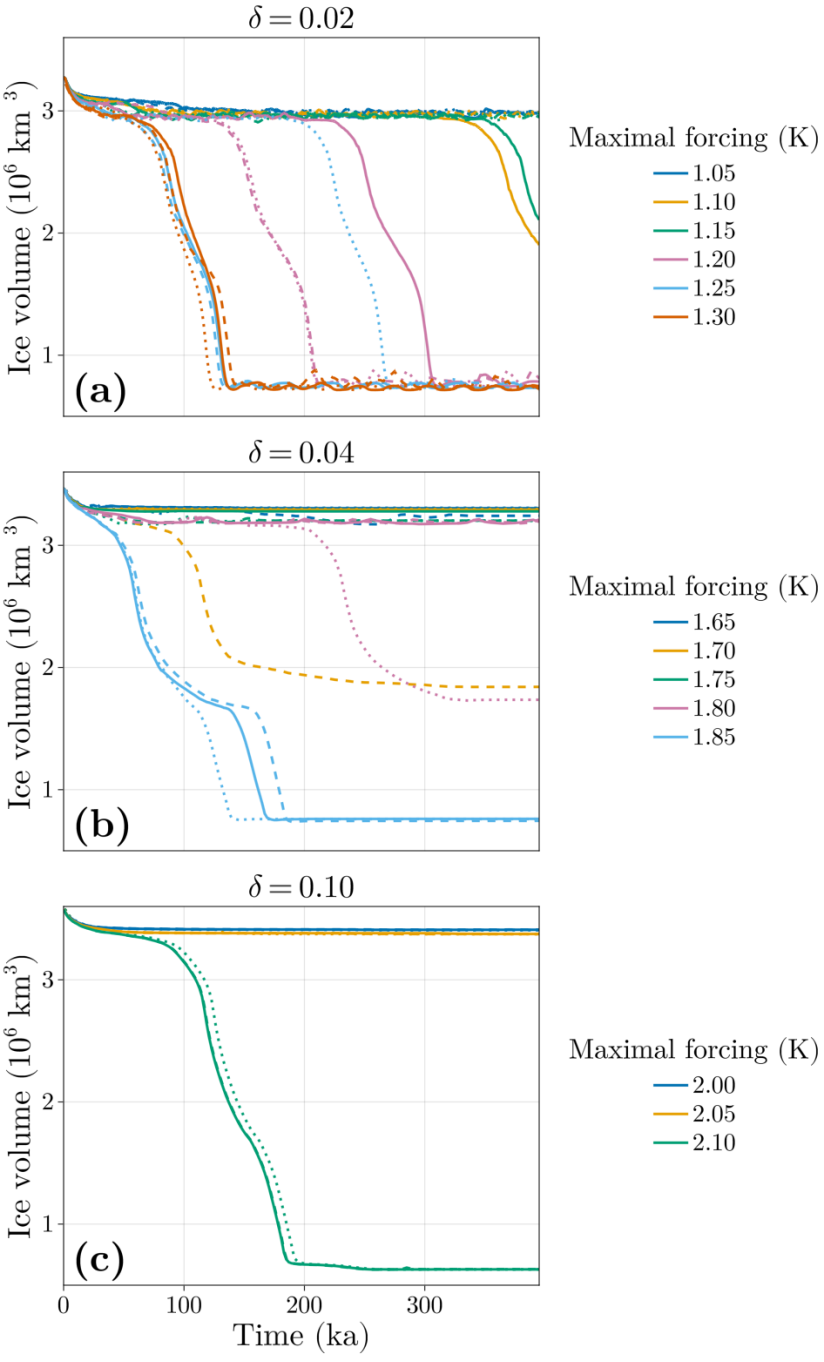

Figure 9. Time series of the ice sheet volume for increasing δ. For each value of δ, a range of forcing magnitudes at three rates starting from initial state A were applied: 10^{-1} (dotted lines), 10^{-2} (dashed lines) and 10^{-3} (solid lines) K a^{-1}.

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studies of Tsai et al. (2017) and Verjans et al. (2025) to be able to account for the variability of the atmosphere and ocean when forcing the ice sheet. This is distinct to the results of this study, where the model is not coupled to an external climate beyond a simple, deterministic diffusive energy and moisture balance atmosphere and thus the variability is internal to the ice sheet itself. As a result of using an ensemble, we see simulations at a given forcing magnitude exhibit significant deviations from each other, indicating a sensitive dependence on the initial state which is a hallmark of a chaotic mode of internal variability.

The chaotic variability has an undeniable effect on the tipping behaviour. In particular, the unpredictable tipping times can be explained as being due to *chaotic transients*, which are described in Appendix B. Whether the chaos is a genuine physical phenomenon or simply a result of parameterization is under contention and will require further investigation. As variability of this type has been reported in many other ice-sheet modelling studies, its existence is not entirely unfounded. What is unique to this study is the period and amplitude of the oscillations in Greenland, as well as its switching between two different modes due to a coupling of nearby ice streams. While oscillations in the GrIS ice volume have been reported in the modelling study of Zeitz et al. (2022), the period and amplitude of those oscillations are an order of magnitude higher than what is seen in the present work. This is due to the fact that glacial isostatic adjustment, which acts on a longer timescale than the melt-elevation feedback, plays a much larger role in their model.

4.2 R-tipping of the GrIS

Accelerating mass loss is typically seen in marine-terminating outlet glaciers due to their sensitivity to oceanic forcing (Howat et al., 2008; Krabill et al., 2004; Rignot and Kanagaratnam, 2006), suggesting rate-induced effects may be more prevalent in these areas. However, they are topographically confined and their impact may be limited under greater forcing (Joughin et al., 2010). For this reason, future loss due to negative surface mass balance forced by increasing atmospheric temperatures could outweigh that of ice-sheet dynamics (Bevis et al., 2019; Enderlin et al., 2014). The ice extent shown in Fig. 3 indicates that the mass loss does not start in regions with many marine-terminating outlet glaciers, specifically the southeast (Van Den Broeke et al., 2009). In fact, many remain after the tipping has completed. This indicates that the possibility of r-tipping of the GrIS primarily by oceanic forcing may not be relevant.

The remainder of the ice sheet interacts predominantly with the atmosphere, and the mass loss occurs either through surface melt or dynamically through ice streams which may not necessarily be marine-terminating. The question of whether the large-scale mass loss of the GrIS can be influenced by the sensitivity of these fast-moving ice streams to the rate of atmospheric warming is greatly obscured by the chaotic variability seen in the model. The simulations indicate that the tipping behaviour is affected by the rate of forcing but in a non-monotonic manner. There is no clear critical forcing rate for which r-tipping occurs, at least not for a rate faster than one Kelvin of warming over 100 ka. However, the long tipping times are not explained by non-monotonic r-tipping, as we would expect any tipping events to occur at roughly the same time at a given forcing magnitude. That is, it is valid to claim that r-tipping does indeed occur since the final state of the GrIS depends on the rate of forcing in some way, but it is not the typical r-tipping as understood in simple non-chaotic systems. It is rather related to sensitive dependence on an initial state rather than any critical rate of forcing, with the different rates of forcing essentially being perturbations of the initial state, resulting in trajectories with unpredictable behaviour.

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4.3 Removal of the oscillations

Removing the oscillations by increasing δ makes the tipping more predictable. At a value of $\delta = 0.1$, the tipping occurs at approximately the same time for both the fast and slow rates of forcing increase. This is in contrast to the simulations with $\delta = 0.02$, where oscillations are present and the tipping at a given maximal forcing can vary by tens to hundreds of thousands of years depending on the rate and in a non-montonic manner. This corroborates the hypothesis that the oscillations cause the delay in tipping. For an intermediate value of $\delta = 0.04$, tipping to an ice-free state only occurs for the largest forcing magnitude of +1.85 K. Interestingly, two simulations at lower forcing magnitudes see a tipping to an intermediate ice-sheet state. Such a state has been seen before in studies such as Ridley et al. (2010) and Robinson et al. (2012), but is not explored further in the present paper.

Increasing δ also reveals some interesting interplay between the parameterization that allows for the oscillations and the tipping. As increasing δ decreases the ice-stream velocity, the ice sheet loses less mass dynamically and thus the forcing magnitude required for tipping increases. However, the tipping is now no longer delayed by the oscillations. Since the lifetime of the transients can be upwards of 100 ka, it may be the case that the tipping occurs later for a lower forcing value. Thus the oscillations simultaneously serve to lower the value of the tipping point as well as to increase the time before tipping occurs.

5 Conclusions and future work

Setting out to identify whether the GrIS is susceptible to r-tipping, we performed warming experiments at different rates using a comprehensive ice-sheet model. In the course of this line of investigation, it was discovered that the coupled model exhibits a mode of variability that has heretofore not been observed in models of the GrIS. This is presumably because, at least for our model, they only exist for a very small range of external temperature forcing between +1.05 and +1.30 K.

This variability appears in the form of oscillations of ice streams in the present-day GrIS due to thermomechanical coupling at the base of the ice. Warming of the ice sheet causes an initial retreat of the ice extent in the northwest region, resulting in the Petermann and Humboldt glaciers entering a configuration where they experience build-up/surge variability. Due to their proximity, they influence each other and the resulting pattern is chaotic. Since the tipping of the ice sheet begins in the region where these ice streams are found, their presence delays the tipping of the ice sheet to an ice-free state.

The conclusions are limited by the amounts and types of simulations conducted. An obvious next step is to repeat experiments using a different grid size to observe the dependence, if any, of these oscillations on the model domain. A full investigation of the phase space and the basins of attraction in the parameter range around the tipping would give a much clearer picture on when the tipping may occur, or if there are multiple closer steady states before a larger tipping. Additionally, an edge-tracking algorithm (Lucarini and Bódai, 2017; Mehling et al., 2024; Börner et al., 2025) can be used to approximate the chaotic non-attracting set, even for a ghost attractor. This can be used to identify whether it is such a ghost attractor, or else a chaotic saddle.

In connection with other elements of the climate system, the oscillations themselves on a shorter time scale are worth studying. For example, they may represent a periodic freshwater forcing condition on the AMOC. If the retreated configuration

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where oscillations occur is considered as a separate state to the current ice-covered GrIS, r-tipping onto this attractor may be investigated.

The implications of chaotic transients on anthropogenic climate change in this context is phenomenological rather than sociologically relevant due to the long time scales. Long tipping times might erroneously suggest that the GrIS is stable, although it eventually tips when keeping the forcing parameter constant. On the other hand, these long transients allow for overshooting of the tipping point, whereafter the forcing parameter may still be reduced in time to prevent the tipping.

Code availability

The source code for Yelmo and REMBO can be found at https://github.com/palma-ice/yelmo and https://github.com/alex-robinson/rembo1, respectively.

Author contribution

KK: Conceptualization, Investigation, Visualization, Writing (original draft). **MM:** Conceptualization, Methodology, Software, Writing (review and editing) **AR:** Conceptualization, Methodology, Software, Writing (review and editing) **JA-S:** Conceptualization, Methodology, Software, Writing (review and editing) **JS-J:** Conceptualization, Methodology, Software, Writing (review and editing) **PD:** Conceptualization, Writing (review and editing)

Competing interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A: Comparison to oscillations of the LIS

Large-scale oscillations of ice streams were first proposed as the reason for Heinrich events (HEs) during the last glacial period (LGP) (Heinrich, 1988; Broecker et al., 1992; MacAyeal, 1993), but these events might be caused instead by external forcing (Alvarez-Solas et al., 2013; Bassis et al., 2017). While the variability of the oscillations seen in this study seems similar to the build-up/surge variability seen in most simulations of HEs in the Laurentide ice sheet (LIS) of the LGM (Calov et al., 2002; Papa et al., 2006; Roberts et al., 2016; Ziemen et al., 2019; Schannwell et al., 2023), they do not match exactly. Most notably, the Hudson ice stream in those studies displays a more gradual increase in ice volume followed by a sudden surge. This is in contrast to the pattern in the retreated HIS and retreated PIS in this experiment, where the build-up is either over the same time period (in the case of the short oscillations) or faster (in the case of the longer asymmetric events) than the subsequent surge. Additionally, in the latter case, the ice loss accelerates over time, rather than being maximal at the beginning of the surge. It may be the case that the variability seen in the GrIS, while having the same physical mechanism of thermo-mechanical coupling, may have a source additional of variability, i.e. the spatial interaction of the retreated PIS and retreated HIS, that causes it to behave differently from these experiments of a single oscillating ice stream.

Common to ice-sheet model simulations of the LIS are oscillations in ice-sheet volume that sometimes show quasiperiodicity or seemingly chaotic behaviour. It would be expected that, due to the large spatial extent of the system and the complex basal topography, the oscillations would not have a near-constant period. Even in the idealized geometry of Calov et al. (2010) there is spontaneous spatial asymmetry that leads to inconsistent oscillatory frequency. Such irregular variability is of special importance when studying the tipping behaviour of a system. Still, without an ensemble of simulations starting from similar initial conditions, it is unknown how irregular the variability seen in these studies is.

Appendix B: Link to nonlinear dynamics

B1 Chaotic transients

Systems that experience chaotic variability have long transients with lifetimes of indeterminate length, and these are hence called chaotic transients (Lai and Tél, 2011). Specifically, the lifetime of any chaotic transient depends sensitively on its initial condition, but the lifetimes of an ensemble are exponentially distributed. These chaotic transients are due to the existence of chaotic non-attracting sets, the most relevant being chaotic ‘ghost attractors’ and chaotic saddles. The distinction is important, as the ghost attractor exists in the monostable parameter regime, whereas the chaotic saddle exists in the bistable parameter regime.

A chaotic ghost attractor materializes as a result of a chaotic attractor undergoing a *boundary crisis* due to a continuously changing parameter (Grebogi et al., 1982), essentially a chaotic analogue of a saddle-node bifurcation. A trajectory that is forced beyond this crisis parameter value will remain around the ghost attractor for some time before eventually tipping, causing the chaotic transient. On the other hand, a chaotic saddle lies on the basin boundary that separates two co-existing attractors. It generates long-lived transients when a trajectory crosses this basin boundary, as solutions are captured by the

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attracting manifolds of the saddle before eventually approaching one of the two attractors. Such a phenomenon in the context of r-tipping has been previously reported in a study by Lohmann and Ditlevsen (2021) for the Atlantic Meridional Overturning Circulation (AMOC). In their study, trajectories forced at different rates are brought close to the basin boundary between the stable 'AMOC on' state and the chaotic saddle that separates it from the stable 'AMOC off' state. As the basin boundary is fractal, differing rates may cause it to tip or not in a non-monotonic way. For a chaotic saddle, the mean lifetime of chaotic transients scales with the fractal dimension of the saddle (Hunt et al., 1996; Mehling et al., 2024; Sweet and Ott, 2000).

Due to being a transient simulation, the AQEF itself may exhibit a chaotic transient. The biggest evidence for this is comparing the simulations ramped to +1.25 K and 1.30 K. If the oscillations are due to crossing a chaotic saddle before the tipping point of 1.275 K is reached (i.e those simulations for +1.25 K), then they must be qualitatively different from the ghost attractor generated by a boundary crisis after the boundary crisis (i.e those for +1.30 K), as the chaotic saddle only exists in the bistable parameter regime and the ghost attractor only exists in the monostable parameter regime. However, the trajectories at both forcing values occupy the same regions of phase space before tipping, implying that they are the same type of chaotic transient. This implies that the bifurcation does not occur between +1.25 K and 1.30 K. Similar reasoning can be applied to all parameter values that result in tipping. Thus, the bifurcation point is either between +1.05 K and 1.10 K and all transients are due to a ghost attractor, or the tipping point is larger than +1.30 K and all of the transients are due to an r-tipping through a chaotic saddle.

The behaviour of the chaotic transients in the present study more closely resembles those due to a boundary crisis than crossing a chaotic saddle. First, scaling laws indicate that the lifetime of the chaotic saddle, and thereby the tipping time, should increase as the maximal forcing approaches the bifurcation point from the left (Mehling et al., 2024). In contrast, the mean lifetime of the chaotic transients scales with the magnitude of the parameter past the crisis value (Grebogi et al., 1986),

$$\langle \tau \rangle \sim (p - p_c)^{-\gamma}, \tag{B1}$$

which mirrors the pattern seen in the tipping times in Fig. 5. Secondly, if r-tipping was present, we may expect to see tipping not occur below some critical rate.

B2 Transient lifetime for a forcing value of 1.05 K

The state of the system at a forcing level of +1.05 K is of interest since it is on the boundary between more predictable patterns and the irregular oscillations of the chaotic transients. None of the simulations at this forcing value tip to an ice free state after 400 ka of model time, although they exhibit the same variability as those that do tip. In this parameter range, the system may either be either on a chaotic attractor, or otherwise a chaotic transient with a lifetime much longer than 400 ka.

Using the mean lifetime of the simulations that tip to an ice-free state, the critical exponent in equation B1 can be estimated. Using a maximum likelihood estimation to fit them to an exponential distribution results in a critical exponent of around $\gamma \sim 9.959$. Using this, the mean tipping time for a trajectory with a maximal forcing of +1.05 K is around 511 ka, which is indeed longer than the 400 ka simulation run time.

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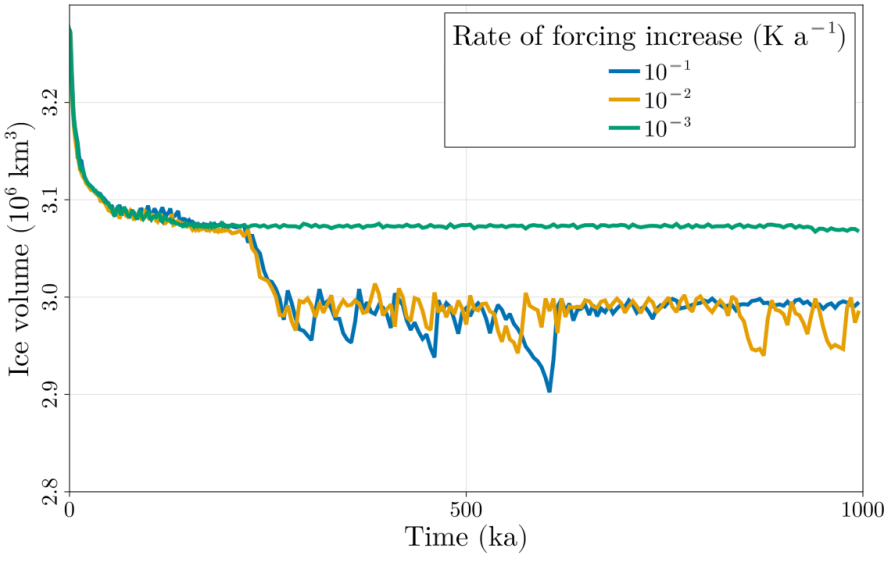

Figure B1. Simulations to 1 Ma for a maximal forcing of +1.05 K at three different rates. This simulation time is estimated to be longer than the mean lifetime of a chaotic transient at this forcing value. The temporal resolution of the output of these simulations for the final 800 ka is 5 ka, in comparison to the 200 a timestep the simulations in Figs. 5, 6 and 8.

A few additional simulations at this forcing level were done going to 1 Ma and are seen in Fig. B1. None of these simulations tip, suggesting that they are not chaotic transients but rather motion on a chaotic attractor, and the ice sheet has a genuine chaotic attractor at this parameter value. This further strengthens the argument that the chaotic transients are generated by a boundary crisis, as the resulting ghost attractor is qualitatively similar to the chaotic attractor that exists before the crisis Lai and Tél (2011).

B3 Intermediate tipping

Within the simulation ensemble, there are a few 'anomalous' runs that do not behave as expected. There are two such types: first, for a forcing level of +1.00 K, one simulation ends up in the retreated configuration with ice-volume variability similar to but slightly smaller in magnitude than those of larger forcing values, as seen in Fig. B2. This might suggest the chaotic attractor associated with the ice-stream oscillations also exists for lower forcing values, albeit with a smaller basin of attraction and thus it has a lower probability of being reached.

Second, there is a simulation with a maximal forcing of +1.15 K that remains in the unretreated configuration. That is, for larger forcing values the less chaotic attractor remains at a diminished size. The 1 Ma simulations for a maximal forcing of

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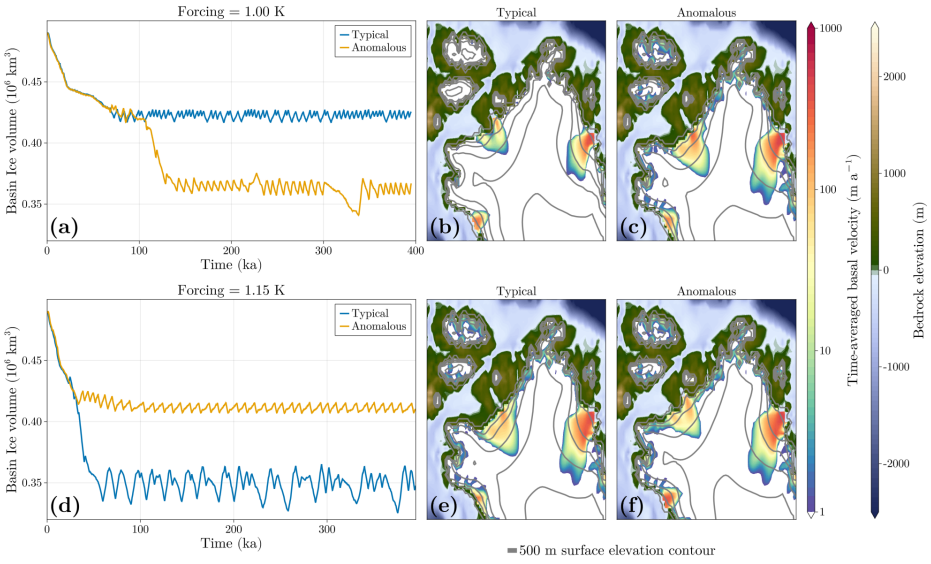

Figure B2. (a): Time series and mean ice thickness for typical and anomalous model trajectories at a forcing of +1.00. (b): Ice sheet extent and time-averaged basal velocity fields of a typical trajectory. (c): Ice sheet extent and time-averaged basal velocity fields of the anomalous trajectory. (d-f): Same as (a-c) but for a forcing of +1.15 K.

+1.05 K in Fig. B1 also show the unretreated configuration is possible at this forcing value. This represents a complication in the analysis, since this parameter value is assumed to be associated the chaotic attractor or its ghost.

In this case, the structure of the attractors may be that there is intermediate tipping similar to Lohmann et al. (2024). This would imply that around a forcing value of +1.00 to +1.05 K, there are two attractors for the ice-covered state, corresponding to the retreated and unretreated configurations. The attractor for the retreated GrIS experiences a boundary crisis between +1.05 and +1.10 K, with corresponding chaotic transients remaining on the associated ghost attractor. On the other hand, the attractor of the unretreated GrIS experiences a bifurcation at a forcing value slightly larger than +1.15 K. This scenario could then have genuine r-tipping onto either the unretreated or retreated configurations, the latter of which experiences an earlier tipping to an ice-free state, resulting in an indirect r-tipping to the ice-free state. The basin boundary between the two ice-covered attractors could be fractal, leading to nearby initial conditions approaching one or the other (McDonald et al., 1985).

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