

## REVIEW egusphere-2025-4098 (2026-03-10)

The authors thoughtfully considered the reviewers comments and overall performed the required adjustments to the manuscript. This improved the manuscript considerably, particularly in terms of structure and clarity.

I only have a few small remaining comments that I would suggest considering before finally accepting the manuscript.

The authors thank the reviewer for the thorough evaluation of our manuscript. We have revised the manuscript accordingly to improve the clarity and robustness of the study. Below, we provide point-by-point responses to the reviewer's comments.

### Small comments:

1. 1.172f: Can you add an explicit equation or reference for this calculation?

The ideal gas law is used to calculate dry air density. We have revised the related sentence which now reads as follows:

“It is noted that a new routine (recipe) has been added to VADER to derive dry air density from air temperature, pressure, and the specific gas constant for the dry air using the ideal gas law. This is applied in cases where the dry air density is not otherwise provided for the PM<sub>2.5</sub> calculation.”

The routine could be found here:

[https://github.com/JCSDA/vader/blob/develop/src/vader/recipes/DryAirDensity\\_A.cc](https://github.com/JCSDA/vader/blob/develop/src/vader/recipes/DryAirDensity_A.cc)

2. 1.256f: As already mentioned in the first review (smaller comment 4, 2nd sub-point): This sentence provides a lot of redundant information with the bullet points in 1.253f. Suggest removing bullet points or shorten text.

We removed the bullet points in the revised manuscript, and rephased the paragraph accordingly.

3. 1.300: Can you add the estimated value for  $s$  for this study?

The value of  $s$  used in the three-hourly cycling experiments is  $\sim 0.81$ . Additional information can be seen in Figure 2, which shows the domain-averaged PM<sub>2.5</sub> error standard deviations for the data assimilation run that assimilated both AirNow and PurpleAir PM<sub>2.5</sub>. Note that the three-week experimental period covers extreme wildfire events, so these values may not be representative for other applications. The value could also be day- or cycle-dependent, although this was not applied in the current study.

4. 1.326f: Related to the authors reply to smaller comment 8 from the first review: Thanks for the information. From your reply I understand that the cutoff scale is related to the background error correlations. Please add this information in the manuscript and specify the resulting correlation lengths of this study, if possible.

In the revised manuscript, we added statements clarifying that the cutoff scales are input parameters and describing them explicitly. The paragraph now reads as follows:

“The background error correlation matrix  $C$  is modeled using a generic diffusion correlation operator designed for short length scales, as implemented in the System-Agnostic Background Error Representation (SABER) repository (Sluka, 2024). The primary input parameters are the

horizontal and vertical cutoff length scales, defined as the distances beyond which correlations are zero.”

5. 1.448: I don't understand what you are referring to with "error patterns" of DA\_AN and DA\_PA. Fig.7d, 7f show the their deviations from CTR, right? Do you mean "deviation patterns" or "analysis increments". Please correct or clarify in the manuscript.

Thanks for catching this. We mean the spatial pattern of large scale adjustments (deviation from the control). The sentence now reads as follows:

“They also produce comparable large-scale adjustments across the Northeast, Midwest, and Southern U.S., with their spatial error patterns (Fig. 7d and 7f) largely opposite in sign to those in the CTR–AirNow difference (Fig. 7b). ”

#### **Technical and formulation-related comments:**

- 1.35: “)” missing after “CONUS”.  
Fixed.
- 1.80: A paper preprint related to Vogel et al.2025 is now available online (under review in NPG): <https://doi.org/10.5194/egusphere-2025-6386>  
Thank you for pointing this out.We have replaced the previous citation with the preprint of the above paper.
- 1.229: “and correlation” it looks like this was moved to the subsequent subsection, suggest removing from section title  
Accepted.
- 1.293: You mean “equation 8”?  
You are correct. We fixed the typo in the revised manuscript.