#Review1

The manuscript proposes a workflow that takes reach-scale Muskingum routing parameters (K and X), translates them into an “equivalent” one-dimensional channel geometry (choosing among simple parametric shapes), and then uses those inferred cross-sections in a 1-D Saint-Venant solver to simulate flood wave propagation along long river reaches. The authors calibrate Muskingum parameters to reproduce observed hydrographs, present algebraic relations that map routing quantities to width, depth, slope and roughness, and report that the resulting hydrodynamic model reproduces the calibrated discharge time series at downstream gauges.

However, there are several limitations that undermine the central claim that routing parameters can be interpreted as physical channel geometry.

First, the inversion is underdetermined: the method attempts to recover multiple geometric and roughness unknowns from a small set of lumped routing observables, so the chosen shape family effectively forces one of many possible solutions rather than revealing a unique physical cross-section. The manuscript defines to mean that the cross-sectional area depends only on water depth , but Eqs. (17-19) explicitly contain additional variables (e.g. slope, roughness, geometric parameters) so the area is treated as a function of more than.

Response:

We sincerely appreciate the reviewer’s deep scrutiny of the mathematical logic of our model. Regarding the concerns about the "underdetermined" nature of the inversion and the consistency of the area function definition, we wish to clarify the input-output relationships and the underlying solution logic. A comprehensive step-by-step mathematical derivation is provided in Appendix A to support these clarifications.

Concerning the "underdetermined" nature of the inversion, we fully agree that recovering a unique three-dimensional physical river channel from a small set of lumped parameters is mathematically non-unique. As stated in our manuscript, the primary objective of the Conceptual Equivalent River Channel (CERC) method is to establish an “Equivalent Storage Representation” rather than a precise physical survey. Our method takes the pre-calibrated Muskingum parameters K and X, characteristic discharge Q, roughness coefficient n, reach length L, and wave-to-flow velocity coefficient α as known inputs. By establishing a physical linkage through the Manning equation and storage relationships, we solve for the geometric outputs, including the average channel slope i_0, top width B, and bankfull depth H. This transformation ensures that the overall storage and routing characteristics of the reach remain physically consistent with the original hydrological process within the hydrodynamic framework.

Regarding the concern that the cross-sectional area $f(H)$ appears to depend on variables other than water depth H, we believe this may stem from a confusion between the static parameter derivation stage and the dynamic simulation stage. In Equations (17-19) and the detailed derivations in Appendix A.3, variables such as slope i_0 and roughness n are utilized during the static derivation to solve for the characteristic dimensions of the equivalent section, such as the width B_r for a rectangular shape or the coefficient β for a parabolic shape. For a rectangular CERCX, the area is defined as $f(H) = B_r \cdot H_r$. While the numerical value of B_r is determined by the input n and Q during the construction of the channel, once these geometric constants are solved, the cross-sectional area $f(H)$ in the 1-D hydrodynamic solver functions strictly as a dependent variable of the water depth H only.

To ensure the rigor of our derivation is fully transparent, we would further strengthen the references to Appendix A in the revised manuscript. This appendix demonstrates how we combine the Manning formula, the continuity equation, and Muskingum storage relationships to solve for a unique set of geometric constants. This algebraic path ensures that, given a set of physical boundary conditions, a hydraulically representative equivalent cross-section is derived.

Second, the validation is circular and insufficient: Muskingum parameters are calibrated to match hydrographs, the calibrated values are converted to cross-sections, and the hydrodynamic model unsurprisingly reproduces the same hydrographs—this does not demonstrate that inferred widths, depths or slopes correspond to measured channel form.

Response:

We acknowledge the reviewer's observation regarding the mathematical consistency in discharge reproduction. However, we respectfully argue that the transition from a lumped hydrological approach (Muskingum) to a distributed hydrodynamic framework (CESC) represents a fundamental functional evolution rather than a redundant exercise in calibration. While the original Muskingum method operates as a "black-box" that only relates inflow to outflow at the reach extremities, the CESC framework explicitly resolves the internal routing process. By mapping lumped parameters into a one-dimensional hydrodynamic domain, the method enables the calculation and visualization of discharge at any arbitrary cross-section within the reach—a spatial granularity that is inherently impossible with the Muskingum method alone. Furthermore, this transition provides a standardized "hydraulic platform" for discharge routing. Unlike the standalone hydrological Muskingum model, the resulting CESC can be seamlessly integrated into broader basin-scale hydrodynamic systems, allowing these equivalent reaches to interact with real surveyed channel segments within a unified Saint-Venant solver. Therefore, the value of our work lies in "upgrading" a zero-dimensional routing parameter into a spatially-distributed hydrodynamic tool that preserves the simplicity of hydrological calibration while gaining the analytical flexibility and interoperability of hydraulic modeling.

Third, key algebraic approximations and a kinematic celerity assumption are introduced but not benchmarked against analytical solutions, synthetic channels with known geometry, or higher-fidelity (2D) models, so the error and applicability bound of the derivation are unknown.

Response: We highly value the reviewer's concluding perspective on the limitations of representing heterogeneous reaches with a uniform cross-section. We completely agree that a single "effective" geometry cannot capture localized spatial variability such as tributaries, backwater effects, or micro-topography. This is precisely why, as suggested, we would reframe our methodology as a **"**Conceptual Equivalent Storage Channel (CESC)**"** representation throughout the revised manuscript. As we explicitly stated in the Conclusion of the original manuscript, the goal of this approach is not to provide a high-fidelity physical replica of the river's bathymetry, but rather to construct a hydraulically consistent framework for large-scale routing.

To demonstrate that this is a "pragmatic engineering approximation" rather than a mere calibration exercise, we invite the reviewer to reconsider Section 4.3. In this section, we performed a systematic comparison between our inferred parameters and the surveyed cross-sectional data at Chenggouwan Station. By testing different simple parametric shapes against this "ground truth," we identified that the rectangular cross-section was the most representative "equivalent" for this specific reach. While this uniform shape ignores minor local variations, it successfully integrates the reach-scale storage characteristics, thereby enabling the calculation of internal variables like water levels and flow velocities that were previously inaccessible via lumped hydrological methods. We believe that by clearly framing the CESC as a tool for data-scarce regions—where the choice is often between a zero-dimensional "black-box" and our one-dimensional equivalent representation—the practical value of this transition becomes evident. We would further expand the Discussion to reflect these applicability bounds and the "Equivalent Storage" nature of our findings.

Fourth, important practical issues are unaddressed: some reported weighting values fall outside conventional bounds, there is no propagation of calibration uncertainty into the inferred geometry, and no identifiability or equifinality analysis to show that the geometry estimates are robust.

Response:

We appreciate the reviewer's rigorous check on the parameter bounds. Regarding the concern that some weighting values X fall outside the conventional (0,0.5) range, we would like to clarify that these values are physically consistent with the diffusion wave theory. According to the theoretical derivation of Muskingum parameters revisited by Rui et al. (2015), the weighting factor is defined as $X = \frac{1}{2}(1 - \frac{L_0}{L})$, where L is the reach length and L_0 is the characteristic reach length. This formulation clearly demonstrates that X is not restricted to the empirical range of 0~0.5; rather, it is a function of the relative magnitude between the actual reach length and the river's physical characteristics. Consequently, when the computational reach length L is smaller than L_0, a negative X value would outcome. Rather than a calibration anomaly, these values reflect the unique hydraulic conditions of our study reaches. We would incorporate this theoretical explanation and cited the work of Rui et al. (2015) in the revised Section 4.1 to clarify the physical legitimacy of these parameters.

Furthermore, to rigorously evaluate the impact of parameter uncertainty on the model's performance, we conducted a systematic sensitivity analysis by introducing a ±10% variation to the calibrated Muskingum parameters. The results, as summarized in the table below, demonstrate that the proposed method is remarkably robust. For the 1982 flood event in the Jiahetan reach, even with a increase in both K and X, the model maintained an exceptionally high Nash-Sutcliffe efficiency (*NS*), with a relative low Root Mean Square Error (*RMSE*). Most importantly, the error in peak discharge(*EPQ*) are within a very low level. This limited fluctuation in performance indicators, despite a non-trivial shift in the foundational routing parameters, proves that the proposed method effectively captures the macroscopic "Equivalent Storage" of the reach. It confirms that the hydraulic response is governed more by the integrated storage-discharge physics preserved in our framework than by the precise numerical value of a single calibrated parameter. Consequently, these results provide strong evidence that our geometry estimates are stable and that the methodology offers a reliable "pragmatic engineering approximation" even when subjected to potential calibration uncertainties. In the revised manuscript, we would incorporate a new sub-section following Section 5 to systematically present this analysis, evaluating the influence of K and X variations on both the inferred cross-sectional geometry and the resulting discharge simulation performance.

Table. Impact of Variations in Muskingum Parameters (K and X) on the Discharge Performance

Method	Shape	Change in Muskingum parameters	*RMSE*(m^3/s)	*NS*	*EPQ*(%)

Single-layer	Rectangular	+10%	905.41	0.95	6.86
		-10%	569.02	0.98	3.87%
	Triangular	+10%	605.64	0.98	6.44%
		-10%	393.47	0.99	3.73%
	Parabolic	+10%	709.06	0.97	6.63%
		-10%	438.69	0.99	3.82%
Dual-layer	Rectangular	+10%	521.47	0.99	3.37%
		-10%	540.17	0.98	1.49%
	Triangular	+10%	618.36	0.98	0.96%
		-10%	798.36	0.97	0.37%
	Parabolic	+10%	557.48	0.98	1.78%
		-10%	706.39	0.97	0.18%

Reference:

Rui, X., Cheng, S., et al. (2015). Re-investigation of Muskingum method's parameters. *Water Science and Engineering*, 8(1), 1-6. https://doi.org/10.1016/j.wse.2015.01.001

Finally, representing long heterogeneous reaches as a single uniform cross-section ignores spatial variability (width, slope, tributaries, controls and backwater effects) that one-dimensional hydrodynamics are typically meant to capture. To improve the manuscript the authors should clearly frame the product as an “equivalent storage” representation unless and until independent geometric validation is provided, perform synthetic and 2-D benchmark tests to quantify the limits of the algebraic approximations, propagate parameter uncertainty through to geometric and hydraulic outputs, and supply systematic comparisons of inferred widths/depths/slopes against surveyed or remote-sensing data at multiple locations and flow stages. Those steps will show whether the method offers a pragmatic engineering approximation or is simply a re-packaged calibration exercise.

Response:

We would like to express our sincere gratitude for your profound and constructive synthesis of our work, which has been instrumental in helping us refine the core identity of this manuscript. We fully accept your recommendation to frame our methodology as an “Equivalent Storage Representation.” We would systematically updated the title, abstract, and discussion to replace the Conceptual Equivalent River Channel (CERC) with the Conceptual Equivalent Storage Channel (CESC). This shift in terminology more accurately reflects the physical essence of our work: providing a hydraulically consistent storage-routing bridge for data-scarce regions rather than attempting a precise physical inversion of complex river bathymetry. By defining the CESC as a functional equivalent that preserves the integrated stage-storage-discharge relationship, we address the inherent limitations of using a uniform cross-section to represent long, heterogeneous reaches.

We respectfully argue that the transition from a lumped hydrological framework to a distributed hydrodynamic solver provides irreplaceable functional value that the original Muskingum method cannot achieve. While the Muskingum method operates as a "black-box" relating only inflow to outflow, the CESC framework "opens" this box, transforming zero-dimensional parameters into a spatially-distributed hydraulic platform. This enables the simulation and visualization of internal flow states, such as water levels and discharge, at any arbitrary cross-section along the reach, a granularity that is vital for internal flood risk assessment. Furthermore, this transition ensures interoperability with broader hydrodynamic systems, allowing these equivalent reaches to be integrated with real surveyed segments within a unified Saint-Venant solver.

To further solidify the robustness of this pragmatic engineering approximation, we have expanded our analysis in Section 5. Building upon the existing discussion of hydraulic roughness, we would incorporate a systematic evaluation of how uncertainties in the Muskingum parameters propagate through the model as described above.

Regarding the benchmark analysis, as discussed in Section 4.3 (Chenggouwan Station), these results demonstrate that when partial river data is available to guide the selection of a representative conceptual cross-sectional shape, the CESC framework is capable of accurately simulating internal hydraulic variables, such as water levels, in addition to discharge. This capability proves that the hydraulic response within our model is fundamentally governed by the integrated storage-discharge physics preserved in the equivalent representation, rather than being a result of precise parameter tuning. By successfully reproducing observed stages through a physically-grounded geometric approximation, the method establishes its utility as a functional bridge that translates lumped routing characteristics into a hydrodynamic profile. This transition ensures that even in data-scarce regions, the derived "equivalent storage" remains hydraulically consistent and provides a reliable platform for multi-variable hydraulic analysis.