

The Cryosphere: Reply to Reviewer 2

“Simulating landfast sea ice breakage due to ocean eddies using a discrete element mode”

Comments Part 2

Overall comments

A very interesting paper that presents an analytic model for the breaking of an ice floe under uniaxial tension. The paper uses a bonded particle model under idealized conditions to validate the analytic model. Next, the authors use MODIS imagery to generate initial conditions to study landfast ice with the bonded particle model with ocean forcing via a QG model. They run this model for 48 days to show that the breakage properties depend upon the deformation radius and bond strength. The paper represents a significant contribution to our knowledge of the ice dynamics in the context of landfast ice, but requires a major revision. The paper also has numerous typographical errors that require correction.

Thank you very much for your valuable comments. They have allowed us to make additional improvements to clarify several points of the paper. We address the revisions below in blue:

Major comments/concerns

1. Critical fracture force.

The authors use a method in which the bond is broken when the absolute value of the normal force exceeds a critical value which implies there would be the same threshold for both compression and tension. The author notes that they disable compressive failures, however it is well known that compressive failures are important to ice dynamics. However as authors note ice is much stronger under compression than tension, so they will need to update equation (3) and the their breakage criteria to reflect this.

To clarify, the bonds in our simulations may break under tension, shear and compression. What we had previously reported as tensile failure included both tension and compression, but the number of bonds breaking due to compression was rather small compared to tension. We now make the distinction between these modes of failure more clearly in the updated manuscript. Note that we now also use different values for the bond strength (tension strength is much less than compression and shear strength, which for simplicity were made to be equal), we get the following approximate proportions for bond breakage: 97-99% in tension, <1% in compression and <2% shear. The ratio between these compressive to tensile strengths is about 4-10, being in line with literature (West et al. 2022).

2. Dominant Balance Assumption

In section 3.1 the authors say for strong enough oceanic forcing, we may assume that the magnitude of the drag force is much larger than the bond force such that the bond force can be ignored in the momentum balance. The authors never give a justification for this, and I wonder how strong the ocean forcing would have to be. When the authors run the models to compare their analytic solution, the bond stiffness they use is so large that given the values they present in the tables at the strongest ocean forcing of 1m/s the bonds would stretch with a relative $1/10^{\text{th}}$ of a millimeter for a 500m radius disk to have the bond

and drag forces in balance with each other. For the more realistic forcing the relative displacement is order of microns.

Thank you for bringing this to our attention. The main reason we can ignore the net bond force in the momentum equation of the idealized system described in Section 3 is that each floe experiences two bond forces, due to the two floes it is connected with. For a smooth enough divergent oceanic forcing, these two forces are almost equal and opposite, such that they cancel each other out in the momentum balance. For typical parameter values, we have indeed verified that the drag force is considerably larger than the net bond force. This is consistent with the physical intuition that the ocean is driving the motion of the ice, while the net bond force provides some resistance to this motion, but its magnitude cannot exceed the magnitude of the driving force, by definition.

We also note that we have adapted the DEM parameters including stiffness and critical tensile strength to be in line with past literature (Damsgaard et al., 2018; West et al., 2022), such that breakage typically occurs at a critical strain of approximately $1e-3$. For our bond radius of 1000m, this corresponds to a critical displacement of approximately 1m. The typical sea ice velocities before breakage are on the order of 0.08-0.12 m/s with a gradient of $1e-4$ 1/s, such that the characteristic breakage time is about 20 s. Given our time step of 0.0125 s, the breakage occurs within 1600 steps, so we can appropriately resolve this process in our simulations.

The assumption of much larger bond force comes from considering the bond to be completely unstrained before applying the drag force. So, before the bond force starts acting, the drag force is significantly larger. Of course, this can be a relatively short interval, but this simplification is done to arrive at an approximate analytical expression that can be better explored. Otherwise, we would be required to do the non-linear solving and dynamic equilibration of the system, leading to analytically intractable expressions, which would prevent clear analysis. We have also modified particle stiffness to 500-5000 MPa based on values in the literature (Damsgaard et al., 2018; West et al., 2022). Given that the material is brittle, the stretch will still be small before breakage occurs regardless, but not as small given the updated bond stiffness value. We have now considered a critical strain of $1e-3$ with respect to a bond radius of 1000 m, to avoid having extremely small displacements. Our critical displacement would now be 1 m instead.

3. Numerical Stability

The authors use very spring stiffnesses, especially in the eddy model. These stiffnesses are accompanied by a large timestep 10s. At these large timesteps, I worry that the authors are not fully resolving the waves propagating through their DEM.

Thank you for your observation; we have addressed this concern in the reply to Review 1. We have now rerun the simulations with the updated parameters ($E = 500-5000$ MPa, $k_{\text{bond}} = 12.5-125$ MN/m, bond strength = 10-40 kPa) and a time step $dt = 0.0125-0.09$ s. Due to the high variability of sea ice strength and the low gradient for the ocean current, we use sea ice critical strength in the lower range of the literature. The timestep choice and scheme are within the stability limit according to the CFL condition for DEM (Damsgaard et al. 2018).

4. Eddy damping scheme

Y^{edge} is chosen by the authors to be 30km and is shown in figure 7. The authors state that the results are not sensitive to the definition of it. However figure 7 seems to show breakage in many of the figures along this line that is plotted. So which results are the author saying do not depend on this definition?

We have modified the attenuation scheme for ocean currents to take account of the local sea ice edge location rather than the mean sea edge. We believe this scheme is more realistic than what was used previously. We detail this in Section 5.1. This comment has been removed from the new version.

We agree with the reviewer's comment. In the previous version of the manuscript, the attenuation scheme depended on the mean sea ice edge location. Since our ice edge is irregular, some parts of the pack did not experience much attenuation and could break easily with or without the attenuation, which likely reduced somewhat the sensitivity of the subsequent breakage behavior to that parameter. Upon reflection, we believe that it is likely more reasonable to base our attenuation scheme on the local rather than the mean sea ice edge. The new simulations include this new scheme. We have also conducted some experiments with different attenuation values and indeed find sensitivity to that parameter. This prescribed attenuation scheme is certainly an idealization, and a fully coupled model would be required to accurately account for it. We mention this in the discussion section.

Minor comments and suggestions

Abstract

Line 4: Maybe want to indicate here or somewhere else that the LS stands for level set

Line 7: Power law exponent. Only slope when plotted on a log-log plot

For Line 4: Will provide full name "Level Set Discrete Element Model"

For line 7: Agreed, will use the term power law exponent to provide a more general description of the FSD characterization

Introduction

Line 28: You introduce FSD in abstract so not sure you need to define it again here.

Given that the definition is in the abstract, we will remove the full definition here.

Line 41: ", than waves that affect smaller scales..." should be "than **by** waves"

Change done.

Line 54: I'm not sure geometry is the best choice of words here

We have changed the word geometry to concentration.

Line 69: "break mode and the evolution of the pack." Missing comma "break mode, and the evolution of the pack."

Change done.

The bonded particle method for sea ice

Fig. 2: I would consider moving figure 2b to 2c since the reference to it comes so much later.

Figures 2b and 2c have been flipped as indicated.

Line 95: How are you calculating ocean velocity when the centroid of the ice does not exactly match up with the ocean grid?

When the centroid of the ice does not exactly match up with the ocean grid, the nearest ocean grid cell center to the centroid is set as the ocean velocity. Bilinear interpolation was tried and results are the same, but performance is slowed down, so the nearest grid cell center is used.

Uniaxial bond force

General comment for this section: Missing subscript i on many of these equations

Eq 7: Does γ represent anything physically or was this just a mathematical convenience?

Use of subscript in this section was inspected to keep consistent notation. γ is just a constant of material properties used for notational convenience.

Eq 8: I feel like the symbol ϵ is typically used to represent strain, where here this is a displacement. So not sure this is the best choice of a symbol in the context Eq 8: I know for your tests you run later the ocean is set to a specific velocity, but can we generally assume that it is independent of time like you did here?

We can assume it is independent of time. Since we do not use strain in other parts of the document and we clearly indicate that ϵ_i means displacement with respect to initial position for floe " i ", it should not be a problem to use this notation.

Breakage length scale under a pulse forcing

Line 146: Say here that $x=0$ is located at the center of the bond

Change done.

Eq 12: Missing $\frac{1}{4}$ in the x^c equation, but appears to have been accounted for Eq 13. Also feels strange that this is dependent upon t . So this integration starts from the moment the forcing is applied to move it out of an unstretched neutral position?

Equation 12 has been corrected to match Equations 11 and 13. As you just indicated, the use of " t " (or $t_{\text{current}} - t_0$) represents the time elapsed since ocean current forcing has been applied to the ice from rest or neutral position.

Eq 13: Looking at fig 3a, the two parts of the bond force greater than F^c are not continuous, but it seems here are adding them together?

Total breakage length (trying to relate it to total area) is the summation of both segments where F_{bond} exceeds F_c , even if both parts are not continuous, this has now been clarified in the text.

Fig 3b: Something is off with this plot. l^{br} should equal 0 at λ_{cut} . The way λ_{cut} is defined, Eq 13 should have an $\arcsin(1)$, which means the right hand side should equal 0.

We have found a plotting issue with $\arcsin$. It has now been identified and solved and now a new Figure 3b. has been added to the paper.

Setup

Line 182: How frequently are bonds evaluated to see if they exceed the critical fracture value? This will impact x^c from your analytic solutions for validation. Any significance to 11 hours and how this value was chosen for the duration of the simulation?

Bonds are evaluated at every time step. Hence, coarsening of time does not affect these results. The value of 11 hours was chosen as a convenient number of timesteps to complete the smaller simulations using less computational resources and is a sufficiently long time to reflect eddy formation in ocean currents. Given the changes in material properties the time for breakage has been changed to 70 seconds.

Line 184: The way you say you evaluate the simulated breakage length, would this include the central $2x^c$ value shown in figure 3a?

Yes, breakage length is based on the total length across the sea ice pack, considering the discontinuity of l_{br} . This has been further clarified in Section 4.

Table 1: include values for ice and ocean density.

Sea ice and salt water density have been added to Table 1.

Results

Line 188-194: what's the diffuse ocean forcing? 100km or 200km because it is not consistent between the text and the figure.

Typo in line 190, it has been changed to $\lambda = 200$ km as indicated in Figure 4b.

Fig 4: A video of this simulation would be nice to have in supplemental material. This will allow us to see any leads form and how fractures appear. I assume we would see the fractures appear at first as we see this is the highest bond force in equation 11. I am curious if we see $\pm$ a stress/8 wave propagate out from when the fractures happen. It is this wave of stress in spaghetti that causes it to fracture in multiple places and this is reminiscent of that process.

Videos for simulations of Section 4 and Section 5 have been added to an open-source online repository for viewing added at the end of the paper. It is possible to observe how bond stresses start building up, leading to breakage and following the Feynman spaghetti patterns depending on λ . Lead formation from Section 5 simulations resembles satellite observations.

Fig 5: Given there seems to be issues with figure 3, check to see if analytical solutions

Figure 5 has been updated to confirm the right analytical solution is being plotted.

Eddying current properties

239-240: Are these run to quasi-equilibrium with the damping in place or is that added after?

Ocean currents are generated with a quasi-geostrophic model first. Then attenuation due to ice presence is added, as there is not a fully resolved ocean-ice dual coupling.

Results

Fig 8: A video of this simulation would be nice to have in supplemental material. This will allow us to see any leads form and how fractures appear. Since we see the large changes to the ice area in 8a, it would be good to see where these losses appear.

Videos for simulations of Section 4 and Section 5 have been added to an open-source online repository for viewing.

Fig 8c-d: These are never specifically mentioned in the text though you definitely talk about them. So put these references in.

Figures are now referenced in the main text.

Fig 8 caption: Cite the paper with the method used for calculating FSD since there are different methods.

Paper to calculate the FSD coefficient has been added to the caption of Figure 8 (Virkar and Clausen, 2014).

Line 293: What is the bin for your smallest FSD? Since we see fracturing down to the smallest grain, I worry that we might see some edge effects in the power-law you find if you include single grains since those are no longer able to fracture into smaller ones.

Single grains were not factored to calculate the FSD, but considered as fines or slurry. To avoid edge effects, the smallest size of the FSD was set to bonded clusters of at least 4 or more individual grains. Thus, the minimum bin size used was about 3 km in diameter.

Code and data availability:

Line 340: Do you need a zenodo DOI or anything like that?

We have now added a Zenodo DOI to link videos of simulations.