Review of **"Implementation of a Multi-resolution Analysis Method to Characterize Multi-Scale Wave Structures in Lidar Data"** by Samuel Trémoulu et al. (2025)

General comments

The authors apply multi-resolution analysis (MRA) using the 8th-order Daubechies wavelet to lidar measurements of the middle atmosphere at La Réunion. While MRA was originally developed more than three decades ago, its application to lidar data in the present form is novel. The authors compare MRA-filtering to several commonly used filtering techniques:

- (A) nightly mean subtraction
- (B) polynomial fitting in the vertical
- (C) spectral filtering in the vertical
- (D) a variance method

This comparison is, in principle, of interest. However, the study lacks a clearly defined benchmark or metric that allows a systematic evaluation of the different approaches. The manuscript states that it "presents a method based on MRA to characterize multi-scale GWs in observational data" and that "in section 3, the characteristics as well as the comparative performance of the four methods are discussed." Yet, the overall aim remains ambiguous. What exactly is being characterized? Is it GW amplitudes, wavelengths, periods, localization, GWPE/GWKE? How are these aspects systematically compared across methods?

In section 3.2, the authors present background temperature profiles, GWPE profiles, and time–height sections of perturbations filtered by different techniques. Section 3.3, however, focuses solely on MRA-filtering. Why is there no systematic comparison here? In particular, for the hodograph analysis, it would be valuable to assess how the choice of filtering method influences the outcome and physical interpretation.

For clarity, I prepared a table summarizing the spectral properties of the different methods (E = MRA, details 3–6):

	A)	B)	C)	D)	E)
period	<4h 45min?	all	all	???	All
wavelength	<7.5km	<26km	<9km	2.5-6km?	0.8 - 12.8km

Figure 1 illustrates the spectral response of several filters, but responses for nightly mean and polynomial subtraction are missing. Please include them for completeness. A normalized FFT spectrum would also help to indicate where wave energy is expected. Either show spectral responses of *all* filters in Figure 1, or summarize their passbands in a table for reader convenience.

Specific comments

Section 1 Introduction

- Very well written and nicely funnels down from "GWs are important" to "we need Multi-resolution analysis to study GW observations".
- Please sharpen the aim of the study. Construct a proper working hypothesis or state a research question.

Section 2.2 (GW analysis techniques)

- Derive and define GWPE and GWKE (without the "density" term) here, stressing the importance of proper background–perturbation separation.
- Clarify the meaning of the brackets in Eqs. (5) and (6); these should represent averages over at least one vertical wavelength.

2.2.1 Nightly mean subtraction (A)

- Subtracting the nightly mean can strongly reduce stationary wave structures (over the ~4h45min observation window). Presumably, this motivated the Hann smoothing step.

Please justify the chosen window size (7.5 km). How general is this choice? Would you increase it (e.g., to 15 km) for quasi-stationary mountain waves?

2.2.2 Polynomial fit subtraction (B)

- The statement that an nth-order polynomial removes perturbations longer than a fraction of the height range is problematic: polynomials are spectrally broad, not trigonometric. Please provide a reference.
- Why was a 4th-order polynomial chosen (implying a cutoff at >26 km)? Was the fit weighted by temperature uncertainties? Why not use higher orders to align the cutoff with other filters?

2.2.3 Spectral filtering (C)

- The vertical cutoff is clear, but what is the impact in the temporal domain? Was the cutoff chosen to match MRA detail 5?

2.2.4 Variance method (D)

- The parameter $N_z = 21$ is only revealed in the caption of Fig. 1. Combined with the vertical resolution, this suggests sensitivity to structures >3.15 km. What limits amplitudes beyond ~6 km? What about temporal averaging (N_t)?

2.2.5 MRA filtering (E)

- Details 3–6 capture wavelengths 0.8–12.8 km. Again, what are the implications in the temporal domain?
- Why assume that noise is contained solely in detail 1? White noise is expected across all scales.

Section 3.1 Case study

- This section mainly confirms, via ERA5, the presence of GWs. Mentioning tides is misleading and should be removed.
- Instead of a 200 hPa map of zonal wind, show ERA5 time–height cross sections of temperature/wind perturbations at La Réunion. The current map does not allow the reader to locate the island.

Section 3.2 Temperature profiles

- Include an ERA5 temperature profile in Fig. 4 for reference.
- Add subpanels in Fig. 5 showing the subtracted background (and possibly noise). Use a diverging colormap for perturbations.
- The oscillatory behavior in GWPE (30–45 km, Fig. 6) likely stems from Hann smoothing. A boxcar filter may mitigate this. Also correct the unequal scaling of altitude in the y-axis.
- Conservative growth is only discussed above 50 km, but deviations below that altitude are also worth analyzing.
- The claim that MRA is less sensitive to noise requires explanation. Why is this the case relative to other methods? For fairness, low-order details (1–2) should be included in the comparison.

Section 3.3 Wind profiles

- The sudden introduction of hodograph analysis is not well motivated. If retained, it should be applied systematically across all methods to reveal differences.
- Variability of GWKE below 15 km is not discussed. Larger scales contain more energy, but one might expect maximum energy for detail 5.
- The GWPE/GWKE ratio is computed only for MRA. This should be repeated for all methods to evaluate potential misinterpretations caused by their differing spectral passbands.

Section 4 Summary and perspectives

- Provide stronger evidence of wave activity (e.g., ERA5 profiles at the lidar site).
- Clarify that discrepancies between methods primarily arise from their different spectral passbands.
- Emphasize that while the choice of wavelet and sampling defines the MRA bands, this is conceptually no different from setting bandpass filters.

- Highlight more clearly the advantage of MRA: the orthonormal decomposition allows energy-conserving reconstruction of GW-induced perturbations.

Recommendation
The manuscript introduces a potentially valuable approach. However, substantial clarifications and additional analyses are required before the work can be considered for publication.