Responses to comments from Referee 2:

Dear Prof. Ian Willis,

Thank you very much for your careful and thorough review of this manuscript. Your professional insights have been highly constructive for improving the paper. In your review, you provided five major comments along with many minor, detailed suggestions. Below, we have basically revised and corrected the manuscript according to your suggestions and have responded to each of your comments in detail, including revisions to the textual descriptions, redesign of relevant sections, and updates to the experimental content. In the document, your comments and suggestions are marked in black, while our responses and the revised sections of the paper based on your suggestions are marked in blue.

Main Comments

Q1: The paper does not adequately acknowledge all the work that has been done by many people over the last decade +, which uses a physically-based algorithm to calculate water depths on the GrIS or Antarctic ice shelves from optical imagery (MODIS, Landsat, ASTER, Sentinel-2). For example, none of the following papers is mentioned or acknowledged in this context.

……

The well used 'Philpot' approach should be acknowledged and it would also be useful to mention (or show) how the 'Philpot' equation varies from the classic and modified Lyzenga models. Of course, it would also be valuable to actually use the 'Philpot' equation (e.g. with the parameters advocated by Pope et al 2016) to compare against the 2 calibrated versions of the Lyzenga model and the ArcticDEM.

A1: Thank you so much for your comments. We acknowledge that the manuscript did not sufficiently recognize previous contributions in this field, which was an oversight on our part. We sincerely thank you for pointing this out and apologize for the omission. Due to space limitations, the specific references you mentioned during your review are not all listed in this document, and we kindly ask for your understanding. We have carefully revised the introduction to incorporate relevant descriptions of prior work, including those you highlighted. The revised paragraph is as follows:

For the bathymetry inversion in polar lakes, Lin et al. (2012) used multibeam bathymetric data and Landsat TM data to invert the bathymetry of lakes in the Arctic Alaska Coastal Plain. Pope et al. (2016) also utilized the Landsat satellites with the OLI sensor, applying both the Philpot radiative transfer equation (RTE) model (Philpot 1987) and a semi-empirical model based on partial in situ measurements to estimate the water volume of SGLs in western GrIS. The results were evaluated and validated using satellite stereo-derived elevation data. Similarly, Moussavi et al. (2016) also utilized the stereoscopic imaging capability of Worldview-2 data to estimate and validate the bathymetry of SGLs of the GrIS, achieving high accuracy. Williamson et al. (2018) applied this RTE-based approach to Sentinel-2 multispectral imagery. Through the synergistic use of Sentinel-2 and Landsat 8 satellites, they identified numerous drained lakes, providing algorithmic support for water depth and volume estimation. Melling et al. (2024) used Sentinel-2 data to construct RTE for different bands and validated them with ICESat-2 and ArcticDEM (Arctic Digital Elevation Model) data. Based on a rigorous adherence to physical principles, they evaluated the applicability of the RTE model to SGLs.. Fricker et al. (2021) utilized ICESat-2 data to estimate the meltwater depth of the Antarctic ice sheet (AIS) and Greenland, providing a reference for the GrIS and AIS lake water depth inversion. Datta and Wouters (2021) proposed the Watta algorithm, which automatically calculates SGLs bathymetry and detects potential ice layers along tracks of the ICESat-2, focusing on the drainage situation of arctic lakes by utilizing ICESat-2 data and multispectral data. Lv et al. (2024) used the Stumpf model, combined with ICESat-2 and Sentinel-2 imagery, to invert the bathymetry of some SGLs on the GrIS from 2019 to 2023. Lutz et al. (2024) integrated ICESat-2 altimetry, in situ sonar measurements, and the RTE to establish four depth estimation methods, validated against TanDEM-X elevation models, providing a systematic methodological comparison for supraglacial lake depth and volume estimation in GrIS. Feng et al.

(2025) integrated ICESat-2 and Sentinel-2 data using a multi-layer perceptron neural network for depth inversion, achieving volumetric evolution monitoring of SGLs throughout the 2022 melt season in southwestern GrIS.

Reference:

[1] Lin, Zheng, Xia Li, and Jigang Qiao. "Polar lake bathymetry retrieval from remote sensing data of the arctic coastal plain in Alaska." Zhongshan Daxue Xuebao/Acta Scientiarum Natralium Universitatis Sunyatseni 51.3 (2012): 128-134.

[2] Pope, Allen, et al. "Estimating supraglacial lake depth in West Greenland using Landsat 8 and comparison with other multispectral methods." The Cryosphere 10.1 (2016): 15-27.

[3] Philpot, William D. "Radiative transfer in stratified waters: a single-scattering approximation for irradiance." Applied Optics 26.19 (1987): 4123-4132.

[4] Moussavi, Mahsa S., et al. "Derivation and validation of supraglacial lake volumes on the Greenland Ice Sheet from high-resolution satellite imagery." Remote sensing of environment 183 (2016): 294-303.

[5] Williamson, Andrew G., et al. "Dual-satellite (Sentinel-2 and Landsat 8) remote sensing of supraglacial lakes in Greenland." The Cryosphere 12.9 (2018): 3045-3065.

[6] Melling, Laura, et al. "Evaluation of satellite methods for estimating supraglacial lake depth in southwest Greenland." The Cryosphere 18.2 (2024): 543-558.

[7] Fricker, Helen Amanda, et al. "ICESat - 2 meltwater depth estimates: application to surface melt on amery ice shelf, East Antarctica." Geophysical Research Letters 48.8 (2021): e2020GL090550.

[8] Datta, Rajashree Tri, and Bert Wouters. "Supraglacial lake bathymetry automatically derived from ICESat-2 constraining lake depth estimates from multi-source satellite imagery." The Cryosphere Discussions 2021 (2021): 1-26.

[9] Lv, Jinhao, et al. "Long-term satellite-derived bathymetry of Arctic supraglacial lake from ICESat-2 and Sentinel-2." The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences 48 (2024): 469-477.

[10] Lutz, Katrina, et al. "Assessing supraglacial lake depth using ICESat-2, Sentinel-2, TanDEM-X, and in situ sonar measurements over Northeast and Southwest Greenland." The Cryosphere 18.11 (2024): 5431-5449.

[11] Feng, Tiantian, Xinyu Ma, and Xiaomin Liu. "Volumetric evolution of supraglacial lakes in southwestern Greenland using ICESat-2 and Sentinel-2." The Cryosphere 19.7 (2025): 2635-2652.

In addition, you suggested comparing the Philpot RTE model with our spectrally stratified Lyzenga model. We have adopted your valuable recommendation. Following the approaches of Lutz et al. (2024), who applied Philpot’s RTE model to supraglacial lakes on the GrIS, we performed bathymetric inversion for the four lakes in this study and compared the results with those obtained using both the original and the optimized Lyzenga model.

Q2: The paper remains rather limited in scope. In essence it has applied two versions of an empirical model: one, the 'classic' Lyzenga model, that has already been applied to a GrIS lake by Lv et al 2024; and a modified version of it that uses different spectral bands, that has so far only been applied outside of the context of GrIS lakes. The models were calibrated using ICESat data and validated against ArcticDEM data. There appears to be quite marginal improvement in using the modified version of the model. It is a methods paper, and it doesn't tell us anything about the glaciology or hydrology of GrIS surface lakes. It would be a more compelling paper if it had gone further.

It could have remained methods based but compared also the 'Stumpf' model and the 'Philpot' model for the 4 lakes. It could have told us how transferable the model is – it appears as though the model was calibrated separately for each lake? What were the different parameter values associated with these calibrations? What happens if you apply the model calibrated for one lake to the other lakes? What errors occur? How universal is the model? What happens if you combine all the ICESat-2 data with all the corresponding spectral data to produce an 'all lake' model? How transferable is that? The model needs testing against data that are outside of the lakes that were used for calibration.

It could also have been expanded to tell us about the hydrology of the lakes by calculating their volumes over summer seasons and between years.

A2: Thank you so much for your very insightful comments, and each point you made is reasonable. You have raised several key issues, and we will answer them one by one below.

Q2 (1): The models were calibrated using ICESat data and validated against ArcticDEM data. There appears to be quite marginal improvement in using the modified version of the model. It is a methods paper, and it doesn't tell us anything about the glaciology or hydrology of GrIS surface lakes. It would be a more compelling paper if it had gone further.

A2 (1): We acknowledge that the manuscript lacks a direct discussion of the hydrological characteristics of lakes on the GrIS, and relevant descriptions have been appropriately added to the Introduction. The present study provides a precise remote-sensing-based approach for deriving SGLs bathymetry on the GrIS, which can serve as a valuable tool for investigating more in-depth hydrological and glaciological processes. We sincerely appreciate your suggestion, which indeed represents a promising research direction, and it will be an important focus of our future work.

Q2 (2): It could have remained methods based but compared also the 'Stumpf' model and the 'Philpot' model for the 4 lakes. It could have told us how transferable the model is – it appears as though the model was calibrated separately for each lake?

A2 (2): We have adopted your advice and added a comparative experiment using the Philpot RTE model (Philpot, 1987), which is a purely physics-based approach that differs in algorithmic principle from the model proposed in this study. Including it as a comparison is indeed meaningful. As you pointed out, the model parameters established in this study were individually calibrated for different lakes. Regarding the model's transferability, although the specific parameters in this study cannot be directly transferred, the workflow of the active and passive bathymetric model with spectral stratification is transferable, that is, dividing the optical imagery into zones based on the penetration differences among spectral bands and constraining the model parameters using a subset of in situ bathymetry data. This workflow can be applied to parameter calibration in the Stumpf model (Stumpf et al, 2003) and even integrated into other machine learning methods, as it also accounts for the physical penetration characteristics of electromagnetic waves and performs zonal modeling. This aspect ensures conceptual consistency across different active and passive bathymetry models.

Reference:

[3] Philpot, William D. "Radiative transfer in stratified waters: a single-scattering approximation for irradiance." Applied Optics 26.19 (1987): 4123-4132.

[12] Stumpf, Richard P., Kristine Holderied, and Mark Sinclair. "Determination of water depth with high-resolution satellite imagery over variable bottom types." Limnology and Oceanography 48.1part2 (2003): 547-556.

Q2 (3): What were the different parameter values associated with these calibrations? What happens if you apply the model calibrated for one lake to the other lakes? What errors occur? How universal is the model?

A2 (3): The model parameters indeed vary among different lakes. Since the parameters for each lake were individually calibrated, the mathematical relationship between water depth and spectral reflectance may not be entirely consistent across lakes. Consequently, directly transferring the model from one lake to another would inevitably introduce errors, meaning that the current algorithm does not yet possess large-scale transferability. Nevertheless, the workflow of spectral stratification proposed in this study is generalizable, as mentioned in our response to Comment A2 (2). Thank you again for your gentle comments. We will continue to research more precise algorithms to obtain the bathymetric information of GrIS. If further explanation or modification is needed, please feel free to let us know, we will be grateful.

Q2 (4): What happens if you combine all the ICESat-2 data with all the corresponding spectral data to produce an 'all lake' model? How transferable is that? The model needs testing against data that are outside of the lakes that were used for calibration.

A2 (4): This is indeed an inspiring and innovative perspective. Undoubtedly, developing an "all-lake" model based on extensive ICESat-2 data would substantially enhance the generality and transferability of the approach. In contrast, this study focuses on improving the inversion accuracy of individual lakes through an optimized spectral stratification algorithm, which necessitates separate calibration for each lake. Since the core objective of this work is accuracy improvement, the model's generality is not as strong as that of an all-lake model. We have added relevant discussion to the revised manuscript, and enhancing the model's general applicability will be an important direction for our future research.

Q2 (5): It could also have been expanded to tell us about the hydrology of the lakes by calculating their volumes over summer seasons and between years.

A2 (5): Your comment highlights a valuable prospect for the application of our algorithm. Any optimization method must ultimately be validated through practical implementation. Regarding this point, we have previously conducted some preliminary work (Lv et al., 2024), in which the Stumpf model was used to estimate the interannual variations in the total water volume of a subset of SGLs on the GrIS over five years (Figure 1), and possible related environmental factors, such as temperature, were also discussed. This is indeed an interesting direction. The point you raised is very meaningful, and since the present study improves the model accuracy, we believe our approach could be valuable for such related research. We plan to further explore this aspect in our future work based on the method proposed in this study.

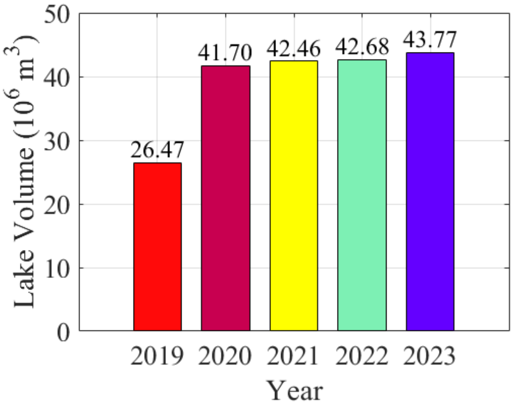

Figure 1. Volume change of a subset of SGLs on the GrIS from 2019 to 2023 (Lv et al, 2024).

Reference:

[9] Lv, Jinhao, et al. "Long-term satellite-derived bathymetry of Arctic supraglacial lake from ICESat-2 and Sentinel-2." The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences 48 (2024): 469-477.

Q3: The work uses the NDWI (which uses the Green and NIR bands) to separate water from ice. This is unusual as virtually all the papers I know about (see e.g., those listed above) use the variant of NDWI tailored for glacier ice and water separation that uses the blue and red bands instead of green and NIR:

NDWIice = (Blue-Red)/(Blue+Red)

Did the authors consider using this? Why did they opt for the Green and NIR bands? Could we be shown whether it makes a difference to the results?

A3: Thank you so much for your comments. We greatly appreciate your careful and professional review. We acknowledge that our consideration in this part was insufficient. The NDWI applied in our previous work (based on the green and near-infrared bands) is primarily designed for inland lakes (e.g., for water extraction over land or forested areas) and is not the optimal index for detecting water on glaciers. In glacial regions, both the green and NIR reflectance are relatively high, and their difference is much less pronounced than over soil or vegetation. This may introduce some inaccuracies when extracting glacial lake water. To address this, we have re-extracted the lake water using NDWIice, and the corresponding revisions have been made in the manuscript.

Q4: There is confusion throughout the paper about the definition of what the authors refer to as "a satellite-derived bathymetry (SDB) method". See lines 12-15 where it is first mentioned and defined, but then all subsequent references to it as well. Need to clarify that this is the complete method that is being used in this paper, i.e. the Lyzenga model applied to optical satellite data (Sentinel-2), calibrated using altimetry data (ICESat-2). And the paper uses two versions of the Lyzenga equation. It is not just the use of optical data alone as is sometimes implied in the manuscript.

A4: Thank you so much for your comments. We acknowledge that the definition and description of SDB in the manuscript were not sufficiently clear, which was an oversight on our part. We would like to clarify this section as follows:

SDB can encompass various approaches. It may involve only optical remote sensing data (e.g., Philpot's RTE method), or it may combine active and passive remote sensing techniques (such as the approach in the original manuscript, which integrates optical imagery with ICESat-2 data). Both the Lyzenga model and its optimized version used in the manuscript rely on two data sources: optical imagery and ICESat-2 altimetry. Therefore, they do not use optical imagery alone. Relevant descriptions in the manuscript have been revised to prevent any potential confusion.

Q5: The methodology is not clearly articulated in a consistent way throughout the paper, and parts of the methods appear in the results (see line by line comments below). A key aspect of the methodology that is not articulated clearly up front is that the ICESat-2 data are being used to calibrate the parameters of the two versions of the Lyzenga model. The equation for the 'traditional' version of the Lyzenga model and how it is applied are not explained. Does the traditional version use just one waveband from the optical data? If so which one?

A5: Thank you so much for your comments. We acknowledge that the description of the methods in the manuscript was not sufficiently clear. In combination with our follow-up experiments, we have revised this section to make each method more clearly defined. In the original manuscript, both the traditional Lyzenga model and the optimized Lyzenga model use the same spectral bands, namely the blue and green bands. The difference between them is that the optimized Lyzenga model employs a zonal modeling approach, using different model parameters for each partition to partially address potential errors arising from the varying penetration capabilities of electromagnetic waves at different wavelengths.

Line by line comments

Q6: 15 'verify' => 'validate' [as 'validate' is used subsequently in the paper].

A6: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q7: 17. say you're using 'time stamped' DEM strips here.

A7: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q8: 19-21. Do you need a sentence before this one explaining what you did to enable you to make this statement? How did you prove it is 'scalable' and what do you mean by that? I'm not convinced you've proved it's scalable, have you?

A8: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly. We have removed the description of "scalable" from the abstract.

Q9: 26. Is Luthje et al a good reference here? Does it explicitly consider 'ecological' issues?

A9: Thank you so much for your valuable comments. The study by Luthje et al. does not explicitly address ecological issues; we have revised the manuscript accordingly.

Q10: 28-30. Beckmann and Winkelmann, 2023 do not consider lakes in their paper as implied. Clarify what you mean by this sentence and use an appropriate ref.

A10: Thank you so much for your valuable comment, which has helped make the manuscript more rigorous. The sentence was intended to convey that Arctic glacier melt has negative impacts on the global ecosystem. We have accordingly revised the references in the manuscript.

Q11: 32. You can't sail ships on ice sheet surface lakes so delete this! Your 3 refs all relate to lidar. Could you quote studies using bathymetry from small boats, e.g.:

1. Box, J. E., & Ski, K. (2007). Remote sounding of Greenland supraglacial melt lakes: implications for subglacial hydraulics. Journal of Glaciology, 53(181), 257–265. https://doi.org/10.3189/172756507782202883. Cambridge University Press & Assessment
2. Tedesco, M., & Steiner, N. (2011). In-situ multispectral and bathymetric measurements over a supraglacial lake in western Greenland using a remotely controlled watercraft. The Cryosphere, 5, 445–452. https://doi.org/10.5194/tc-5-445-2011. Copernicus TC

Both papers include in-situ depth sounding (Box & Ski with a raft + depth sounder; Tedesco & Steiner with a remotely controlled boat equipped with GPS and sonar).

A11: Thank you so much for your valuable comments. We sincerely apologize for our oversight. The relevant statements in the manuscript have been revised to accurately reflect the facts. The corresponding parts in the manuscript have been updated accordingly as follows:

Accurately estimating the volume of these lakes requires detailed bathymetry data, which is particularly challenging to obtain due to the harsh climatic conditions in the Arctic. Although conventional bathymetric methods, including bathymetric airborne lidar and shipborne sonar, have achieved high levels of maturity and accuracy, They have also been successfully applied over the GrIS (Box and Ski, 2007; Tedesco and Steiner, 2011); however, due to limitations in cost and timeliness (Li et al., 2022; Qi et al., 2022; Qi et al., 2024), these methods cannot meet the monitoring demands of rapidly changing SGLs on GrIS.

Reference:

[13] Box, Jason E., and Kathleen Ski. "Remote sounding of Greenland supraglacial melt lakes: implications for subglacial hydraulics." Journal of glaciology 53.181 (2007): 257-265.

[14] Tedesco, Marco, and N. Steiner. "In-situ multispectral and bathymetric measurements over a supraglacial lake in western Greenland using a remotely controlled watercraft." The Cryosphere 5.2 (2011): 445-452.

[15] Li, Shaoyu, et al. "Bathymetric LiDAR and multibeam echo-sounding data registration methodology employing a point cloud model." Applied Ocean Research 123 (2022): 103147.

[16] Qi, Chao, et al. "A method to decompose airborne LiDAR bathymetric waveform in very shallow waters combining deconvolution with curve fitting." IEEE Geoscience and Remote Sensing Letters 19 (2022): 1-5.

[17] Qi, Chao, et al. Analysis and correction in the airborne LiDAR bathymetric error caused by the effect of seafloor topography slope, National Remote Sensing Bulletin, (2024)26, 2642-2654.

Q12: 32. You say 'in polar regions' here. But I'd suggest focussing on the GrIS earlier in your introduction and stop referencing Arctic / Polar regions after that.

A12: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly and carefully checked the entire text to ensure consistency and accuracy.

Q13: 41. These references are not on the GrIS so make that clear in the sentence. You could reference other papers relevant for the GrIS and AIS.

A13: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have updated the relevant references accordingly.

Q14: 42-3. "…integrated multispectral technology with ICESat-2 to conduct bathymetric detection and inversion, leveraging both active and passive remote sensing…" Here and elsewhere in the paper (e.g. 47-8, 64) it would be useful to stick to one order and not switch the order you mention these two types of satellite data. So here you could write "integrated multispectral technology with ICESat-2 to conduct bathymetric detection and inversion, leveraging both passive and active remote sensing". It's a trivial point but it makes for a clearer, more logical read.

A14: Thank you for your suggestion, which has helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q15: 44. You refer to 'the mainland' but you need to be clearer in this section that the methods you're using were first applied in settings outside the GrIS. Tell us what mainland here as it reads like you're talking about mainland Greenland!

A15: Thank you for your suggestion, which has helped make the manuscript more rigorous. We have revised the manuscript accordingly. To avoid confusion, the phrase “far away from the mainland” has been removed.

Q16: 46. "…leverages active and passive remote sensing techniques…" You told us that 3 lines up so delete. More useful to tell us what sensors .

A16: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly, making some reductions while retaining more concise and effective information. The revised contents are as follows:

Cao et al. (2016) developed a high-precision bathymetry model for Ganquan Island by combining laser bathymetry and WorldView-2 imagery, leveraging both active and passive approaches.

Reference:

[18] Cao, B., Z. G. Qiu, and B. C. Cao. "Comparison among four inverse algorithms of water depth." Journal of Geomatics Science and Technology 33.04 (2016): 388-93.

Q17: 50. Full stop after 'Bermuda."

A17: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q18: 50. 'proposed' is the wrong word. Do you mean 'divided'? Or 'classified'?

A18: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly and removed this citation, as including too many examples from non-polar regions was deemed unnecessary.

Q19: 54. You say "improving the inversion accuracy" but compared to what?

A19: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly and removed this citation, as including too many examples from non-polar regions was deemed unnecessary.

Q20: 56-7. Before the sentence spanning these lines, I'd specify that you're focussing on surface lakes on ice masses. Note it's not just for 'Arctic regions' as you include Fricker et als work on Antarctic surface lakes in your references.

A20: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have replaced 'Arctic regions' with 'polar lakes'.

Q21: 61. "Watta algorithm". This is the 3rd algorithm that's been introduced now with no details. It'd be more important to tell us the basis of this (and earlier) algorithms not what Datta and Wouters called it. I assume they called it this because it is an amalgamation of their names, but this is not really crucial information here.

A21: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly. The revised content is as follows:

Datta and Wouters (2021) proposed the Watta algorithm, which automatically calculates SGLs bathymetry and detects potential ice layers along tracks of the ICESat-2, focusing on the drainage situation of arctic lakes by utilizing ICESat-2 data and multispectral data.

Reference:

[7] Datta, Rajashree Tri, and Bert Wouters. "Supraglacial lake bathymetry automatically derived from ICESat-2 constraining lake depth estimates from multi-source satellite imagery." The Cryosphere Discussions 2021 (2021): 1-26.

Q22: 61. YOU say 'arctic lakes' but it was just for GrIS lakes. Having already focussed down on the GrIS in your review, I'd stick to referencing work relevant to the GrIS here and not keep mentioning 'Arctic' or 'Polar regions' etc.

A22: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly and have made clearer distinctions in the manuscript, no longer conflating the GrIS with the broader Arctic Polar region.

Q23: 63. Delete 'some' and delete 'from 2019 to 2023' This latter not relevant.

A23: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q24:65. 'polar' See my comment above for line 61.

A24: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q25:65-7. Is this just the Lv et al (2024) paper that's relevant here? If so say so. But you should also acknowledge Mousavi et al, Pope et al, Williamson et al, Melling et al, etc. who did consider different bands using the 'Philpot' algorithm.

A25: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly and have fully cited the previous studies you mentioned; the revisions have already been provided in our response to the previous comment.

Q26:68 Suggest "…Chu et al. (2023) to offshore islands…"

A26: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q27: 68-70. So you're building on Lv et al 2024 which so far is the only paper to apply what you're calling the SDB method to GrIS lakes, but you're adding spectral stratification as used by Chu et al in a different context? Could state this more clearly

A27: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have provided more detailed descriptions in the manuscript to more clearly illustrate the relationships among these methods. The revised content is as follows:

In this study, we further extend previous methods for retrieving SGLs bathymetry. Inspired by Chu et al. (2023) on offshore islands and reefs, we applied an improved bathymetric inversion approach for SGLs that combines active and passive remote sensing.

Reference:

[19] Chu, Sensen, et al. "Shallow water bathymetry using remote sensing based on spectral stratification." Haiyang Xuebao 45.1 (2023): 125-137.

Q28: 72-3. Ok so you're 'calibrating' using ICESat-2 and validating with ArcticDEM. This could be expressed more clearly.

A28: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. The section you mentioned indeed required clarification, and we have revised the manuscript accordingly. The revised content is as follows:

Specifically, ICESat-2 lidar data were integrated with Sentinel-2 multispectral imagery, taking into account the varying penetration abilities of different spectral bands (i.e., red, green, blue, and near-infrared). The Sentinel-2 imagery was divided into multiple spectral layers using the Otsu algorithm to construct a spectral stratification-based Lyzenga model. In this framework, ICESat-2 lake bottom photons were used as training samples to build semi-empirical models for each spectral layer, thereby improving the accuracy of SGL bathymetric estimation. For comparison, we also applied the Philpot RTE model and the traditional Lyzenga model without spectral stratification optimization to the same study area. All results were validated against high-resolution ArcticDEM data.

Q29: 74 'Arctic' => 'GriS'

A29: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q30: 74. "…offers effective technical support for predicting Arctic glacier melt and global climate change." This is too grand. I'd delete this. You can't do this based on your work.

A30: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. The previous statement is overstated. The manuscript has been revised accordingly; the revised contents are as follows:

This study provides a new and more accurate approach for monitoring the volumes of SGLs and offers methodological guidance for SGL bathymetry research.

Q31: 75-7. Suggest delete.

A31: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q32: 80-2. Suggest delete "in the Arctic, the second-largest ice sheet in the world, surpassed only by the Antarctic Ice Sheet. However, the GrIS is more fragile and sensitive to temperature changes than the Antarctic Ice Sheet (Robinson et al., 2012)." Everybody knows the first statement and the 2nd is a bit vague.

A32: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q33: 85. 'aimed => 'aims' and "verify" => 'validate'

A33: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q34: 86 "bathymetry data …were used" [data are plural]

A34: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q35: 91. Its blue green red yellow respectively

A35: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q36: Fig 1. Your Arctic inset is rather ugly. I suggest use another inset - just for the GrIS - and remove the words 'study area' from the map.

A36: Thank you so much for your comments. We have revised the manuscript accordingly. The inset has been changed to show Greenland, and the updated study area map is presented below:

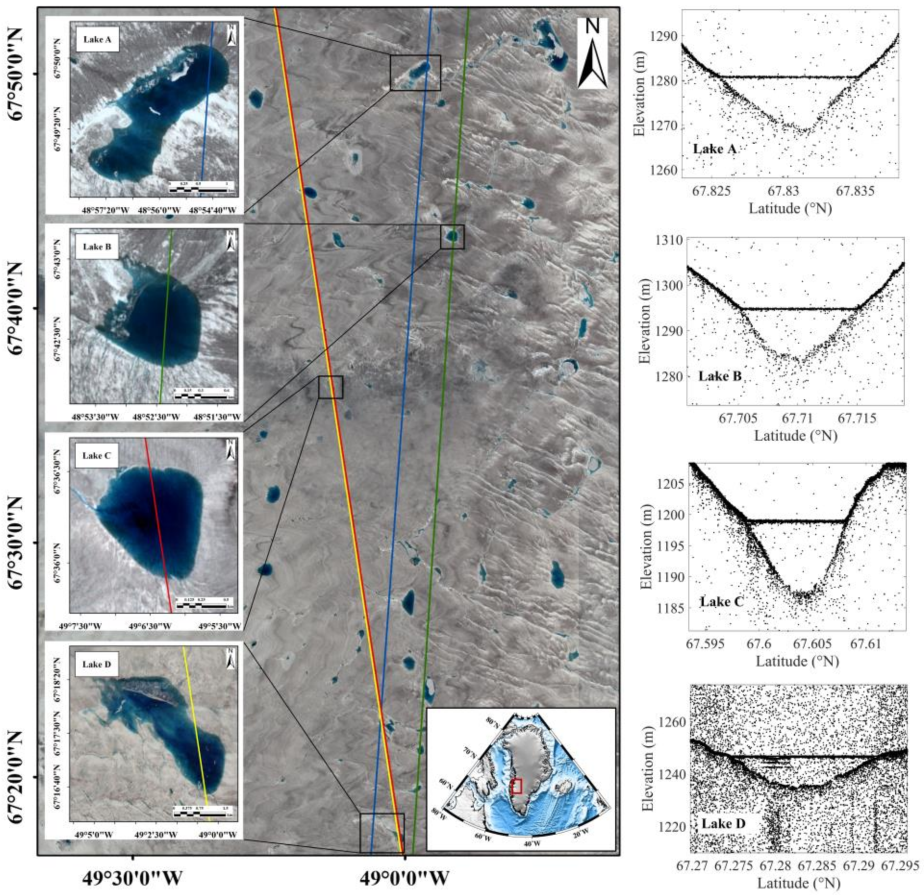

Figure 2. Revised SGLs insets

Q37: 100-1. Suggest change to "Sentinel-2 imagery was obtained for lakes A and B on 4 July 2020, lake C on 17 July 2022, and lake D on 15 July 2021."

A37: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly. The related statements have been removed, and the acquisition dates are now directly presented in Table 1 to avoid redundant descriptions. The revised table is as follows:

Table 1. Detailed information of the datasets used in this study, the acquisition dates are highlighted in bold within the dataset names in the format yyyy/mm/dd.

Study area	Datasets	Data filename
Lake A	Sentinel-2	T22WEA_**20200704**T145921
	ICESat-2	ATL03_**20200706**005932_01630805_005_01_gt2l
	ArcticDEM	SETSM_s2s041_WV01_**20200511**_1020010094C9D900_1020010098791800_2m_lsf_seg3_dem
Lake B	Sentinel-2	T22WEA_**20200704**T145921
	ICESat-2	ATL03_**20200706**005932_01630805_005_01_gt3l
	ArcticDEM	SETSM_s2s041_WV01_**20200511**_1020010094C9D900_1020010098791800_2m_lsf_seg3_dem
Lake C	Sentinel-2	T22WEA_**20220717**T150811
	ICESat-2	ATL03_**20220714**010847_03381603_006_02_gt2r
	ArcticDEM	SETSM_s2s041_WV01_**20220420**_10200100C131A200_10200100C42D4300_2m_lsf_seg1_dem
Lake D	Sentinel-2	T22WEV_**20210715**T151911
	ICESat-2	ATL03_**20210715**182907_03381203_006_01_gt2r
	ArcticDEM	SETSM_s2s041_WV02_**20210312**_10300100BB24B100_10300100BBC0A100_2m_lsf_seg1_dem

Q38: 102. 'can be' => 'was' [say what you did not what is possible to do]

A38: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly and carefully checked the entire text.

Q39: 110-11. Suggest "The left and right points of each beam pair are approximately 90 m apart in the transverse track direction and about 2.5 km apart in the along-track direction."

A39: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q40: 112-13. Suggest : "The ICESat-2 data used in this study were acquired on 6 July 2020 (lakes A and B), 15 July 2021 (lake C), and 14 July 2022 (lake D)."

A40: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly. The related statements have been removed, and the acquisition dates are now directly presented in Table 1

Q41: 115 "data, and"

A41: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q42: 115-16. "Please note that this study assumes" => "We assume"

A42: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q43: 118 "can be" => "data were"

A43: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q44: 119 delete "and has the characteristics of a large coverage area and high spatial resolution."

A44: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q45: 121. Delete "high-resolution Arctic digital elevation model ArcticDEM"

A45: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q46: 122-4. Ambiguous. Tell the reader what you did - what data you used.

A46: Thank you so much for your valuable comments. We have revised the manuscript accordingly as follows:

In this study, the most recent version (s2s041) of the ArcticDEM strip was used to validate the accuracy of the bathymetry and volume estimated results.

Q47: 124. Delete "It should be pointed out that"

A47: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q48: 124-5. Explain why these dates. Presumably the first data to cf. with your derived bathymetry along the ICESat-2 line for lakes A and B, 2nd for bathymetry of lake C, 3rd for lake D?

A48: Thank you so much for your valuable comments. We removed this part from the text description and instead presented the data correspondence more intuitively in Table 1.

Q49: 130-1. You say "To address the challenges and the limitations of traditional bathymetric methods, which do not consider the varying penetration of electromagnetic waves of different wavelengths into water…" But this is not true as many previous studies using the 'Philpot' method have done this.

A49: Thank you so much for your valuable comments. We apologize for our oversight and have revised the manuscript accordingly. Previous studies based on the Philpot RTE method have indeed considered the differences in water penetration among spectral bands. What we intended to emphasize here is the approach of dividing the entire lake into different zones according to variations in electromagnetic wave penetration, and performing bathymetric inversion separately in each zone. This process is implemented by combining ICESat-2 bathymetric data with the zonal Lyzenga model. We have thoroughly revised this part accordingly as follows:

To improve the accuracy of SDB for SGLs of GrIS, this study compares three approaches: the physically-based RTE model, the traditional Lyzenga model, and a novel spectrally stratified optimized Lyzenga model. While the RTE model relies on physical parameterization of water column properties, and the traditional Lyzenga model combines multiple spectral bands empirically, while both approaches incorporate spectral information through wavelength-dependent attenuation coefficients, they generally do not stratify the retrieval model based on the varying effective depth ranges of different spectral bands.. Inspired by the spectral stratification method applied to shallow coral reefs by Chu et al. (2023), this study adapts and extends this approach to SGLs. The proposed method combines ICESat-2 data with Sentinel-2 multispectral imagery. Using the Otsu algorithm (Otsu, 1975), spectral stratification is performed based on radiance differences at various water depths across different bands. The stratified spectral layers are then combined with ICESat-2 bathymetric data to construct optimized Lyzenga models for each spectral layer.

Reference:

[19] Chu, Sensen, et al. "Shallow water bathymetry using remote sensing based on spectral stratification." Haiyang Xuebao 45.1 (2023): 125-137.
[20] Otsu, Nobuyuki. "A threshold selection method from gray-level histograms." Automatica 11.285-296 (1975): 23-27.

Q50: Figure 2. This implies you're producing two versions of the Lyzenga model - traditional and spectral stratified. And you'll compare them both against Arctic DEM data? This was not mentioned as part of your methodology earlier in the abstract or in your brief overview of the methodology for the work on lines 68-73. It should be clearer earlier that this is part of your work.

A50: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly. We have clearly stated in both the abstract and the introduction that two methods were used. In addition, we have added relevant descriptions of the RTE method. The revised flowchart is as follows:

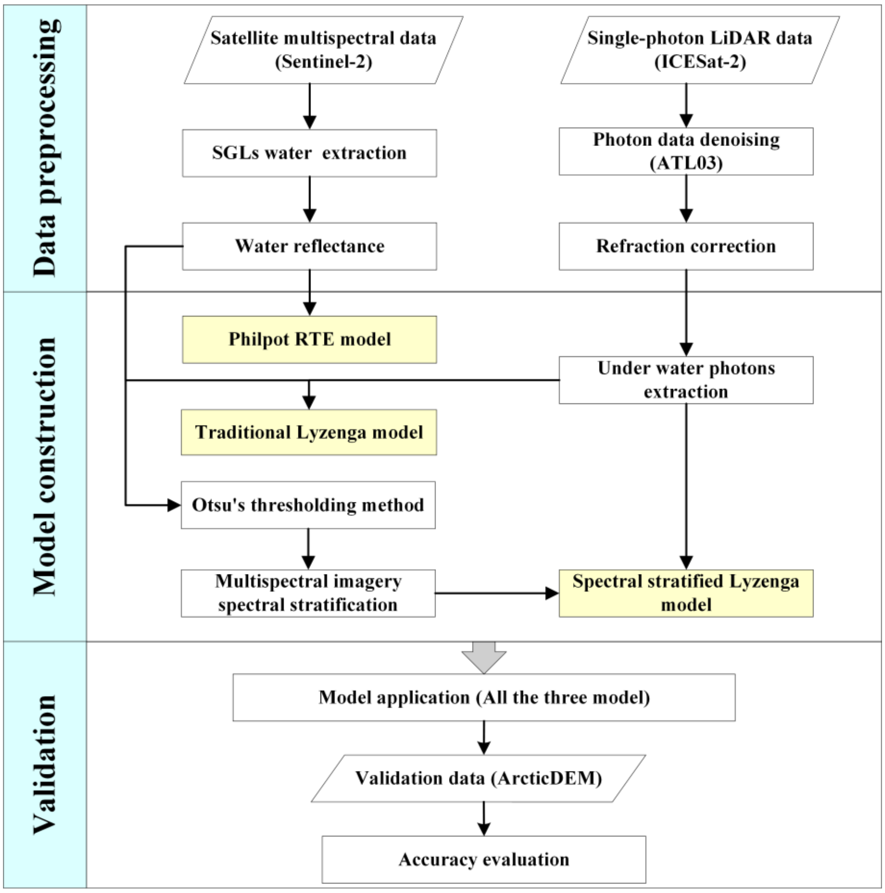

Figure 3. Revised flowchart, with the SDB model used in this study highlighted in yellow in the illustration.

Q51: 140. Delete "provided by the ESA" and say "…data are…"

A51 Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q52: 142. '…can be processed…' Again, tell us what you did not what can be done.

A52: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q53: 144-6. Why did you not use the version for separating water from ice? NDWI_ice? there's another variant of NDWI tailored for glacier ice detection that uses the blue and red bands instead of green and NIR.

That formulation is:

NDWIice =(Blue-Red)/(Blue+Red)

• Blue: reflectance in the blue band (~0.45 µm, e.g., Sentinel-2 Band 2)

• Red: reflectance in the red band (~0.65 µm, e.g., Sentinel-2 Band 4)

A53: Thank you so much for your valuable comment. We acknowledge that our consideration in this part was insufficient. The NDWI version using the Green and NIR bands is typically applied for water extraction in vegetation or soil environments rather than in glacial settings. Therefore, in this study, we have replaced it with NDWIice for water extraction and revised the subsequent experimental sections accordingly. The revised text is as follows:

The Sentinel-2 data are divided into L1C and L2A levels. The L1C level data product is a geometric precision correction radiographic product that has not undergone radiometric correction. L2A products are products that undergo radiation correction processing based on L1C. The water column was extracted from multispectral imagery using water-land separation methods, i.e., the Normalized Difference Water Index (*NDWI*), specifically its variant adapted for water extraction in ice–snow covered environments, Eq. (4) (originally Equation 1 in the manuscript) (McFeeters, 1996; Yang et al., 2012)，combined with threshold-based grayscale segmentation.

$$NDWI_{Ice} = \frac{Blue - Red}{Blue + Red} \quad (1)$$

Where *Blue* represents the reflectance at the blue band (corresponding to Sentinel-2 Band 2) and *Red* represents the reflectance at the red band (corresponding to Sentinel-2 Band 4).

The image was divided into water and non-water parts using the $NDWI_{Ice}$, with the non-water portion—including lake ice—applied as a mask to extract the open-water body. As the multispectral imagery contained partially unmelted ice on the lake, this masking process effectively removed ice-covered areas and ensured accuracy during the water column extraction.

References:

[21] McFeeters, Stuart K. "The use of the Normalized Difference Water Index (NDWI) in the delineation of open water features." International journal of remote sensing 17.7 (1996): 1425-1432.

[22] Yang, Kang, and Laurence C. Smith. "Supraglacial streams on the Greenland Ice Sheet delineated from combined spectral–shape information in high-resolution satellite imagery." IEEE Geoscience and Remote Sensing Letters 10.4 (2012): 801-805.

Q54: 151. 'data are'

A54: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q55: 152. Delete "In this study"

A55: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q56: 154 and 155 "should say 'the four'

A56: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q57: 155 "Due to the fact that" => 'Because"

A57: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q58: 157-8. "Finally, more accurate bathymetric photons were obtained for constructing a bathymetry inversion model." I don't understand this sentence and it splits up the two either side of it that are related. I suggest delete this sentence (or clarify what it means if it's important).

A58: Thank you so much for your valuable comments. We have revised the manuscript accordingly. We agree that this sentence may be confusing. Our intended meaning was that the ICESat-2 data, after refraction correction, can more accurately represent the underwater topography. To avoid ambiguity, this sentence has been deleted.

Q59: 169. Delete "It should be noticed that"

A59: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q60: 176-7. Suggest " This study applied the spectral stratification method using the Otsu algorithm, which automatically determines thresholds without requiring input parameters (Otsu, 1975)."

A60: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q61: 177-8. This is not a sentence. The main clause is missing a finite verb. Right now, "multispectral images of water stratified into four layers" is written as if it's a complete statement, but it lacks a clear subject performing an action. Should it say '...water were stratified..." ? But 'stratified into 4 layers' implies lake depth layers, but you don't mean this - you're giving 4 wavebands. This is confusing.

A61: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q62: 179. Could delete 'layer'

A62: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q63: 189. It'd be useful to show the equation for this "traditional Lyzenga model" too.

A63: Thank you so much for your valuable comments. We have revised the manuscript accordingly and briefly derived its relationship with the Philpot RTE model to make the logic more coherent. The revised content is as follows:

3.2.2 Traditional Lyzenga model

Both Lyzenga multi-band logarithmic linear model (Lyzenga, 1978, 1985) and Philpot RTE models exploit the exponential attenuation of light in water following Beer-Lambert's law, with Philpot's RTE approach providing explicit physical parameterization of bottom reflectance and water column properties that were empirically combined in Lyzenga's earlier formulation. Unlike the purely theoretical RTE approach, which derives water depth only from optical imagery, this method introduces empirical constraints using a limited number of measured depth values to estimate model parameters. In this formulation, the bottom reflectance (A_d) and the diffuse attenuation coefficient function (g) are treated as constants, where $a_0=ln(A_d-R_\infty)/g$, $a_1=-1/g$. At this stage, the model equation is written as:

$$Z = a_0 + a_1 \left(R_\omega - R_\infty \right) \tag{2}$$

By integrating the spectral reflectance information from multiple bands, the following expression can be obtained:

$$Z = a_0 + a_i \sum_{i=1}^{n} \ln\left[R_\omega(\lambda_i) - R_\infty(\lambda_i) \right] \tag{3}$$

This represents the commonly used Lyzenga model. In this study, ICESat-2 bathymetric points are incorporated to constrain the empirical parameters of the Lyzenga model, and the parameter estimation is performed using the Levenberg–Marquardt (LM) algorithm.

Reference:

[23] Lyzenga, David R. "Passive remote sensing techniques for mapping water depth and bottom features." Applied optics 17.3 (1978): 379-383.

[24] Lyzenga, David R. "Shallow-water bathymetry using combined lidar and passive multispectral scanner data." International journal of remote sensing 6.1 (1985): 115-125.

Q64: 191. You say "…combining the traditional Lyzenga model…" But with what? This is unclear.

A64: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly. What we intended to express here is that different spectral layers are combined with ICESat-2 data to establish zonal Lyzenga models for different regions. The revised content is as follows:

By leveraging the varying penetration abilities of electromagnetic waves in water, the multispectral images were segmented into different spectral layers, and a spectral-stratified Lyzenga model was established by integrating ICESat-2 data with water reflectance in each layer.

Q65: 192-3. "the near-infrared layer, the red layer, and the green layer were combined for processing. In other words, Arctic SGLs were divided into green and blue layers." This is contradictory. Did you use the NIR and Red or just the Green? Explain more precisely what you did.

A65: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly. Since the threshold-segmented areas of the NIR and Red bands are very small, the available

ICESat-2 control points are extremely limited. Therefore, in this study, the NIR, Red, and Green spectral layers were combined and collectively referred to as the "green layer".

Q66: 197-8. So what values do these parameters take? You haven't really explicitly stated you're fitting these equations using the ICESat-2 data to derive Z, then you're using these calibrated equations to derive bathymetry for the whole lake. Please in your methods explain the calibration process explicitly. In the results it would be useful to know what values these parameters take for the two equations for the 4 lakes. It'd also be useful to know what the equation for the original Lyzenga model is, how that works, and what the parameter values are for that and how they vary between lakes.

A66: Thank you so much for your valuable comments. Accordingly, we have revised the manuscript to specify that the parameter estimation method used in this study is the Levenberg–Marquardt (LM) algorithm. In addition, all model parameters applied in this study will be listed in a newly added appendix or supplement to help readers better understand the proposed algorithm.

Q67: 210 'performed' => 'performs'

A67: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q68: 211 'was' => 'is'

A68: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q69: 213. It's only here that it becomes apparent that you're constructing 2 models. The original Lyzenga and a spectral stratified version of Lyzenga? This has not been clear throughout your methodology sections so far. For example, you do not mention this in Section 3.

A69: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have added a description of the traditional Lyzenga model.

Q70: 216-22. This should all have been stated above in methods. Not here. Start the results section with the results!

A70: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly and removed these irrelevant descriptions.

Q71: 229. Where have you used the active (ICESat-2) data to derive Fig 5? This was not adequately explained.

A71: The ICESat-2 data were used as control points to constrain the construction of the Lyzenga model, and this has been clarified in the revised manuscript.

Q72: 231. You say "…underscoring the reliability and feasibility of the spectral stratified model". But you cannot conclude this until you've compared with the Arctic DEM. Delete this.

A72: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q73: 232. You don't need any of these phrases ending with the word 'that', e.g. 'It should be noted that…'. Just delete this and all such phrases.

A73: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q74: 232-3. You say " the ArcticDEM data only contains spatial information of the lake bottom, and lacks water surface elevation information when obtaining bathymetry benchmark data". Is that true? Esp as you use early season strips. How do you know the lakes were empty at that time rather than containing water and frozen over?

A74: Thank you so much for your valuable comments, which are well justified. The ICESat-2 and ArcticDEM profiles used in this study show good agreement. ICESat-2 is capable of detecting lake bottom information, and if the lakes contained water covered by ice, the DEM would display a flat surface. However, this is not the case with the ArcticDEM data used here. Therefore, it can be reasonably assumed that the ArcticDEM represents empty lake basins, consistent with the assumptions in Melling et al. (2024) and related studies.

Q75: 236-7. You say "However, the sediment in the lakes of the experimental area primarily consists of bedrock, a type of material that remains stable and does not undergo significant changes over short periods." This makes no sense to me at all.

A75: Thank you so much for your valuable comments. We are sorry that the statement in this section lacked sufficient scientific basis and theoretical support, and we have supplemented and clarified the description in the revised manuscript, as follows:

The GrIS is covered by ice, and both the ice surface and supraglacial lakes evolve, making short-term morphological changes unavoidable. However, as noted by Echelmeyer et al. (1991), many large supraglacial lakes remain fixed in space because their surface depressions are dynamically supported by irregularities in the underlying bedrock. This bedrock control limits large-scale spatial shifts within short periods, although minor variations are inevitable. Consequently, some discrepancies between ICESat-2-derived bathymetry and ArcticDEM data collected several months apart are expected. Nevertheless, the overall consistency between the two datasets remains strong, as also demonstrated in the experiment by Melling et al. (2024). These differences fall within an acceptable range and do not compromise the reliability of ArcticDEM as a high-quality reference dataset.

Reference:

[5] Melling, Laura, et al. "Evaluation of satellite methods for estimating supraglacial lake depth in southwest Greenland." The Cryosphere 18.2 (2024): 543-558.

[25] Echelmeyer, Keith, T. S. Clarke, and Will D. Harrison. "Surficial glaciology of jakobshavns isbræ, West Greenland: Part I. Surface morphology." Journal of Glaciology 37.127 (1991): 368-382.

In addition, to clarify this point, we have excerpted relevant descriptions from these two papers:

'*Due to the sparse temporal sampling of ArcticDEM and the need to resolve empty basins, the DEMs are not temporally concurrent with the ICESat-2 and Sentinel-2 data. As a result, the smallest period between the ArcticDEM and ICESat-2 acquisition dates was approximately 2 months (Lake 4), and the largest period was approximately 11 months (Lake 5) (Table A1). As the location and shape of supraglacial lakes are determined by bedrock topography (Echelmeyer et al., 1991), we assume there should be little change in the bathymetry of the lake basins between the data acquisition dates (see Sect. 3.1 for further details).*' Melling et al. (2024)

'Repeated photogrammetry and surveying shows that many large lakes remain fixed in space and that they are not advected along with the moving ice. This seems to imply that the larger surface depressions are tied to bedrock irregularities and are thus dynamically supported.' Echelmeyer et al. (1991)

Q76: 239. Having read to here, I'm not convinced you needed any of the previous section 4.1 on the qualitative analysis. Consider deleting it as the quantitative analysis is what is needed.

A76: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have added spatial maps of the inverted bathymetric errors for the lakes in this section to support the qualitative analysis. The analysis figures are shown as follows:

To further evaluate the performance differences among the models, residuals between the bathymetry derived from ArcticDEM and that obtained from each model were calculated for qualitative comparison. The spatial distribution of the residuals is illustrated in Figure 4 (corresponding to Figure 6 in the original manuscript). In addition, since the ArcticDEM data only contains spatial information of the lake bottom, and lacks water surface elevation information when obtaining bathymetry benchmark data. Therefore, in this study, the edge position of the ArcticDEM lake was determined using ICESat-2 data, with this elevation serving as the water surface elevation.

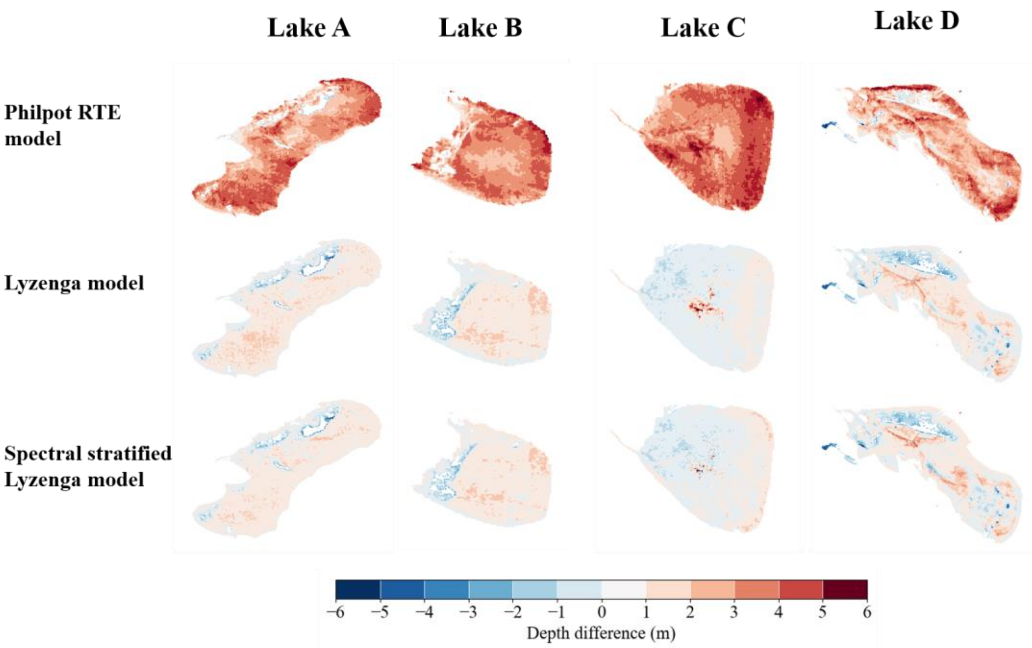

Figure 4. Differences between the water results derived from the three SDB models and those obtained from ArcticDEM. Red areas indicate overestimation, while blue areas indicate underestimation.

Q77: 242. "spectral stratified Lyzenga model" . Check entire paper and refer to this model in the same way throughout. This is quite clear here but you've not so far ever referred to it in this way.

A77: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q78: 246. 'visually demonstrate' => "illustrate"

A78: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q79: 247-9. This sentence is obvious and could be deleted.

A79: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q80: 249-50. Do you mean to refer to Fig 7 here as it doesn't show ArcticDEM validation. Do you mean Fig 8? If so refer to Fig 7 earlier.

A80: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. After considering the revised experimental content and the overall structure of the manuscript, we believe that the contents described in Figures 7 and 8 (originally L246–L260) are no longer necessary to retain. This is because we have supplemented spatial error maps for all lake model inversion results relative to ArcticDEM, and all results have been quantitatively evaluated for accuracy. These additions fully cover the information previously presented in those figures.

Q81: 251. " spectral stratified model" See my comment for line 242. Try to refer to this model consistently throughout the paper.

A81: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q82: 256-7. You say "…are primarily concentrated in the 2-6 m range, with more pronounced differences at the transitions between different spectral layers. This is hard to see, esp. the last point as you don't mark on where the different spectral layer data were used to construct the bathymetry.

A82: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly. We agree that this part makes it difficult to discern the differences clearly. Moreover, after reviewing the context before and after this section, we found that the description does not contribute significantly to the overall logic, so it has been removed. Regarding your comment that "different spectral layer data were used to construct the bathymetry," we would like to clarify that in this study, spectral stratified Lyzenga models were established for different spectral layers, and this has been clearly described in the revised methodology section.

Q83: Fig 8. I'd assumed this would be a subset of Fig 6a, but for the data in the 2 sub-areas shown in Fig 7. But it's 'density' . Why are you showing this? It needs explaining.

A83: Thank you so much for your valuable comments. This part has been removed from the revised manuscript, as the differences between lakes can already be clearly seen from the difference maps and scatter plots, making the additional description redundant.

Q84: 276-7. Delete this sentence from here. This belongs in the Abstract and the start of the Conclusion.

A84: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q85: 278. You say "volume changes" but this implies you're going to determine volume change through time, i.e. applied your final model over more time periods. But you don't do this, which is a shame. Clarify what you mean by 'volume changes' here.

A85: Thank you so much for your valuable comments. We acknowledge that our study focused solely on volume estimation and did not include an analysis of volume variation, and apologize for the inappropriate wording. Accordingly, the term "change" has been removed from the revised manuscript, and long-term variation analysis will be addressed in our future research.

Q86: Figure 9. I'm not fully convinced these add anything although they're nice visually. Suggest put in Supplementary Materials. But what would be more useful to know is what are the volumes of the 4 lakes derived by the traditional Lyzenga method and from the Arctic DEM? How do the 3 estimates compare? Which overestimates vs. underestimates cf. others?

A86: Thank you so much for your valuable comments. We have calculated the lake volumes derived from each model and compared them with those obtained from ArcticDEM. The corresponding analysis has been added to the Discussion section of the manuscript as follows:

5.1 Evaluation of the lake volume estimation

Based on the inverted bathymetry, the lake volume can be readily obtained by integrating the product of pixel area and water depth. In this study, the volumes derived from the Philpot RTE model, the Lyzenga model, and the spectral stratified Lyzenga model were calculated and compared with those obtained from ArcticDEM, as summarized in Table 2.

Table 2. Volume estimation based on the three SDB model and comparison with ArcticDEM.

Study area	SDB model	ArcticDEM volume (m^3)	Estimated volume (m^3)	Relative difference (m^3)
Lake A	Philpot RTE model		15 700 000	+6 690 000 (+74%)
	Lyzenga model	9 010 000	9 620 000	+610 000 (+7%)
	Spectral stratified Lyzenga model		9 480 000	+470 000 (+5%)
Lake B	Philpot RTE model		5 870 000	+2 750 000 (+88%)
	Lyzenga model	3 120 000	3 360 000	+229 000 (+8%)
	Spectral stratified Lyzenga model		3 350 000	+235 000 (+7%)
Lake C	Philpot RTE model		6 560 000	+2 720 000 (+71%)
	Lyzenga model	3 840 000	3 650 000	–189 000 (–5%)
	Spectral stratified Lyzenga model		3 710 000	–127 000 (–3%)
Lake D	Philpot RTE model	16 200 000	26 100 000	+9 900 000 (+61%)

Lyzenga model	16 800 000	+600 000 (+4%)
Spectral stratified Lyzenga model	16 500 000	+300 000 (+2%)

As shown in Table 2, the Philpot RTE model exhibited a consistent overestimation of lake volume across all four sites, ranging from 61% to 88%, which is considerably higher than that obtained from the Lyzenga and spectral stratified Lyzenga models. The latter two produced comparable results, with only minor differences observed for some lakes. Although the overall volume estimations derived from the spectral stratified Lyzenga model are close to those from the conventional Lyzenga model, the stratified approach offers a more physically interpretable framework by considering the spectral heterogeneity within the water body. In particular, this method allows each optical subset to be characterized by a separate empirical relationship, which may better reflect the inherent variations in water optical properties. While the improvement in total volume estimation is limited in this study, the model design provides a conceptually sound extension of the traditional Lyzenga model, and its potential advantages may become more evident in environments with stronger spatial or spectral variability.

Q87: 289-90. You say "While the spectral stratification-based method enhances the accuracy of bathymetric inversion compared to traditional approaches, such as the Lyzenga model…" But you can't say this. You can't generalise. You can only refer to the traditional Lyzenga model here, not ALL 'traditional approaches' by which I assume you also mean the Stumpf model? As that is the only other one you mentioned in your Intro. Did you ever consider applying this one and comparing it? And of course it'd have been valuable and interesting to have applied the 'Philpot' method too and evaluated that.

A87: Thank you so much for your valuable comments. The point you raised is indeed crucial. It is true that we previously generalized all traditional methods and claimed that the spectral stratification algorithm improved upon them, which was inaccurate. Accordingly, we have revised both the text and the experimental design, adding the Philpot RTE model for bathymetry inversion as a reference. The experimental results are shown in Figures 5 and 6.

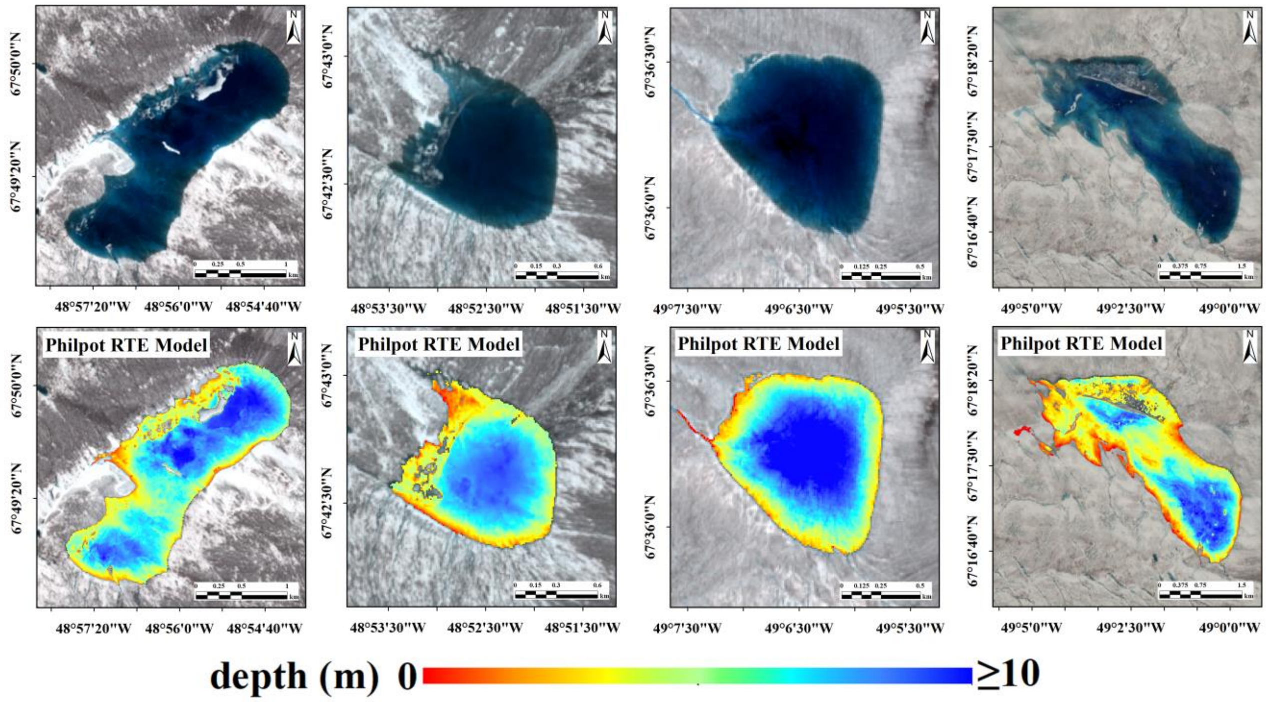

Figure 5. Bathymetry results using the supplementary Philpot RTE method.

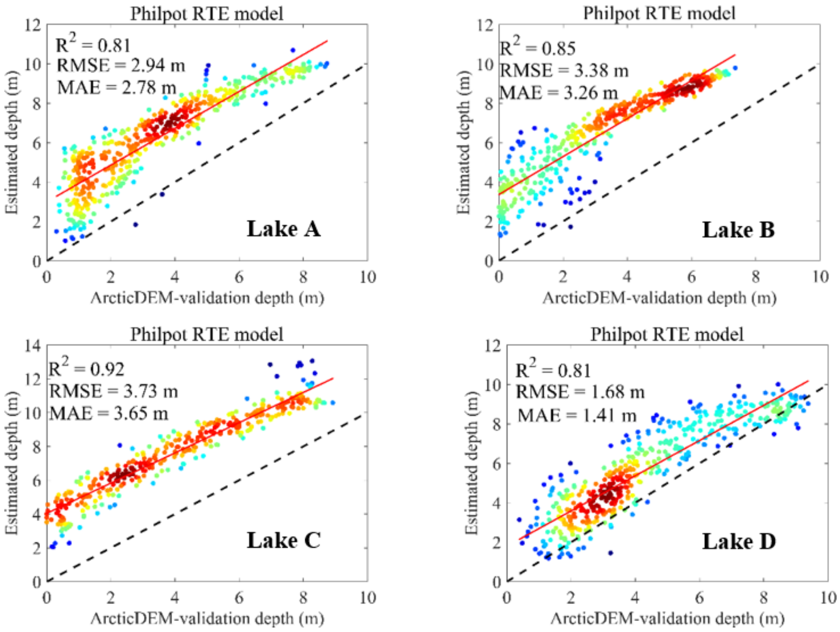

Figure 6. Accuracy validation of the supplementary Philpot RTE method.

Q88: 291. 'amount' => 'number'

A88: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

Q89: 291. What do you mean 'strip-shaped data'? Do you mean " orbital track" ?

A89: Thank you so much for your valuable comments. The term we described indeed refers to the orbital track, and “strip-shaped data” in the manuscript has been replaced with “orbital track data.”

Q90: 291-2. You say there were not enough training sample data. But each lake seems well sampled along the tracks according to Fig 3. What was the total number of sampling points for each lake? Are you saying you need more? Is that really the case?

A90: Thank you so much for your valuable comments. We acknowledge that our original statement was unclear and may have caused confusion. What we intended to convey is not that the total number of ICESat-2 points is insufficient, but rather that their spatial distribution leads to uneven coverage of training points. ICESat-2 provides sufficient points only along its tracks, leaving other areas of the lakes often underrepresented. After spectral stratification, some spectral layers, such as the red band layer, cover very small areas, resulting in even fewer matching ICESat-2 points. Therefore, these layers had to be combined with other spectral layers; this is indeed a major challenge. The revised content is as follows:

Due to the distribution characteristics of ICESat-2 orbital tracks, the available bathymetric samples were not spatially well distributed but were confined along the ICESat-2 orbital tracks. This limitation restricted the full training of the model. Consequently, in this study, the NIR, Red, and Green bands had to be combined for model construction. If additional spectral stratification based on the red band could be introduced, the detection accuracy might be further improved.

Q91: 292. You say the lack of training data "...hindered further improvement in model accuracy" which is poorly phrased but more importantly, you can't conclude that. Perhaps extra data would not have improved accuracy (when cf. ArcticDEM). It MAY have improved the estimation of the model parameters and thereby led to greater accuracy, but you don't know this.

A91: Thank you so much for your valuable comments. We agree that this statement lacked rigor; we have removed the relevant description to avoid any possible misunderstanding.

Q92: 293-4. You say "…where bathymetric variations are minimal, the impact of spectral penetration differences is limited, resulting in only marginal improvements in accuracy." Did you point out such areas? Looking at Fig 6 I do not see greater 'improvements' at high depths cf. shallow depths.

A92: Thank you so much for your valuable comments. Your comment is well taken. The differences between the Spectral Stratified Lyzenga model and the original Lyzenga model are indeed minimal, making it difficult to highlight such regions. In the original manuscript, Figures 7 and 8 were used to visualize relatively noticeable areas and assess their accuracy; however, they still appear very similar visually. Therefore, we have revised the relevant description to make it more rigorous, as follows:

Additionally, because the SGLs are characterized by clear water, shallow depth, and stable substrates, the reflectance of the lake water shows no significant overall variation across different regions. Therefore, the impact of spectral penetration differences is limited, resulting in only marginal changes and improvements in accuracy between the Lyzenga model and the spectral stratified model, as shown in Figures 5 and 6.

Q93: 294. 'Second,…'

A93: Thank you so much for your valuable comments, which have helped make the manuscript more rigorous. We have revised the manuscript accordingly.

At last, we would like to once again express our heartfelt thanks for your professional and detailed review. Your comments are of great importance to the optimization of this manuscript. As you rightly pointed out, there are indeed many shortcomings in this work. We have made every effort to revise it, and those aspects that we were not able to fully address will become important objectives of our future work.

Best regards.