Reply to referee #1

We thank Referee #1 for reviewing the manuscript and the valuable comments and suggestions which we address below. The responses to the referee comments are given in blue italic letters.

This study presents a novel retrieval method based on multi-angle polarimetry to derive the ice fraction near the cloud top. The method involves simulating polarized signals within the cloudbow and slope ranges of the scattering angle. The retrieval was applied to synthetic cloud scenes representing Arctic mixed-phase clouds, as well as to measurements taken during the HALO-(AC)³ campaign. Uncertainties in the retrieval were attributed to 3D radiative effects and the assumption of a vertical ice fraction profile.
Overall, the authors did a very good job of presenting their new approach. The manuscript is well structured. Some suggestions for improving the content are provided below. I can recommend publication once clarification has been provided.

General Comments

1. Retrieval results for the horizontal distribution of the thermodynamic phase of clouds near the cloud tops have shown systematic features that do not necessarily provide a consistent picture of slope and cloud bow range (see Figures 10a, 10b and 13a). In any case, the spatial variability of the ice fraction is significantly impacted by retrieval uncertainty, both quantitatively and qualitatively. Would it be more accurate to claim that the method works well on average for complete cloud scenes rather than for individual cloud elements?
   Thank you very much for noting that. We agree that the spatial variability of the ice fraction is Influenced by the retrieval uncertainties, that the retrieval results are best suited for statistical analyses, and that this should be mentioned. We added a discussion about it:
   "The spatial variability of the ice fraction is affected by the retrieval uncertainty due to the impact of the 3D cloud geometry and the assumption of a vertical ice fraction profile. The validation based on synthetic data, however, showed that the method works well on average for complete cloud scenes. Thus, the results can, for example, be used for a statistical analysis of the cloud thermodynamic phase partitioning of a cloud scene."
2. The authors have shown the I and Q components of the Stokes vector for 550 nm to illustrate the sensitivity to the ice fraction. The polarization camera moreover gives I and Q for a broader spectral range (R, G, B channels). How are the spectral signatures affected by this fact? Is there a lower sensitivity when accounting for the spectral resolution of the camera? Which of the three spectral bands (R, G or B) is finally used for retrieval? From the different figures, I guess it is the green channel. Perhaps I missed a discussion about which of these channels is most appropriate.
   The sensitivity was shown for 550nm, which is very close to the center wavelength of the green color channel and used here as a representative wavelength. However, the same simulations for the different color channels of specMACS look very similar. The retrieval itself can be applied to all three color channels. In Sect. 4 to 6, however, only the results for the green color channel are shown. The blue color channel is generally more influenced by Rayleigh scattering due to the smaller wavelength, which is why the green or red channel should be preferred. The green channel, however, has a better spatial resolution than the red channel due to the Bayer pattern of the polarization filters on the sensors of the polarization-resolving cameras. Therefore, we show the results of the green color channel. We added an additional discussion about that to the paper draft, e.g.:
   "The shown simulations are for a wavelength of 550nm, which is close to the center wavelength of the green color channel of the polarization-resolving cameras of specMACS

and was used here as a representative wavelength. Simulations for the broader color channels of specMACS look very similar."
"The retrieval can, in general, be applied to the red, green, and blue color channels of the polarization-resolving cameras of specMACS. However, in the following sections of this paper, only results for the green color channel are shown, which should be preferred due to the smaller influence of Rayleigh scattering at this wavelength range and the higher spatial resolution of the measurements for this channel (Weber et al., 2024). The results for the other channels look similar."

3. Why is the shadow masking performed after the retrieval of the ice fraction? Would it not be more efficient to mask this data in advance?
Thank you very much for noting that. You are absolutely right, it would be more computationally efficient to apply the shadow mask before the phase partitioning retrieval. We applied it afterwards to be able to have a look at the retrieval results and potential effects of shadows for all data points first and then filtered the results afterwards. We added an explanation to the corresponding section noting that the shadow mask could of course also be applied before, which would speed up the computation time:
"The shadow mask could also be applied before the phase partitioning retrieval to further reduce computation time."

Minor/Specific Comments

1. The method is sensitive to the penetration depth of the radiation into the cloud. The retrieved ice fraction corresponds to the upper most cloud layer (optical thickness between 1 and 2). What are realistic values of the optical depth of the liquid layer at cloud top? Could you be more specific in the introduction about what mixed-phase clouds typically look like in the Arctic?
We added more details about low-level Arctic mixed-phase clouds to the introduction. The typical geometrical thickness of the liquid layer is on the order of 100m and typical liquid water paths in these clouds are around 100 g/m^2. Thus, the optical thickness of the liquid water is on the order of 10, and thus larger than the vertical optical thickness the retrieval is sensitive to.
2. P3l82: "10 m resolution" → Specify a flight altitude that complies with this resolution.
The resolution of 10m is for a typical flight altitude of 10km. We added this information.
3. P4 Fig1a: The figure is not discussed in this section, only Fig. 1b.
Thank you very much for noting that. Panel a is referred to but not explained in detail. For completeness, nevertheless, we would like to keep panel 1a.
4. P5l121: "Here, cases where the polarization signal is saturated and not saturated are distinguished." → "Here, cases where the polarization signal of Q is saturated and not saturated are distinguished (see Sec. 3.4)." I found it difficult to understand the meaning of 'saturated' here. This becomes clear in Section 3.4.
Changed as suggested.
5. P6l146: "… from the cloudbow range to cover the complete scattering angle range" Maybe add "available" before "scattering angle"
Changed as suggested.
6. P6l160: "see Fig. 3b" I would suggest referring to Fig. 1b instead, since Fig. 3b has not yet been introduced.
Changed as suggested.
7. P7l163: "In case the geometry does not allow for observing the cloudbow…" This case is not included in Fig. 2. Maybe revise the schematics.
We added this case to the schematics.

8. P7l175: “but the accuracy of the retrieved ice fraction is not affected” Why not?
   The cloudbow detection is meant as a pre-selection of the data. For signals that are classified as cloudbow, the phase partitioning retrieval is performed next and this retrieval determines the ice fraction – not the cloudbow detection. Thus, if too many signals without a cloudbow are falsely classified as cloudbow, the phase partitioning retrieval is applied to more data than necessary, which increases the computation time. But, the ice fraction is not affected. We added more detail about that.
9. P7l192: “Unknown parameters are the total optical thickness” → “Unknown parameters are the total cloud optical thickness”
   Changed as suggested.
10. P8l204: “Thus, the retrieved ice fraction has to be interpreted as an effective ice fraction under the assumption of a homogeneously mixed cloud.” Later, the ice fraction is related to the cloud top layer. This statement may give the reader the wrong impression, of what the effective ice fraction provides. It’s not representing the whole cloud layer.
    We changed the sentence to: “Thus, the retrieved ice fraction has to be interpreted as an effective ice fraction of the upper most part of the cloud under the assumption of a homogeneously mixed cloud.” This should make clear that it is not representing the whole cloud layer.
11. P8l214: “total intensity I are compared to simulations with a worst-case assumption of f_{ice} = 1, since ice clouds are brighter than liquid clouds” I wouldn’t call it a worst-case assumption. It’s rather an extreme case. Further, add “within this range of scattering angles”. Ice clouds are not generally brighter than liquid clouds.
    Changed as suggested.
12. P8l221: “I and Q are converted to reflectivity as in Weber et al. (2025)” Do the previous plots show R_I and R_Q (notation from Weber et al., 2025) or I and Q? Which downward irradiance (E_dw) is used to calculate the reflectivity? E_0*cos(theta) is only a rough estimate for E_dw at flight altitude.
    The plots actually show I and Q as denoted in the figure labels and explained in the text. For the optimization, we convert the measurements to reflectivity using the downward irradiance at the top of the atmosphere from the ATLAS3 data with modtran version 3.5, which is included in libRadtran. It is true that this is only a very rough estimate of the reflectivity at flight altitude. However, the only reason for converting the measurements of I and Q to reflectivity is that the neural network forward operators provide reflectivity. The conversion of simulated I and Q for the forward operators was performed the exactly same way and the the conversion was only performed to obtain roughly normalized values for the neural network training. As the conversion for both, measurements and forward operators is consistent and only a multiplication with a constant factor for every data point, it does not affect the retrieval results and I and Q are in this case equivalent to R_I and R_Q. We changed the symbols in formula 2 to R_Q for consistency and added some information about the reason for the conversion.
13. P10l249: “there are two possible extreme cases” There are certainly more extreme cases. I suggest to rewrite the sentence. “For the phase partitioning in low-level Arctic mixed-phase clouds, we assume two extreme cases for the vertical profile.”
    Changed as suggested.
14. P10 Fig4: I am having difficulty understanding the illustration. Assuming the black line represents the ice fraction, Fig. 4b is understandable. However, the horizontal line in Fig. 4a, which is located between the liquid and ice phases, cannot represent the ice fraction. It must be either 0 for the upper liquid part or 1 for the lower part. Please clarify.

Yes, the black line in Figure 4a and 4b represents the ice fraction at a given height. In Figure 4a the black line denotes an ice fraction of 0 at the larger heights of the upper liquid cloud (the black line overlaps there with the y-axis) and then increases in a single step at the height of the interface between the liquid water and the ice cloud to 1 at lower altitudes. So, the use of the black lines should be consistent between both panels. We added this information to the figure capture and slightly reworded the description.

15. P11l280: "In addition, there is agreement between the threshold values determined for the two-layer cloud and the profile cloud." Does this refer to Fig. 6b? I wouldn't call it an agreement, given that the threshold values derived for the two cloud profiles differ by around 0.5.
    This refers to Fig. 6a and 6b, but you are right that there are differences. We changed the sentence to: "In addition, the threshold values determined for the two-layer cloud and the profile cloud show only small differences between about 0 and 0.5 for the cloudbow angular range in both cases."
16. P13l313: "A realistic cloud field…" The realistic aspect is the geometry; the cloud microphysics is not.
    We changed the wording to "A cloud field with realistic cloud geometry".
17. P14l323, l326: I suggest to combine Fig. 7 and Fig. 12 here, as Fig.12 is discussed here already.
    We have also thought about combing the two figures. Fig. 12 is actually discussed together with Fig. 7 but also in Sect. 6. The question was, where does it fit better. We finally decided to put Fig. 12 into Sect. 6, since it is related to measurement data and also very helpful for the interpretation of the retrieval results in Sect. 6, without having to jump forth and back to a combined Fig. 7.
18. P14 Fig7: Add "Retrieved" in front of "cloud top height" in the figure caption.
    Changed as suggested.
19. P15l355: "The cloudbow range shows a small bias even for the plane-parallel assumption…" Maybe add "mean" in front of bias. Here and elsewhere.
    Changed as suggested.
20. P16l369: "reflectance"→"reflectivity"?
    Changed as suggested.
21. P17l143: "but smaller standard deviation" Actually, the numbers are almost the same.
    We changed the wording to "a slightly smaller standard deviation".
22. P18l417: "In the slope range, there seems to be a compensating effect between the influence of 3D cloud geometry and the assumption of the vertical ice fraction profile, …" I cannot follow the argument here. Compensation may only occur in the 1D case when 3D radiative effects are not considered. However, I think the authors are referring to Fig. 10 d–f and the 3D-based retrieval. Please clarify.
    The statement refers to differences between the mean differences and standard deviations shown for homogeneously mixed 3D clouds and realistic 3D clouds in Fig. 8 and 10, respectively. For the cloudbow angular range, for the IDEFAX and plane-parallel clouds, the errors are increased for realistic clouds. The additional uncertainty due to the vertical ice fraction profile increases the error. In contrast, the errors are slightly decreased in the slope angular range. This is due to the combined influence of 3D cloud geometry and the vertical ice fraction profiles. Therefore, we wrote that the influences of both effects seem to compensate each other. We rewrote the corresponding section to make this clearer.
23. P18l420: "In all cases, however, there is a significant contribution of the 3D cloud geometry to the total retrieval error. The influence of the assumption of the vertical ice fraction profile can also not be neglected." It's a quite general statement here. Can you estimate which one

has the bigger effect on the retrieval?
We tried to quantify the influence of both effects by applying the retrieval to synthetic data of homogeneously mixed 3D clouds and realistic clouds in Sect. 5.2 and 5.3 and compare the mean differences and standard deviations. From the results it is hard to draw a general conclusion, besides the fact that both effects are contributing significantly to the uncertainty. For the cloudbow angular range, the effect of 3D cloud geometry seems to introduce a larger fraction of the error, especially for the 1D forward operator. For the slope angular range, it is hard to tell which effect dominates due to the compensating effects. We added a reference to the differences in the text.

24. P20l455: "… IDEFAX demonstrated a better performance …" It's not completely true for the cloudbow range.
For the cloudbow angular range for realistic clouds, the mean bias is smaller for the IDEFAX and the standard deviation is almost similar to the one for 1D clouds (see Fig. 10). For the slope angular range, however, there is a larger mean bias for the IDEFAX but the width of the distribution is smaller. We added a "generally " to the sentence to make clear that there are in general improvements but in some cases of course also slightly worse results.
25. P20l456: "before applying the shadow mask" Why does Fig. 13 not show the final result after applying the shadow mask?
We have applied the shadow mask to the data, but the results before applying the mask showed a better overview and were easier to understand due to the larger number of data points and smaller "gaps". This is the reason why we showed the retrieval results without the mask and provided additional information about the cloud geometry (i.e. cloud top height) in Fig. 12.
26. P21l460: "Evaluation of the angular range between the slope range and the cloudbow range is not possible…" I'm not sure what is meant here. Is there no comparison between the retrieved ice fractions from slope range and cloudbow range possible?
In Fig. 13, there is a horizontal stripe without any retrieval results. The cloud targets in this area were observed only for scattering angles in between the slope and the cloudbow range, and neither the former nor the latter was completely covered. Therefore, no ice fractions can be derived in this case. This is what the sentence refers to. It should explain the missing data in Fig. 13. We changed and extended the explanation to:
"The results in the upper part of the panels correspond to the slope angular range, whereas the results in the lower part are for the cloudbow angular range. Evaluation of the area in between is not possible since neither the slope nor the cloudbow angular range are completely observed for these cloud targets. The complete angular range from 135.9° to 160° is needed to determine the cloud droplet size distribution for the phase retrieval in the cloudbow angular range. In addition, the retrieval for the slope range is only possible for minimum scattering angles of 80° and smaller since the sensitivity of the slope angular range to cloud thermodynamic phase decreases with increasing minimum scattering angle…"

Technical Comments

1. P5l116: "so-called L1C data" → "so-called level1C (L1C) data".
Changed as suggested.
2. P7l187: I suggest to remove "(unknown)"
Changed as suggested.
3. P7l188: "These include …" Perhaps consider splitting the very long sentence.
We split the sentence as suggested.
4. P21 Fig 13: Number of contour lines are hard to read.
We increased the font size of the contour labels.