Answers to reviewers

January 2, 2026

REV1 The authors have done a nice job responding to the concerns the other reviewer and I had with the initial draft. There is one minor thing that needs adjusting. The middle panel of Figure 20, and the associated text, is (almost certainly) showing the correlation between the uncertainties in the retrieved Dei and OD – not the correlation between the actual retrieved values.

ANSWER
As we have added in the text in Section 5, the correlation between the cloud parameters OD and D_{ei}, as well as the IC and the DOFs, are computed at the end of the first retrieval iteration in order to quantify the intrinsic information content of the spectral measurements independently of retrieval convergence effects. Therefore, these are not correlations between the retrieved cloud parameters, but rather correlations between the parameters at the end of the first iteration of the inversion algorithm. We have also modified the caption of Figure 20 to clarify this point.

REV2 I appreciate the authors' replies to my comments and some added analyses. Here are a few further comments: 1. The finding that temperature information is limited to 1km and water vapour 4km is interesting, and surprising - how can water vapour be constrained if temperature isn't? 2. Fig 20. Define DOF. I'm confused by what each panel shows and means here. Panel a: what state variables (and how many of them) is the IC assessed for? Panel c: how is DF quantified? Why does panel c show little difference while Panel a shows a larger difference? 3. Fig 21: can you clarify whether the "retrieved" states lead to a better closure in MIR than FIR - this isn't so clear concerning the water vapour lines (larger non-closure also noticed at the higher wavenumber end of the spectrum)! Is this due to the water vapour (uncertainty) or residual effect of habit fitting? 4. Regarding the cloud inhomogeneity - what does the in situ data say about this?

ANSWER
1. This difference in sensitivity is not surprising. Our previous work based on REFIR-PAD measurements at Dome-C, Antarctica — particularly Di Natale et al. (2017) — demonstrates, via singular value decomposition (SVD), that temperature sensitivity is mainly confined to the first kilometer above the surface, while water vapour sensitivity extends up to about 4 km. This is precisely the reason why the analysis of REFIR-PAD measurements is performed by limiting the number of fitted vertical levels accordingly. With the current retrieval setup, temperature is more constrained than water vapour, since it is retrieved using fewer fitted levels. Moreover, in both cases the a priori constraints are intentionally chosen to be sufficiently loose with respect to the actual atmospheric profiles, so as to prevent any undue over-constraining of the retrieval.

2. The degrees of freedom (DOFs) are computed as the trace of the averaging kernel matrix restricted to the two cloud parameters, namely the optical thickness (OD) and the effective diameter (D_{ei}), since the objective was precisely to assess the differences with respect to the cirrus analysis. The information content (IC) is computed as Shannon information content, i.e.

$$\mathrm{IC} = \frac{1}{2}\ln\left|\mathbf{S}_a\mathbf{S}_x^{-1}\right| = \frac{1}{2}\ln\left|(\mathbf{I}-\mathbf{A})^{-1}\right| = \frac{1}{2}\sum_{i=1}^{2}\ln\left(\frac{1}{1-\lambda_i}\right) \tag{1}$$

where $\mathbf{S}_a$ is the a priori covariance matrix and $\mathbf{S}_x$ is the posterior covariance matrix for the parameters $\mathbf{x} = (D_{ei}, OD)$ and the vertical bars denote the determinant, $\mathbf{A}$ is the averaging kernel matrix, $\mathbf{I}$ is the identity matrix, the vertical bars denote the determinant, and λ_i are the eigenvalues of $\mathbf{A}$. The second equality follows from the relationship $\mathbf{S}_x = (\mathbf{I}-\mathbf{A})\mathbf{S}_a$, while the last equality is obtained by diagonalising the averaging kernel matrix and expressing the determinant as the product of its eigenvalues. IC and DOFs are evaluated at the first iteration of the retrieval, in order to quantify the intrinsic information content of the spectral measurements independently of retrieval convergence effects. As already mentioned, the degrees of freedom (DF) are computed as the trace of the averaging kernel matrix, restricted to the two cloud parameters:

$$\mathrm{DF} = \mathrm{tr}(\mathbf{A}) = \sum_{i=1}^{2} \lambda_i \tag{2}$$

Although the difference in degrees of freedom between the two spectral configurations is small, the corresponding difference in information content is much larger. This is because the inclusion of far-infrared radiances significantly increases the sensitivity of the measurements to the retrieved parameters, reducing their dependence on the a priori. This effect is evident from the fact that the information content in Eq. 1 is computed as the logarithm of the inverse of (1 minus the eigenvalues of the averaging kernel), so as these eigenvalues approach 1, the information content increases rapidly. We have added the following sentence in Section 5 to clarify the procedure and the indicators used:

To explain this behaviour, we investigated three diagnostic indicators, namely the Shannon information content (IC), the number of degrees of freedom (DOFs), and the correlation between the cloud parameters, namely the optical thickness (OD) and the effective diameter (D_{ei}), since the objective was precisely to assess the differences with respect to the cirrus analysis. The IC is computed as (Rodgers, 2000):

$$\mathrm{IC} = \frac{1}{2} \ln \left| \mathbf{S}_a \mathbf{S}_x^{-1} \right| = \frac{1}{2} \ln \left| (\mathbf{I} - \mathbf{A})^{-1} \right| = \frac{1}{2} \sum_{i=1}^{2} \ln \left(\frac{1}{1 - \lambda_i} \right) \tag{3}$$

where $\mathbf{S}_a$ is the a priori covariance matrix and $\mathbf{S}_x$ is the posterior covariance matrix for the parameters $\mathbf{x} = (D_{ei}, OD)$ and the vertical bars denote the determinant, $\mathbf{A}$ is the averaging kernel matrix, $\mathbf{I}$ is the identity matrix, the vertical bars denote the determinant, and λ_i are the eigenvalues of $\mathbf{A}$. IC and DOFs are evaluated at the first iteration of the retrieval, i.e. around the a priori state, in order to quantify the intrinsic information content of the spectral measurements independently of retrieval convergence effects. The DOFs are computed as the trace of the averaging kernel matrix restricted to the two cloud parameters:

$$\mathrm{DOF} = \mathrm{tr}(\mathbf{A}) = \sum_{i=1}^{2} \lambda_i \tag{4}$$

3. The issue in the FIR portion, as already explained at line 483, is related to the difficulty in achieving an accurate characterisation of the instrument line shape in that spectral region. The residuals increase where CO_2 and H_2O lines are very dense, as shown in Figure 21. They are not due to the presence of clouds; in fact, the degraded spectrum demonstrates that, in general, within the transparency micro-windows the residuals are smaller than or comparable to the noise level.

4. The INCAS KA probe data show that the variability in particle size occurs mainly along the vertical direction, as illustrated in Figure 11, which shows that the effective particle radius varies mostly between 20 and 50 μm with altitude. However, the radiative effect is determined by the integral contribution over the vertical distribution of particle sizes.

References

Di Natale, G., Palchetti, L., Bianchini, G., and Guasta, M. D.: Simultaneous retrieval of water vapour, temperature and cirrus clouds properties from measurements of far infrared spectral radiance over the Antarctic Plateau, Atmos. Meas. Tech., 10, 825–837, 2017.

Rodgers, C. D.: Inverse methods for atmospheric sounding : theory and practice, World Scientific Publishing, 2000.