Review Comments:

This manuscript presents an extension of Dynamic Mode Decomposition (DMD) aimed at better capturing extreme climate anomalies. The proposed "extreme DMD" framework, which incorporates a penalisation term for extremes, is clearly relevant for climate science and has potential for broader applications in analyzing and predicting extreme events. The case studies on the 2003 and 2010 European heatwaves demonstrate the promise of the approach.

Below I list the specific comments:

Major Comments

1. The study employed two heatwave examples to demonstrate the superiority of the extreme DMD framework. Mathematical metrics (MSE, L∞, norms, SSIM) were applied to evaluate the reconstruction. However, both the comparison figures and the metrics do not reveal a clear advantage of extreme DMD over the normal DMD. In particular, for the 2003 heatwave event, the difference in MSE between the two methods is minimal, and the SSIM values are equal. Can these comparisons pass the significance test? How, then, can one convincingly demonstrate that extreme DMD possesses a stronger advantage than the normal DMD approach?
2. Since the study focuses on applying DMD to extreme events, only heatwaves are analyzed. It remains unclear whether the proposed method is applicable to other types of extremes, such as cold spells or heavy precipitation. While the authors briefly acknowledge this limitation in line 364, the issue is particularly important given that the study's title emphasizes "extremes." Without demonstrating applicability beyond heatwaves, the generality and broader relevance of the method remain uncertain. The authors should either provide additional analysis or clearly qualify the scope of their conclusions.
3. The manuscript mentions that the method could potentially be extended to prediction, but this remains unexplored. It is recommended that the authors provide a discussion on how the current results could be applied to forecasting, including potential challenges and considerations for operational implementation. It would help enhance the paper's impact on the climate community, where forecasting extreme events is a central goal.

Minor Comments

1. The introduction could benefit from citing more recent applications of DMD in atmospheric science, which would help better position the study within the broader climate dynamics literature.
2. Some figures (e.g., Figs. 5, 9, 16) may be difficult for readers unfamiliar with DMD to interpret. It is recommended that the authors provide additional explanations and detailed descriptions to improve clarity and accessibility.
3. Several sentences in the manuscript are lengthy and could be tightened for clarity. For instance, in Section 2.2 ("This linearisation holds only locally…"), the phrasing could be simplified to enhance readability, particularly for interdisciplinary audiences.