The manuscript has been revised in accordance with the reviewer's comments. Below we provide a point-by-point response describing the corresponding changes made in the revised manuscript.

Comment 1. The terminology KL and KU following eq. (59) will be unfamiliar to some readers, so perhaps it should be omitted. Otherwise, a reference should be given.

Answer: The text

"The matrix $\mathbb{A}$ has $KL = 3M - 1$ subdiagonals and $KU = 3M - 1$ superdiagonals (excluding the main diagonal) and can therefore be stored in banded form and treated using standard band-matrix techniques."

has been replaced by

"The matrix $\mathbb{A}$ has $3M-1$ subdiagonals and $3M-1$ superdiagonals (excluding the main diagonal) and can therefore be stored in banded form and treated using standard band-matrix techniques."

Comment 2. Just before eq. (82), it says "Assuming that the lower boundary conditions can be expressed ...". This was added in response to one of my comments, but I find it unclear. Presumably, this has to do with the problem of large dissipative vertical wavenumbers in the inviscid region. If so, then more context should be added.

Answer: The text

"Assuming that the lower boundary conditions can be expressed in terms of state variables, we consider at the lower boundary z_1 the localized boundary conditions..."

has been replaced by

"In the following, we assume that the lower boundary conditions are expressed in terms of state variables. This assumption excludes the low-viscosity regime in which the dissipative eigenvalues become excessively large and the modal boundary condition discussed above is preferred. Under this assumption, we consider at the lower boundary z_1 the following localized boundary conditions..."

Comment 3 Lines 643-645: It says "These phase shifts can be attributed to differences in the real part of the gravity- wave eigenvalue, or equivalently, in the vertical wavenumber of the ascending gravity-wave mode." It should say the imaginary part of the eigenvalue or the real part of the vertical wavenumber.

Answer: The text

"These phase shifts can be attributed to differences in the real part of the gravity-wave eigenvalue, or equivalently, in the vertical wavenumber of the ascending gravity-wave mode"

has been replaced by

"These phase shifts can be attributed to differences in the imaginary part of the eigenvalue, or equivalently, in the real part of the vertical wavenumber of the ascending gravity-wave mode.

Comment 4. Figure 2: The sharp changes in the third panel (differences between roots in log scale for the general model) around 105 km and 175 km suggest that the imaginary frequency shift may not be sufficient. To investigate this, the authors should look at the real parts of the ascending and descending vertical wavenumbers to verify that they are continuous. It may be that the roots cross each other, meaning that they trade places at the aforementioned altitudes. Imaginary frequency shifting is supposed to prevent this. Further investigation could be left for future work.

Answer: Following the reviewer's suggestion, we investigated the behavior of the real parts of the ascending and descending vertical wavenumbers as functions of altitude. Our numerical experiments indicate that the real parts of some vertical wavenumbers indeed cross in the vicinity of the altitudes where the sharp variations in the third panel of Fig. 2 occur. This suggests that, in these regions, the imaginary frequency shift used in the present work may not be sufficient to completely prevent branch crossings or root interchange. A detailed analysis of the root-tracking procedure and the influence of the imaginary frequency shift on branch continuity would require a dedicated investigation. We have therefore added a remark to the manuscript indicating that the observed behavior deserves further study and will be the subject of future research.

Specifically, the text

"*Pairwise classification of ascending and descending* modes. Fig. 2 compares the imaginary parts of the vertical wavenumbers associated with the ascending and descending gravity-wave modes. The upper panel shows the results obtained with the general model, including ion drag, while the second panel shows the corresponding results for the simplified model. In the general model, the ascending and descending vertical wavenumbers approach each other in the altitude range from approximately 80 km to 120 km. To demonstrate that the ascending and descending gravity-wave modes remain distinct over the entire altitude range, the third and fourth panels show the quantity $\mathrm{Im}(k_{z4}) - \mathrm{Im}(k_{z1})$ on a logarithmic scale for the general and simplified models, respectively. These plots confirm that the two gravity-wave modes remain separated at all altitudes, although the separation becomes very small in the transition region. Fig. 2 shows the imaginary parts of the vertical wavenumbers for all wave modes ($k_{zn}, n = 1, \ldots, 6$) computed using the general model. The plot illustrates the clear separation between the gravity-wave modes and the viscosity- and thermal-conduction modes. The latter two mode pairs approach each other most closely in the altitude range around 100 km."

has been replaced by

"*Pairwise classification of ascending and descending modes.* Figure 2 compares the imaginary parts of the vertical wavenumbers associated with the ascending and descending gravity-wave modes. The upper panel shows the results obtained with the general model, including ion drag, while the second panel shows the corresponding results for the simplified model. In the general model, the imaginary parts of the ascending and descending vertical wavenumbers approach each other in the altitude range from approximately 80 km to 170 km. To illustrate the degree of separation between these two modes, the third and fourth panels show the quantity $\mathrm{Im}(k_{z4}) - \mathrm{Im}(k_{z1})$ on a logarithmic scale for the general and simplified models, respectively. These plots demonstrate that the imaginary parts of the two gravity-wave modes remain separated at all altitudes, although the separation becomes very small in the transition region. A complete assessment of branch continuity, however, requires consideration of both the real and imaginary parts of the vertical wavenumbers. Additional numerical experiments indicate that the real parts of the ascending and descending gravity-wave modes cross near the altitudes (approximately 105 km and 175 km) where sharp changes in the quantity $\mathrm{Im}(k_{z4}) - \mathrm{Im}(k_{z1})$ are observed. A detailed investigation of the root-tracking procedure and the influence of the imaginary frequency shift on branch continuity will be the subject of future research. Figure 2 shows the imaginary parts of the vertical wavenumbers for all wave modes ($k_{zn}, n = 1, \ldots, 6$) computed using the general model. The plot illustrates the clear separation between the gravity-wave modes and the viscosity- and thermal-conduction modes. The latter two mode pairs approach each other most closely in the altitude range around 100 km.

Comment 5. Lines 690-705: Please mention here that the solutions are not expected to be affected by changes in δ if the LC condition holds and cite Knight et al. (2022, following eq. (2.58)) which discusses this.

Answer: After the sentence:

"All five solutions satisfied this condition and produced nearly identical maximum values of the perturbed horizontal velocity, vertical velocity, and temperature (Table 2)".

the following sentence has been added:

"This behavior is expected because, once the layerwise causality condition is satisfied, the physical solution is not expected to depend on the particular value of the imaginary frequency shift δ, provided that δ remains within the admissible interval (see Knight et al., 2022, following Eq. (2.58))." The cited reference is: H. Knight, D. Broutman, S. Eckermann, Full-wave anelastic and compressible Fourier methods for gravity waves in the thermosphere, Wave Motion 110 (2022) 102894. https://doi.org/10.1016/j.wavemoti.2022.102894.

Comment 6. Figure 9: What are the y-axis units in Figure 9? If they are relative errors, then this should be stated in the figure labels or caption. Also, how are they normalized?

Answer: The text

"Fig. 9 shows the Root-mean-square (RMS) differences in the perturbed horizontal velocity, vertical velocity, and temperature between the selected solution and the remaining solutions. For each altitude, the RMS difference is computed over the entire time interval. The RMS differences remain below 0.003 (0.3%) throughout the altitude range, demonstrating that the computed wave fields are practically insensitive to the selected value of the imaginary frequency shift within the considered interval."

has been replaced by

"Fig. 9 shows the relative Root-Mean-Square (RMS) differences in the perturbed horizontal velocity, vertical velocity, and temperature between the selected solution and the remaining solutions. For each altitude, the relative RMS difference is computed over the entire time interval by dividing the RMS difference by the RMS value of the corresponding field of the selected solution over the same time interval."

Also, the caption of Fig. 9:

"Root-mean-square (RMS) differences in the perturbed horizontal velocity $\overline{u}$, vertical velocity $\overline{w}$, and temperature perturbation $\overline{T}$ between the selected solution ($\delta = 16.62 \times 10^{-6}$ s^{-1}) and the remaining solutions corresponding to different values of the imaginary frequency shift δ. For each altitude, the RMS difference is computed over the entire simulation time interval."

has been replaced by

"Relative Root-Mean-Square (RMS) differences in the perturbed horizontal velocity $\overline{u}$, vertical velocity $\overline{w}$, and temperature perturbation $\overline{T}$ between the selected solution ($\delta = 16.62 \times 10^{-6}$ s^{-1}) and the remaining solutions corresponding to different values of the imaginary frequency shift δ. For each altitude, the relative RMS difference is computed over the entire simulation time interval by dividing the RMS difference by the RMS value of the corresponding field of the selected solution over the same time interval."

Comment 7. Lines 933-937. Where it refers to Section 2.2 and eq. (3.35), it should be made clear that this refers to Knight et al. (2024). This can be done by saying "their Section 2.2" and "their eq. (3.35)".

Answer: The text:

"In Knight et al. [Knight2024], the dispersion relation reported in Eq. (3.35) was derived by setting the determinant of the corresponding coefficient matrix equal to zero and applying the variable transformation described in Section 2.2."

has been replaced by

"In Knight et al. [Knight2024], the dispersion relation reported in their Eq. (3.35) was derived by setting the determinant of the corresponding coefficient matrix equal to zero and applying the variable transformation described in their Section 2.2." The cited reference is: H. Knight, D. Broutman, S. Eckermann, Compressible and anelastic governing-equation solution methods for thermospheric gravity waves with realistic background parameters, Theoretical and Computational Fluid Dynamics 38 (2024) 479–509, https://doi.org/10.1007/s00162-024-00709-X