



Rapid Communication: Middle Pleistocene Transition as a Phenomenon of Orbitally Enabled Sensitivity to Initial Values

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Abstract. The Middle Pleistocene Transition (MPT), i.e., the “fast” transition from ~41- to ~100-kyr rhythmicity that occurred about 1 Myr ago, remains one of the most intriguing phenomena of the past climate. The cause of this period shift is generally thought to be a change within the Earth System, since the orbital insolation forcing does not change its pattern through the MPT. Using a dynamical model rooted in ocean chemistry, we advance three novel concepts here: (a) the MPT could be a dominant-period relaxation process that is strongly dependent on the initial state of the system, (b) this sensitivity to the initial state is enabled by the orbital forcing, and (c) depending on the amplitude of the orbital forcing and initial values, the MPT could have been not just of the 40 – 80 kyr type, as we observe in the available data, but also of a 20 – 40, 80 – 100, 40 – 120, or even 80 – 40 kyr type.

1. Introduction

Around 1 Myr ago, the dominant period of the glacial-interglacial cycles shifted from ~41 to ~100 kyr (see Fig. 1).

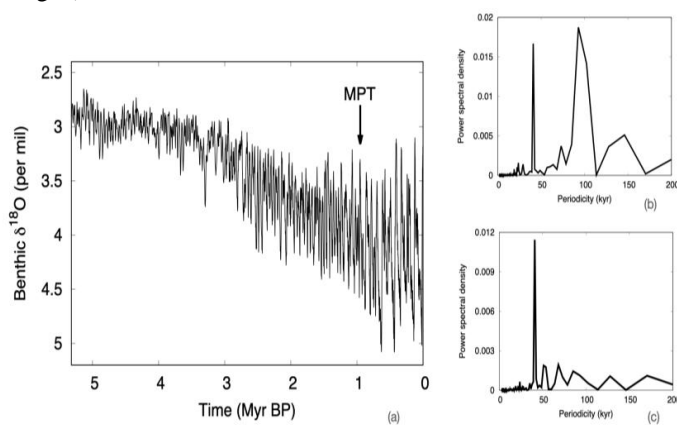


Figure 1. (a) The Mid-Pleistocene Transition (MPT) is visible as a shift in the period and amplitude of the glacial-interglacial cycles around 1 Myr ago in the benthic foraminifera $\delta^{18}\text{O}$ record (Lisiecki and Raymo, 2005), which is a proxy for global ice volume (time goes forward to the right). (b) $\delta^{18}\text{O}$ frequency spectrum between 1 Myr ago and the present (post MPT); note the dominant peak at ~100 kyr. (c) $\delta^{18}\text{O}$ frequency spectrum between 2 Myr and 1 Myr ago (pre MPT); note the dominant peak at ~40 kyr. This figure was reproduced from Shackleton et al. (2023).



The disambiguation of this change in glacial rhythmicity, i.e., the Middle Pleistocene Transition, or MPT hereafter, has been a challenge for the scientific community throughout the last few decades (e.g., Saltzman and Verbitsky, 1993; Clark and Pollard, 1998; Tziperman et al., 2006; Crucifix, 2013; Mitsui and Aihara, 2014; Paillard, 2015; Ashwin and Ditlevsen, 2015; Verbitsky et al., 2018; Willeit et al., 2019; Riechers et al., 2022; Shackleton et al., 2023; Carrillo et al., 2025; Scherrenberg et al., 2025; Pérez-Montero et al., 2025). Since the orbital insolation forcing does not change its pattern through the MPT, several proposed hypotheses included slow changes in governing parameters *internal to the Earth System*. These may define intensities of positive (e.g., variations in carbon dioxide concentration, Saltzman and Verbitsky, 1993) or negative (e.g., regolith erosion, Clark and Pollard, 1998) system feedbacks or a combination of positive and negative feedbacks (e.g., the interplay of ice-sheet temperature vertical advection and the geothermal heat flux, Verbitsky and Crucifix, 2021). The importance of the orbital forcing in generating the pre-MPT ~41 kyr cycles and post-MPT ~100 kyr cycles has widely been acknowledged. In particular, it has been suggested that orbital cycles either directly drive these cycles (Raymo et al., 2006; Bintanja and Van de Wal, 2008; Tzedakis et al., 2017) or synchronize auto-oscillations of the Earth's climate (Rial et al., 2013; Nyman and Ditlevsen, 2019; Shackleton et al., 2023). However, the orbital forcing has not been considered to play a role in the origin of the MPT.

Recently, it has been proposed (Verbitsky and Volobuev, 2024) that the orbital forcing may play a bigger role and can also change the dynamical properties of the Earth's climate system. For example, it may change the vertical advection of mass and temperature in ice sheets and make their dynamics sensitive to initial values. Is ice physics unique in this sense? To answer this question, in this paper we will consider the calcifier-alkalinity (C-A) model that describes entirely different physics, focusing on the interactions between a population of calcifying organisms and ocean alkalinity (Omta et al., 2013). Previously, it has been shown that:

(a) The C-A system relaxes slowly to its asymptotic state, i.e., it has a long memory of its initial conditions (Omta et al., 2013);

(b) The asymptotic state of the orbitally forced C-A system depends on its initial conditions (Omta et al., 2016).

We will demonstrate here that the relaxation of the dominant period of the orbitally forced C-A system from its initial value to the asymptotic value can include a sharp transition similar to the MPT. We will also perform a scaling analysis of the C-A model and demonstrate that the asymptotic dominant periods are defined by conglomerate similarity parameters combining the amplitude of the orbital forcing and the initial values. In other words, *the orbital forcing enables the dominant-period sensitivity to initial values*.

2. Ocean calcifier-alkalinity model

The C-A model was first formulated by Omta et al. (2013) and focuses on the throughput of alkalinity through the World's oceans. The alkalinity is a measure for the buffering capacity of seawater that controls its capacity for carbon storage through the carbonate equilibrium (Broecker and Peng, 1982; Zeebe and Wolf-Gladrow, 2001; Williams and Follows, 2011). Alkalinity is continuously transported into the oceans as a consequence of rock weathering on the continents. When alkalinity is added to the ocean, the solubility of CO₂ increases leading to an uptake of carbon from the atmosphere into the ocean. Removal of alkalinity from the water (through incorporation of calcium carbonate into the shells of calcifying organisms and subsequent sedimentation) leads to a lower CO₂ solubility and thus outgassing of carbon from the ocean into the atmosphere. The C-A model assumes that alkalinity A (mM eq) enters the ocean at a constant rate I_0 (mM eq yr⁻¹). Alkalinity is taken up by a population of calcifying organisms C (mM eq) growing with rate constant k ((mM eq)⁻¹ yr⁻¹) and sedimenting out at rate M (yr⁻¹). Altogether, the model equations are:



$$\frac{dA}{dt} = I_0 - kAC \quad (1)$$

$$\frac{dC}{dt} = kAC - MC \quad (2)$$

with t the time (yr). Since there exists observational evidence of variations in calcifier productivity correlated with Milankovitch cycles (Beaufort et al., 1997; Herbert, 1997), we include a periodic forcing term in the calcifier growth parameter k :

$$k = k_0 \left(1 + \alpha \cos \left(\frac{2\pi t}{T} \right) \right) \quad (3)$$

As in Omta et al. (2016) and Shackleton et al. (2023), k_0 is the average value of k , α is the non-dimensional forcing amplitude, and T (yr) is the forcing period.

Simulations with the C-A model are performed in Julia version 1.11.2. As in Shackleton et al. (2023), we use the KenCarp58 solver (Rackauckas and Nie, 2017) with a tolerance of 10^{-16} (code is available on GitHub).

3. Results and Discussion

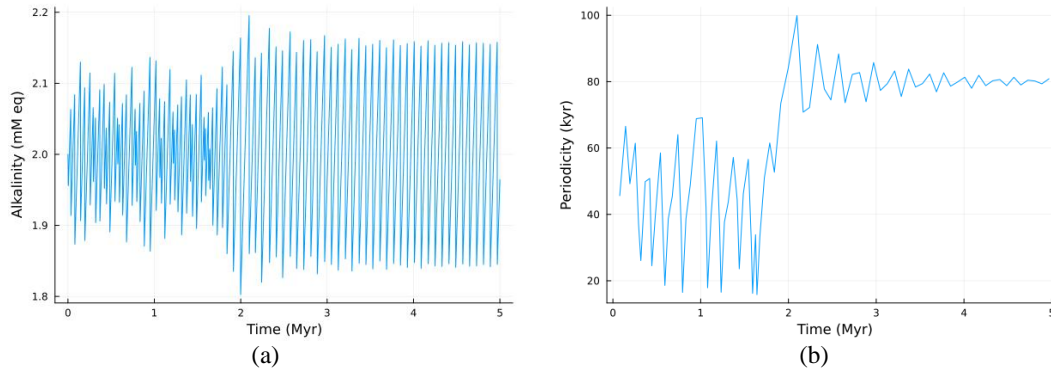


Figure 2. C-A system under orbital forcing ($A(0) = 2.0$ mM eq, $C(0) = 4 \times 10^{-5}$ mM eq, $\alpha = 0.012$, $T = 40$ kyr): (a) alkalinity, (b) dominant period as a function of time

The C-A system (1) – (3) produces sawtooth-shaped cycles in alkalinity, with the alkalinity rising slowly and declining steeply. This corresponds to CO_2 decreasing slowly and increasing rapidly, consistent with the ice-core record (Lüthi et al., 2008). In Fig. 2, a simulation with initial conditions $A(0) = 2.0$ mM eq, $C(0) = 4 \times 10^{-5}$ mM eq, forcing strength $\alpha = 0.012$, forcing period $T = 40$ kyr, and reference values for other parameters (Omta et al., 2016) is shown. The dominant period initially evolves around the forcing period of 40 kyr, then sharply increases (MPT-like) to about 80 kyr (twice the forcing period) and stabilizes at this level. This period shift occurs through a different mechanism than in earlier studies using the C-A model, where period shifts involved noise (Omta et al., 2016) or a positive feedback (Shackleton et al., 2023) to “kick” the system from one dominant period to another one. Here no such kick is imposed: the period shift rather emerges as part of the transient dynamics of the system, as it relaxes from its initial towards its asymptotic state. For the first ~ 1.7 Myr of the simulation, there appears



to be an approximate but not exact frequency lock, from which the system has difficulty escaping. Once the system is out of this approximate frequency lock, its period increases relatively rapidly until it reaches another multiple of the forcing period where the system becomes locked again.

In the following, we analyze how the “pre-MPT” and “post-MPT” periods may depend on the system parameters. In particular, we formulate a scaling law (Section 3.1) that we then investigate in more detail through simulations (Section 3.2).

3.1 Scaling laws

The C-A system of equations (1) – (3) contains seven governing parameters, including the initial conditions. Both the mean initial “pre-MPT” and the asymptotic “post-MPT” periods have to be functions of these seven parameters. Thus, we can write:

$$P = \varphi(I_0, k_0, \alpha, T, M, A(0), C(0)) \quad (4)$$

with P the asymptotic “post-MPT” period. If we take I_0, k_0 as parameters with independent dimensions, then according to the π -theorem (Buckingham, 1914):

$$\frac{P}{\tau} = \Phi \left[\alpha, \frac{T}{\tau}, M\tau, \frac{A(0)}{F}, \frac{C(0)}{F} \right] \quad (5)$$

Here $\tau = (k_0 I_0)^{-1/2}$, $F = \left(\frac{I_0}{k_0}\right)^{1/2}$.

Suppose $T, k_0, I_0, M, C(0)$ are constant; then the adimensional period P is a function of two similarity parameters: the orbital forcing amplitude α and the adimensional initial value $\frac{A(0)}{F}$.

$$\frac{P}{\tau} = \Phi \left[\alpha, \frac{A(0)}{F} \right] \quad (6)$$

Using similar reasoning, we can write for the “pre-MPT” period P_0 :

$$\frac{P_0}{\tau} = \Psi \left[\alpha, \frac{A(0)}{F} \right] \quad (7)$$

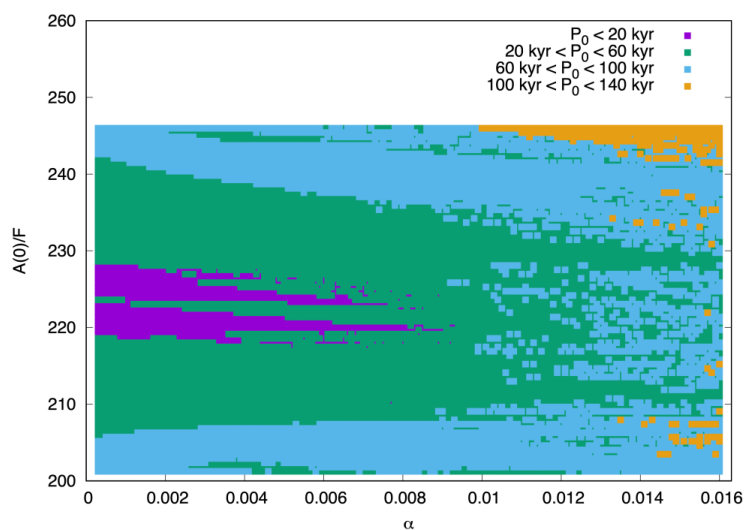
3.2 Scaling law simulations

To investigate the scaling laws (6, 7), we perform a suite of 10-Myr simulations in which we vary α and $\frac{A(0)}{F}$. The average period during the first 1 Myr (P_0 or “pre-MPT”) and the last 1 Myr (P or “post-MPT”)

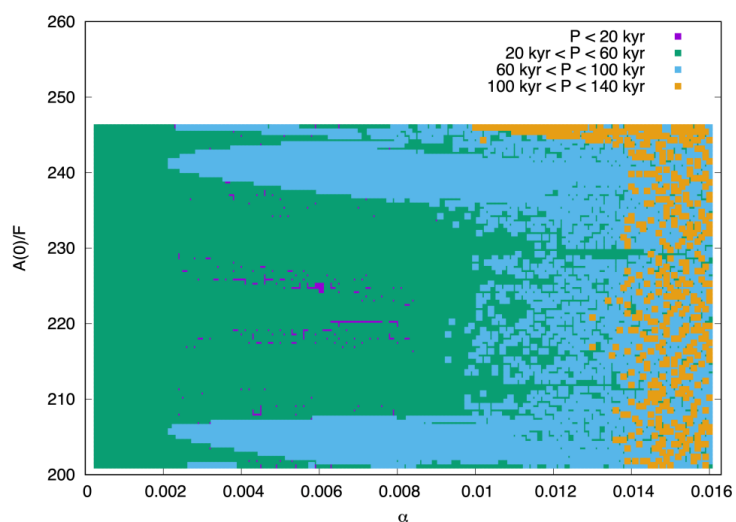
as a function of α and $\frac{A(0)}{F}$ are presented in Figs. 3a and 3b, respectively.



(a)



(b)



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149 **Figure 3.** (a) Pre-MPT periods P_0 (average of first 1 Myr of 10-Myr simulations), and (b) post-MPT
150 periods P (average of last 1 Myr of 10-Myr simulations). Each dot represents one simulation.



From Fig. 3, it can be observed that:

- (a) P_0 and P depend on α and $\frac{A(0)}{F}$ in different manners. Most obviously, $P_0 < 20$ kyr in a significant fraction of the simulations whereas $P > 20$ kyr in almost every simulation. Furthermore, $P > 100$ kyr occurs in many more simulations than $P_0 > 100$ kyr. These differences imply that a period shift emerges in a significant fraction of the simulations.
- (b) When $\alpha \rightarrow 0$, the “post-MPT” period P becomes independent of the initial value $A(0)$ (Fig. 3b). It means that the similarity parameters $\alpha, \frac{A(0)}{F}$ in the C-A system (1) – (3) collide into one conglomerate similarity parameter $\alpha^x \left[\frac{A(0)}{F} \right]^y$ (parameters x and y should be determined through simulations). It provides us with the final form of the scaling law for the “post-MPT” period:

$$\frac{P}{\tau} = \Phi \left\{ \alpha^x \left[\frac{A(0)}{F} \right]^y \right\} \quad (8)$$

The scaling law eq. (8) implies that the orbital forcing affects the dynamical properties of the C-A physics enabling the sensitivity of post-MPT periods to initial values.

- (c) When α increases, the sensitivity of the dominant post-MPT period to the initial conditions $\frac{d\left(\frac{P}{\tau}\right)}{d\left(\frac{A(0)}{F}\right)}$ also increases. Specifically, when $\alpha < 0.002$, as we have already noted, $\frac{P}{\tau}$ is not sensitive to initial values. When $0.002 < \alpha < 0.01$, it takes $\Delta \left(\frac{A(0)}{F} \right) \sim 10$ to obtain a different post-MPT period. Orbital forcing with $0.01 < \alpha < 0.014$ reduces the critical value of initial values changes to $\Delta \left(\frac{A(0)}{F} \right) \sim 1$, and finally for $\alpha > 0.014$ changes as small as $\Delta \left(\frac{A(0)}{F} \right) \sim 0.1$ lead to different post-MPT periods.
- (d) Depending on $\alpha^x \left[\frac{A(0)}{F} \right]^y$, the MPT transition could have been not just 40 – 80 kyr type, but also 20 – 40, 80 – 100, 40 – 120, or even 80 – 40 kyr type (compare Figs. 3a and 3b).

4. Conclusions

The history of climate has been given to us as a single time series. For many years, perhaps somewhat naively, significant efforts have been applied to reproduce this time-series under a unique combination of the governing parameters and thus presumably to explain the history (Peacock al., 2006; Abe-Ouchi et al., 2013; Willeit et al., 2019). The fundamental fact that the dominant-period trajectory is governed by a conglomerate similarity parameter $\alpha^x \left[\frac{A(0)}{F} \right]^y$ (demonstrating a property of incomplete similarity as defined by Barenblatt, 2003) tells us that the MPT could have been produced under very different combinations of the intensity of orbital forcing and initial values. Most intriguingly, the conglomerate similarity parameter $\alpha^x \left[\frac{A(0)}{F} \right]^y$ also tells us that such an “intimate” terrestrial property as the sensitivity of alkalinity-calcination system to initial values manifests itself only under orbital forcing, and thus *MPT exhibits a remarkable physical phenomenon of orbitally enabled sensitivity to initial values.*



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192 **Competing interests:** The authors declare that they have no conflict of interest.

193 **Author contributions:** MYV conceived the research, AWO performed simulations and discovered the

194 MPT-like periodicity relaxation, MYV performed scaling analysis and discovered orbitally enabled

195 sensitivity to initial values. The authors jointly wrote and edited the paper.

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