

1 **Rapid Communication: Middle Pleistocene Transition as a Phenomenon of**
2 **Orbitally Enabled Sensitivity to Initial Values**

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10 **Abstract.** The Middle Pleistocene Transition (MPT), i.e., the “fast” transition from ~41- to ~100-kyr
11 rhythmicity that occurred about 1 Myr ago, remains one of the most intriguing phenomena of the past
12 climate. The cause of this period shift is generally thought to be a change within the Earth System, since
13 the orbital insolation forcing does not change its pattern through the MPT. Using a dynamical model
14 rooted in ocean chemistry, we advance several novel concepts here: (i) the MPT could be a dominant-
15 period relaxation process that may be dependent on the initial state of the system, (ii) this sensitivity to
16 the initial state is enabled by the orbital forcing, (iii) depending on the amplitude of the orbital forcing and
17 initial values, the MPT could have been not just of the 40 – 80 kyr type, as we observe in the available
18 data, but also of a 20 – 40, 40 – 120, or even 80 – 40 kyr type, (iv) when the orbital forcing of the global
19 glaciation-climate model is accompanied by the alkalinity (CO_2) forcing containing a dominant-period
20 shift from 41 kyr to 80 kyr, this ice-climate system produces a 40-to-100 kyr glacial rhythmicity transition
21 resembling the MPT LR04 data, and (v) when the glaciation-climate model is forced by an alkalinity
22 (CO_2) forcing containing a periodicity transition from 20 kyr to 42 kyr, a non-linear interplay of the
23 orbital forcing and of ~40-kyr periods of the alkalinity forcing may produce glaciation periods of ~100
24 kyr that are also consistent with the LR04 data.

25 **1. Introduction**

28 Around 1 Myr ago, the dominant period of the glacial-interglacial cycles shifted from ~41 to ~100
29 kyr. The disambiguation of this change in glacial rhythmicity, i.e., the Middle Pleistocene Transition, or
30 MPT hereafter, has been a challenge for the scientific community throughout the last few decades (e.g.,
31 Saltzman and Verbitsky, 1993; Clark and Pollard, 1998; Tziperman et al., 2006; Peacock et al., 2006;
32 Abe-Ouchi et al., 2013; Crucifix, 2013; Mitsui and Aihara, 2014; Paillard, 2015; Ashwin and Ditlevsen,
33 2015; Verbitsky et al., 2018; Willeit et al., 2019; Riechers et al., 2022; Shackleton et al., 2023; Carrillo et
34 al., 2025; Scherrenberg et al., 2025; Pérez-Montero et al., 2025). Since the orbital insolation forcing does
35 not change its pattern through the MPT, several proposed hypotheses included slow changes in governing
36 parameters *internal to the Earth System*. These may define intensities of positive (e.g., variations in
37 carbon dioxide concentration, Saltzman and Verbitsky, 1993) or negative (e.g., regolith erosion, Clark
38 and Pollard, 1998) system feedbacks or a combination of positive and negative feedbacks (e.g., the
39 interplay of ice-sheet vertical temperature advection and the geothermal heat flux, Verbitsky and Crucifix,
40 2021). The importance of the orbital forcing in generating the pre-MPT ~41 kyr cycles and post-MPT
41 ~100 kyr cycles has widely been acknowledged. In particular, it has been suggested that orbital periods
42 either directly drive these cycles (Raymo et al., 2006; Bintanja and Van de Wal, 2008; Tzedakis et al.,
43 2017, Barker et al., 2025) or synchronize auto-oscillations of the Earth’s climate (Saltzman and Verbitsky,
44 1993, Tziperman et al., 2006, Rial et al., 2013; Nyman and Ditlevsen, 2019; Shackleton et al., 2023).
45 However, the orbital forcing has not been considered to play a role in the origin of the MPT.

46 Recently, it has been proposed (Ma et al., 2024) that the amplitude of the orbital forcing may
47 experience a change on a million-year timescale and this may have its effect on the MPT. Verbitsky and
48 Volobuev (2025) suggested that the orbital forcing may play an even bigger role and can also change the
49 dynamical properties of the Earth’s climate system. For example, it may change the timescale of the

50 vertical advection of mass and temperature in ice sheets and make their dynamics sensitive to initial
51 values. Is ice physics unique in this sense? To answer this question, in this paper we will consider the
52 calcifier-alkalinity (C-A) model that describes entirely different physics, focusing on the interactions
53 between a population of calcifying organisms and ocean alkalinity (Omta et al., 2013). Previously, it has
54 been shown that:

55 (a) The C-A system relaxes slowly to its asymptotic state, i.e., it has a long memory of its initial
56 conditions (Omta et al., 2013);
57 (b) The asymptotic state of the orbitally forced C-A system depends on its initial conditions (Omta et
58 al., 2016).

59 We will demonstrate here that the relaxation of the dominant period of the orbitally forced C-A
60 system from its initial value to the asymptotic value can include a sharp transition similar to the MPT. We
61 will also perform a scaling analysis of the C-A model and demonstrate that the asymptotic dominant
62 periods are defined by a conglomerate similarity parameter combining the amplitude of the orbital forcing
63 and the initial values. In other words, *the orbital forcing enables the dominant-period sensitivity to initial*
64 *values*. We will also prove that what we call an MPT-like event in terms of the alkalinity periodicity can
65 be translated into an MPT event in terms of the glacial rhythmicity.

66 2. Ocean calcifier-alkalinity model

67 The C-A model was first formulated by Omta et al. (2013) and focuses on the throughput of alkalinity
68 through the World's oceans. The alkalinity is a measure for the buffering capacity of seawater that
69 controls its capacity for carbon storage through the carbonate equilibrium (Broecker and Peng, 1982;
70 Zeebe and Wolf-Gladrow, 2001; Williams and Follows, 2011). Alkalinity is continuously transported into
71 the oceans as a consequence of rock weathering on the continents. When alkalinity is added to the ocean,
72 the solubility of CO_2 increases leading to an uptake of carbon from the atmosphere into the ocean.
73 Removal of alkalinity from the water (through incorporation of calcium carbonate into the shells of
74 calcifying organisms and subsequent sedimentation) leads to a lower CO_2 solubility and thus outgassing
75 of carbon from the ocean into the atmosphere. The C-A model assumes that alkalinity A (mM eq) enters
76 the ocean at a constant rate I_0 (mM eq yr^{-1}). Alkalinity is taken up by a population of calcifying
77 organisms C (mM eq) growing with rate constant k ((mM eq) $^{-1}$ yr^{-1}) and sedimenting out at rate M (yr^{-1}).
78 Altogether, the model equations are:

$$80 \frac{dA}{dt} = I_0 - kAC \quad (1)$$

$$81 \frac{dC}{dt} = kAC - MC \quad (2)$$

82 with t the time (yr). Since there exists observational evidence of variations in calcifier productivity
83 correlated with Milankovitch cycles (Beaufort et al., 1997; Herbert, 1997), we include a periodic forcing
84 term in the calcifier growth parameter k :

$$85 k = k_0 \left(1 + \alpha \cos \left(\frac{2\pi t}{T} \right) \right) \quad (3)$$

86 As in Omta et al. (2016) and Shackleton et al. (2023), k_0 is the average value of k , α is the non-
87 dimensional forcing amplitude, and T (yr) is the forcing period.

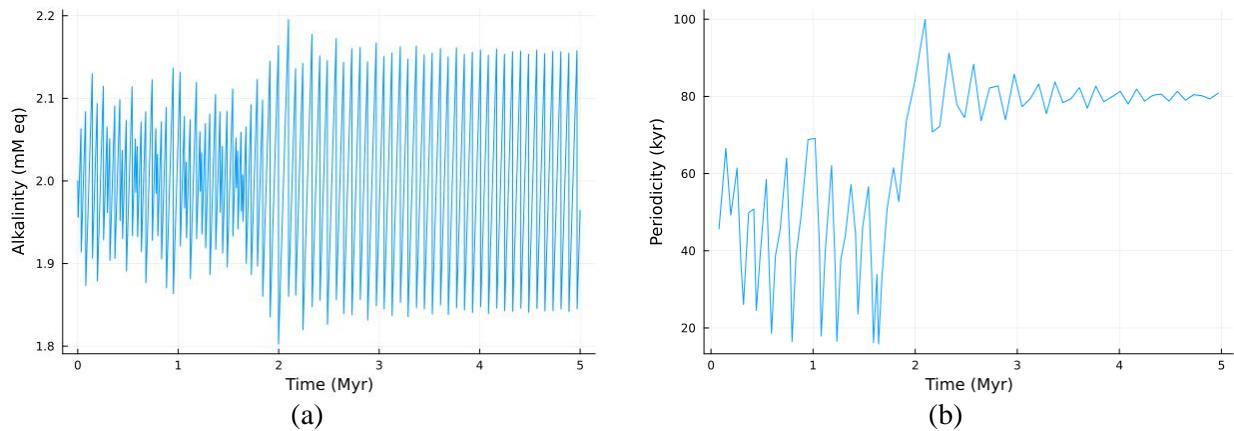
88 Generally speaking, the alkalinity budget is also affected by the seawater carbonate saturation state.
89 In particular, calcite preservation tends to increase with increasing carbonate ion concentration (Broecker
90 and Peng, 1982; Archer, 1996). This carbonate compensation feedback was included in the detailed multi-
91 box version of the calcifier-alkalinity model (Omta et al., 2013). Essentially, carbonate compensation

98 acted as a negative feedback that enhanced the damping of the cycles. If the periodic forcing was
 99 sufficiently strong to overcome this damping, then the model behavior was very similar to the behavior of
 100 the model without carbonate compensation (see Fig. 5 in Omta et al., 2013). Here we chose to use the
 101 simpler, more parsimonious model.

102 Simulations with the C-A model are performed in Julia version 1.11.2. As in Shackleton et al. (2023),
 103 we use the KenCarp58 solver (Rackauckas and Nie, 2017) with a tolerance of 10^{-16} (code is available on
 104 GitHub – <https://github.com/AWO-code/VerbitskyOmta>).

105 3. Results and Discussion

106 The C-A system (1) – (3) produces sawtooth-shaped cycles in alkalinity, with the alkalinity rising
 107 slowly and declining steeply. This corresponds to CO_2 decreasing slowly and increasing rapidly,
 108 consistent with the ice-core record (Lüthi et al., 2008). In Fig. 1, a simulation with initial conditions
 109 $A(0) = 2.0 \text{ mM eq}$, $C(0) = 4 * 10^{-5} \text{ mM eq}$, forcing strength $\alpha = 0.012$, forcing period $T = 40 \text{ kyr}$ (the
 110 obliquity period, i.e., year-average insolation), and reference values for other parameters (Omta et al.,
 111 2016) is shown.



115
 116 **Figure 1.** C-A system under orbital forcing ($A(0) = 2.0 \text{ mM eq}$, $C(0) = 4 * 10^{-5} \text{ mM eq}$, $\alpha = 0.012$, $T =$
 117 40 kyr): (a) alkalinity, (b) dominant period as a function of time

118 The dominant period initially evolves around the forcing period of 40 kyr, then sharply (MPT-like)
 119 increases to about 80 kyr (twice the forcing period) and stabilizes at this level. This period shift
 120 occurs through a different mechanism than in earlier studies using the C-A model, where period shifts
 121 involved noise (Omta et al., 2016) or a positive feedback (Shackleton et al., 2023) to “kick” the system
 122 from one dominant period to another one. Here no such kick is imposed: the period shift rather emerges
 123 as part of the transient dynamics of the system, as it relaxes from its initial towards its asymptotic state.
 124 For the first $\sim 1.7 \text{ Myr}$ of the simulation, there appears to be an approximate but not exact frequency lock,
 125 from which the system has difficulty escaping. Once the system is out of this approximate frequency lock,
 126 its period increases relatively rapidly until it reaches another multiple of the forcing period where the
 127 system becomes locked again.

128 In the following, we analyze how the *initial* and *asymptotic* periods may depend on the system
 129 parameters. In particular, we formulate a scaling law (Section 3.1) that we then investigate in more details
 130 through simulations (Section 3.2). In Section 3.3 we project the discovered alkalinity dynamics onto the
 131 glacial rhythmicity.

3.1 Scaling law

The C-A system of equations (1) – (3) contains seven governing parameters, including the initial conditions. Both the mean initial and the asymptotic periods have to be functions of these seven parameters. Thus, we can write:

$$P = \varphi(I_0, k_0, \alpha, T, M, A(0), C(0)) \quad (4)$$

with P the asymptotic period. If we take I_0, k_0 as parameters with independent dimensions, then according to the π -theorem (Buckingham, 1914):

$$\frac{P}{\tau} = \Phi \left[\alpha, \frac{T}{\tau}, M\tau, \frac{A(0)}{F}, \frac{C(0)}{F} \right] \quad (5)$$

Here $\tau = (k_0 I_0)^{-1/2}$, $F = \left(\frac{I_0}{k_0}\right)^{1/2}$.

In this study, we will focus just on two similarity parameters $\alpha, \frac{A(0)}{F}$ leaving $\frac{T}{\tau}, M\tau, \frac{C(0)}{F}$ to remain constant:

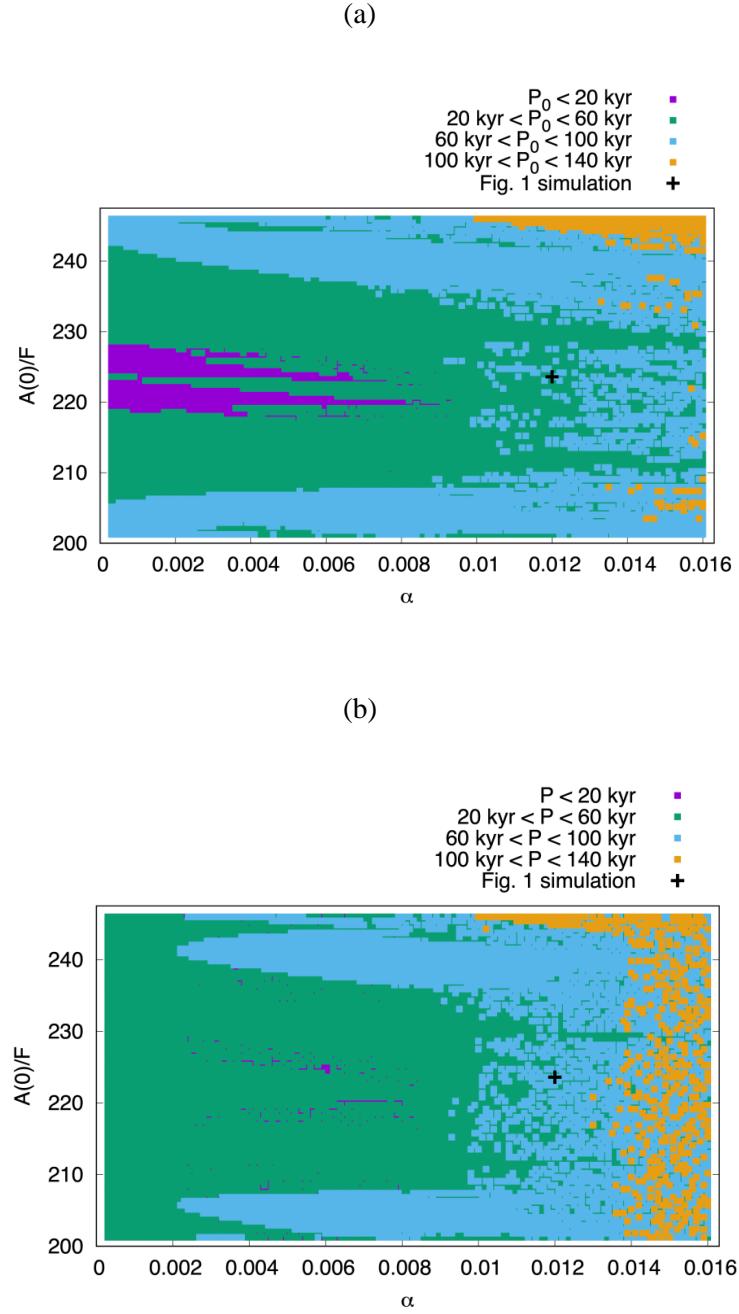
$$\frac{P}{\tau} = \Phi \left[\alpha, \frac{A(0)}{F} \right] \quad (6)$$

Using similar reasoning, we can write for the initial period P_0 :

$$\frac{P_0}{\tau} = \Psi \left[\alpha, \frac{A(0)}{F} \right] \quad (7)$$

3.2 Scaling law simulations

To investigate the scaling laws (6, 7), we perform a suite of 10-Myr simulations in which we vary α and $\frac{A(0)}{F}$. The average periods during the first 1 Myr (P_0) and the last 1 Myr (P) as a function of α and $\frac{A(0)}{F}$ are presented in Figs. 2a and 2b, respectively. The range in $A(0)$, which determines the vertical axis range in Fig. 2, was chosen based on the estimated total weathering input of CaCO_3 (Milliman et al., 1999), which could give rise to alkalinity variations of up to ~20% on ~100-kyr timescales (Omta et al., 2013). The lower and higher ends of the range are probably a bit less likely than the middle part of the range. There is no obvious constraint on α (horizontal axis in Fig. 2), which is why we varied that parameter by two orders of magnitude. In total, Fig. 2 encompasses the results of 12,798 simulations.



171 **Figure 2.** (a) Initial periods P_0 (average of first 1 Myr of 10-Myr simulations), and (b) asymptotic periods
 172 P (average of last 1 Myr of 10-Myr simulations). Each dot represents one simulation; the black cross
 173 indicates the parameter values for the simulation shown in Fig 1. Two similarity parameters are varied:
 174 the non-dimensional forcing strength α (horizontal axis) and the scaled initial alkalinity $\frac{A(0)}{F}$ (vertical
 175 axis). In total, Fig. 2 encompasses the results of 12,798 simulations. In all simulations, $T = 40 \text{ kyr}$ and
 176 $C(0) = 4 * 10^{-5} \text{ mM eq}$. Other parameters are kept constant at their reference values (Omata et al., 2016):
 177 $M = 0.1 \text{ yr}^{-1}$, $k_0 = 0.05 \text{ (mol eq)}^{-1} \text{ m}^3 \text{ yr}^{-1}$, $I_0 = 4 * 10^{-6} \text{ mol eq m}^{-3} \text{ yr}^{-1}$, i.e., $F = \left(\frac{I_0}{k_0}\right)^{\frac{1}{2}} = 0.00894$.

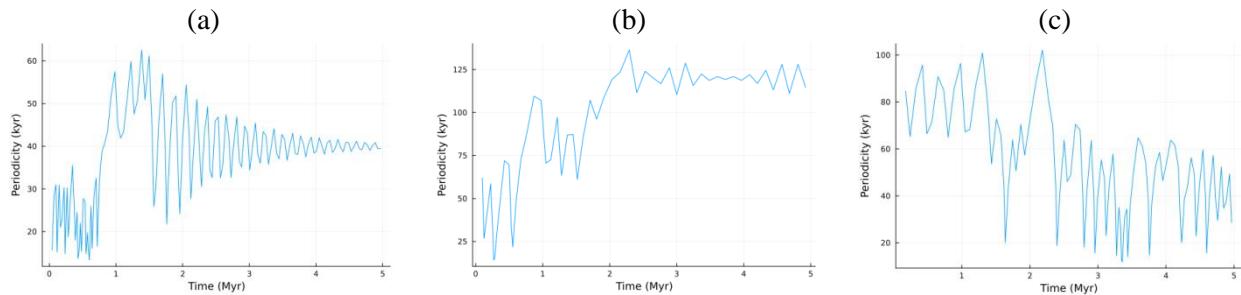
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179 From Fig. 2, it can be observed that:

180 (a) P_0 and P depend on α and $\frac{A(0)}{F}$ in different manners. Most obviously, $P_0 < 20$ kyr in a significant
181 fraction of the simulations whereas $P > 20$ kyr in almost every simulation. Furthermore, $P > 100$ kyr
182 occurs in many more simulations than $P_0 > 100$ kyr. These differences imply that a period shift
183 emerges in a significant fraction of the simulations.184 (b) When $\alpha \rightarrow 0$, the asymptotic period P becomes independent of the initial value $A(0)$ (Fig. 2b), which
185 means that the similarity parameters $\alpha, \frac{A(0)}{F}$ in the C-A system (1) – (3) collide into one conglomerate
186 similarity parameter $\alpha^x \left[\frac{A(0)}{F} \right]^y$ (the parameters x and y should be determined experimentally). This then
187 provides us with the final form of the scaling law for the asymptotic period:
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$$\frac{P}{\tau} = \Phi \left\{ \alpha^x \left[\frac{A(0)}{F} \right]^y \right\} \quad (8)$$

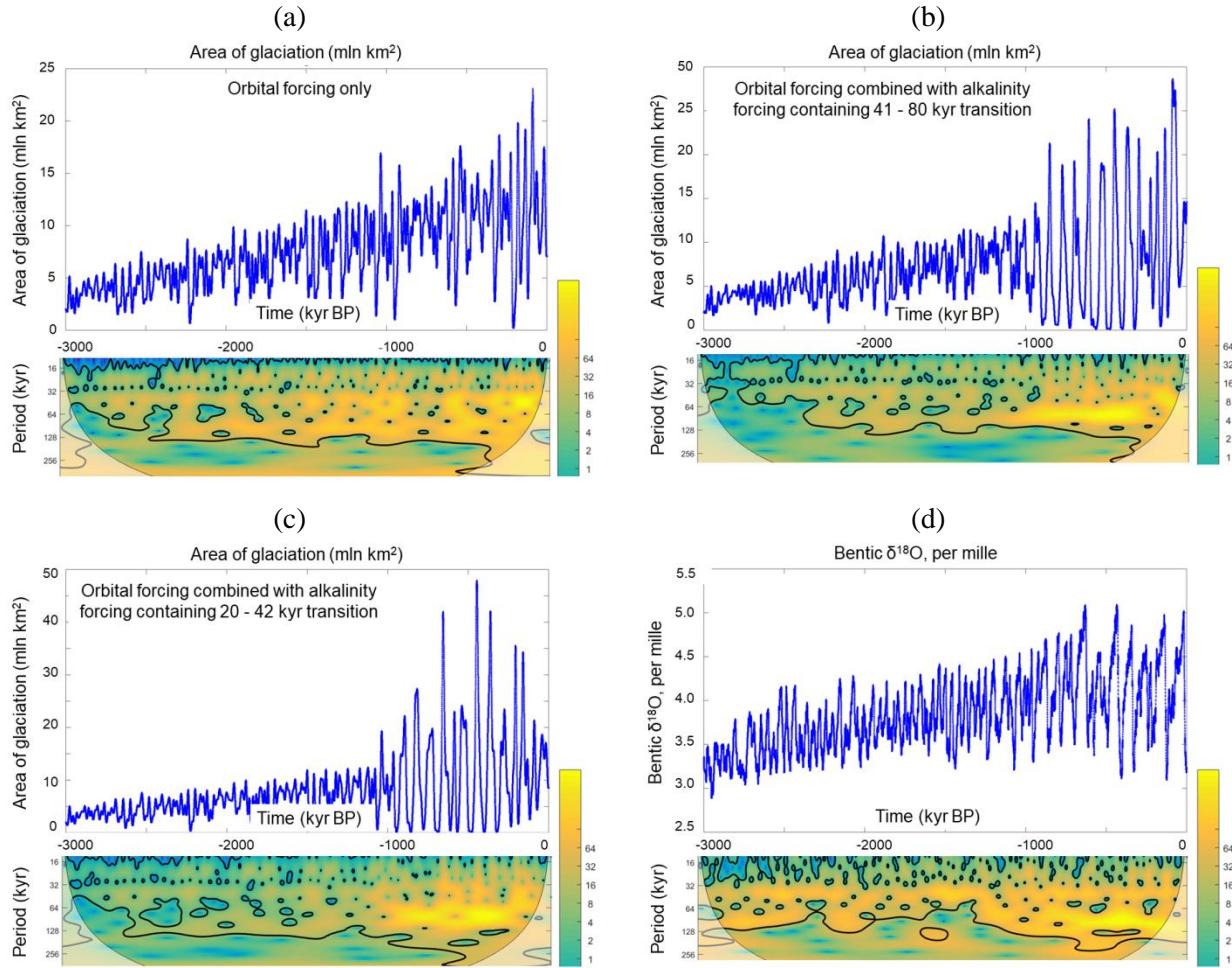
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191 The scaling law (8) implies that the *orbital forcing affects the dynamical properties of the C-A physics*
192 *enabling the sensitivity of asymptotic periods to initial values.*193 (c) When α increases (e.g., Gough, 1981, Ma et al., 2024), the sensitivity of the dominant asymptotic
194 period to the initial conditions $\frac{d(P)}{d(\frac{A(0)}{F})}$ also increases. Specifically, when $\alpha < 0.002$, as we have already
195 noted, $\frac{P}{\tau}$ is not sensitive to initial values. When $0.002 < \alpha < 0.01$, it takes $\Delta \left(\frac{A(0)}{F} \right) \sim 10$ to obtain a
196 different asymptotic period. Orbital forcing with $0.01 < \alpha < 0.014$ reduces the critical value of initial
197 values changes to $\Delta \left(\frac{A(0)}{F} \right) \sim 1$, and finally for $\alpha > 0.014$ changes as small as $\Delta \left(\frac{A(0)}{F} \right) \sim 0.1$ lead to
198 different asymptotic periods.
199 (d) Depending on $\alpha^x \left[\frac{A(0)}{F} \right]^y$, the periodicity transition could have been not just of the 40 – 80 kyr type (as
200 shown in Fig. 1), but also of a 20 – 40, 40 – 120, or even 80 – 40 kyr type (Fig. 3).
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202203 **Figure 3.** Alkalinity dominant-period transitions of (a) 20 – 40 kyr ($A(0) = 2.0$ mM eq, $\alpha = 0.008$); (b)
204 40 – 120 kyr ($A(0) = 2.0$ mM eq, $\alpha = 0.0149$); and (c) 80 – 40 kyr ($A(0) = 1.82$ mM eq, $\alpha = 0.0122$). In
205 all experiments $C(0) = 4 * 10^{-5}$ mM eq, $T = 40$ kyr.206 Most of the simulations reach their asymptotic periods within the first 1 Myr. A period shift after
207 1 Myr occurs in 3,217 out of the 12,798 simulations (about 25%) represented in Fig. 2. However, it is
208 impossible to infer from the proxy data how common or rare a shift in the dominant period of the glacial-
209 interglacial cycles actually is in the real World, since the observed Pleistocene climate is essentially a
210 single time series.211 Classical phase locking (e.g., Tziperman et al., 2006) requires some kind of dissipation in the
212 dynamical system that erases the memory of its initial values. Obviously, this is not the case with the
213 dominant-period trajectories we observe in Figs. 1, 2, and 3. At the same time, the asymptotic periods are
214 multiples of the forcing period. We therefore suggest calling this phenomenon a *delayed* phase locking.

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3.3 Translating alkalinity dynamics into glacial rhythmicity

To investigate the link between the modelled relaxation process and the climate system, we applied some alkalinity time series to the Verbitsky et al. (2018) model as additional forcings for the ice mass balance. This model has been derived from the scaled mass-, momentum-, and heat-conservation equations of non-Newtonian ice flow combined with an energy-balance model of global climate. In our experiments, all reference parameters of the Verbitsky et al. (2018) model remain the same, except one parameter that affects the intensity of positive feedbacks. On its own accord, the Verbitsky et al. (2018) model can produce a period shift if a positive feedback is sufficiently strong. We now set this positive feedback weaker to deprive the Verbitsky et al. (2018) model of this ability to produce MPT-like events.



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Figure 4. Ice–climate system (Verbitsky et al., 2018) response to a pure orbital (a) and to a combination of orbital and alkalinity (CO_2) forcing (b - additional alkalinity (CO_2) forcing contains a periodicity transition from 41 kyr to 80 kyr, c - additional alkalinity (CO_2) forcing contains a periodicity transition from 20 kyr to 42 kyr) presented as time series and evolutions of wavelet spectra over 3 Myr for calculated ice-sheet glaciation area (10^6 km^2) (a, b, c) and for the Lisiecki and Raymo (2005) benthic $\delta^{18}\text{O}$ record (d). The vertical axis of wavelet spectra is the period (kyr); the horizontal axis is time (kyr before present). The color scale shows the continuous Morlet wavelet amplitude, the thick line indicates the peaks with 95 % confidence, and the shaded area indicates the cone of influence for the wavelet transform.

235 In Figure 4a, we show the weak-positive-feedback area-of-glaciation evolution under the imposed
 236 cooling trend without additional alkalinity (CO_2) forcing. This time series does not exhibit MPT-like
 237 periodicity changes. When an additional alkalinity (CO_2) forcing containing a period shift from 41 kyr to
 238 80 kyr is applied, the glaciation-climate system produces a 40-to-100 kyr glacial rhythmicity transition
 239 resembling the LR04 data (Figure 4b vs 4d). This is the case of the direct alkalinity-forced period
 240 transition that could probably be anticipated. Yet, it is quite remarkable and very unintuitive that the
 241 alkalinity forcing may entertain a more subtle interplay with the direct orbital forcing. This becomes
 242 evident in the experiment when we forced the Verbitsky et al. (2018) model with an alkalinity (CO_2)
 243 forcing containing periodicity transitions from 20 kyr to 42 kyr. A non-linear interplay of the direct
 244 orbital forcing (i.e., mid-July insolation at 65°N , Berger and Loutre, 1991) and of \sim 40-kyr periods of the
 245 alkalinity forcing may produce glaciation periods of \sim 100 kyr also consistent with the LR04 data (Figure
 246 4c vs 4d).

247 In this paper, we do not aspire to precisely reproduce the empirical time series and by doing so to
 248 claim any specific attribution. However, with the above experiments, we demonstrate that the calcifier-
 249 alkalinity dynamics may have a profound effect on the climate system, and what we call an MPT-like
 250 event in terms of the alkalinity periods can be translated into an MPT event in terms of glacial
 251 rhythmicity.

252 Generally speaking, it would be indeed interesting to explore possible interactions of different
 253 initial-value-sensitive systems such as the glaciation-climate system (Verbitsky and Volobuev, 2025), the
 254 calcifier-alkalinity system (this presentation), or, possibly, carbon cycle system (Carrillo et al., 2025).
 255 Since the MPT was a global, almost synchronous, event, the discovering of the synchronization
 256 mechanism may be an important next step. In our experiments, presented in Fig. 4, the alkalinity (CO_2),
 257 together with the direct orbital forcing, acted as the external synchronizing force for the glaciation-climate
 258 system, but many other scenarios are, indeed, possible.

260 4. Conclusions

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 262 The history of climate has been given to us as a single time series. For many years, perhaps
 263 somewhat naively, significant efforts have been applied to reproduce this time-series under a unique
 264 combination of the governing parameters and thus presumably to explain the history. The fundamental
 265 fact that the dominant-period trajectory is governed by a conglomerate similarity parameter

266 $\alpha^x \left[\frac{A(0)}{F} \right]^y$ (demonstrating a property of incomplete similarity as defined by Barenblatt, 2003) tells us that
 267 the MPT could have been produced under very different combinations of the intensity of orbital forcing
 268 and initial values. Furthermore, the scaling laws (7) and (8), as they are presented in Fig. 2, show that not
 269 only periodicity transitions of the 40 – 80 kyr type (as we observe in the available data), but also of 20 –
 270 40, 40 – 120, or even 80 – 40 kyr types would be possible. Some of these transitions, i.e., 40 – 80, 40 –
 271 120, and, remarkably, 20 – 40 kyr types, produce glaciation MPT events consistently with the data. Most
 272 intriguingly, the conglomerate similarity parameter $\alpha^x \left[\frac{A(0)}{F} \right]^y$ implies that such an “intimate” terrestrial
 273 property as the sensitivity of alkalinity-calcination system to initial values manifests itself only under
 274 orbital forcing, and thus *the MPT exhibits a remarkable physical phenomenon of orbitally enabled*
 275 *sensitivity to initial values.*

276 We focused our paper on the past, MPT, event. Nevertheless, since we force our model with a
 277 generic obliquity, 40-kyr, forcing without a particular connection to the celestial time, any time series out
 278 of 12,798 simulations can be assumed as starting at present, and any observed periodicity transition can
 279 be considered not just as possible past transition but as potential future transition as well. Therefore, the
 280 complexity of Fig. 2 demonstrates not just the empirical data disambiguation challenge, but also the
 281 difficulty of the future climate prediction.

282 In this paper, when we establish a consistency between model results and empirical data, we are
 283 talking about periodicity transitions only, leaving purposely amplitude-periodicity relationship outside the

scope. Verbitsky and Crucifix (2020) demonstrated that in the short-memory ice-climate system, independent on initial values, the relationship between the glacial area amplitude S' and duration of glacial cycles P is governed by a property of scale invariance, such that $S' \sim P^2$. For the calcifier-alkalinity system, which has a long memory and dependence on the initial values, there exists a linear relationship between the amplitude and the period of the cycles. As was explained in Omta et al. (2016), this property emerges because a longer duration of the slow linear increase in alkalinity (determined by the constant term I_0 in equation (1)) implies proportionately larger amplitude of the cycle and vice versa. In the future, it will be highly desirable to compare predictions from different models regarding the periodicity and amplitude of variations in ice volume and global mean surface temperature against the newly available sea-level and global mean surface temperature data (Clark et al., 2025).

Competing interests: The authors declare that they have no conflict of interest.

Code availability. Simulations with the C-A model are performed in Julia version 1.11.2 using the KenCarp58 solver (Rackauckas and Nie, 2017) with a tolerance of 10^{-16} (<https://github.com/AWO-code/VerbitskyOmta>).

Data availability. This paper refers exclusively to published research articles and their data. We refer the reader to the cited literature for access to the data.

Author contributions: MYV conceived the research, AWO performed the simulations and discovered the MPT-like periodicity relaxation, MYV performed the scaling analysis and discovered the orbitally enabled sensitivity to initial values. The authors jointly wrote and edited the paper.

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