Dear Anonymous Reviewer,

Thank you for reviewing our manuscript and providing insightful feedback. Below, we reply to your comments (marked as **bold blue**) and propose several changes to the manuscript motivated by your suggestions.

Summary:

The Rapid Communication article by Verbitsky and Omta presents a sensitivity study carried out on a simple conceptual ocean chemistry model. The underlying model used in this study is the calcifier-alkalinity (CA) model, as described by Omta et al. (2013, doi: 10.1002/gbc.20060). The authors apply an obliquity-paced orbital forcing to the calcifier growth parameter and demonstrate that the system exhibits long equilibrium times, with transitions from an initial dominant period to an asymptotic one that can occur abruptly. The authors find that this transition in the period can be highly sensitive to the initial conditions and that this sensitivity depends on the amplitude of the orbital forcing. Based on their results, the authors state that the MPT could be the result of a relaxation process resulting in a sharp transition in the dominant period and that it could have resulted from different sets of initial values and orbital forcing amplitudes. Therefore, the observed 41-kyr to 100-kyr shift in the periodicity just resembles one specific instance, but different initial values or an altered orbital forcing amplitude could have led to a completely different pattern in this period shift.

In my view, this article presents an interesting view on the MPT. Especially demonstrating that the C-A model can produce abrupt, MPT-like jumps in periodicity purely driven by orbital pacing alone, without the need for any change in parameters, is significant. Furthermore, the result that the sensitivity of the asymptotic state depends on the amplitude of the orbital forcing is very interesting. My primary concern with this work lies in the very conceptual view of the model, and how the results relate to the MPT and the real world. Strengthening the link between the modelled relaxation processes and the real climate system would enhance the significance of the results for the MPT.

Response

We are grateful that you consider our results to be significant and very interesting.

Your suggestion to strengthen the link between the modelled relaxation process and the real climate system has steered us to one more novel result. Specifically, as a step in this direction, we applied some of the modeled alkalinity time series, containing periodicity transitions, as additional forcing to the glacial mass balance of the Verbitsky et al (2018, VCV18 hereafter) model. VCV18 model is a dynamical system, not postulated, but derived from the scaled mass-, momentum-, and heat-conservation equations of non-Newtonian ice flow combined with an energy-balance model of global climate. In our additional experiments, all reference parameters of the VCV18 model remain the same, except one parameter that affects the intensity of positive feedbacks.

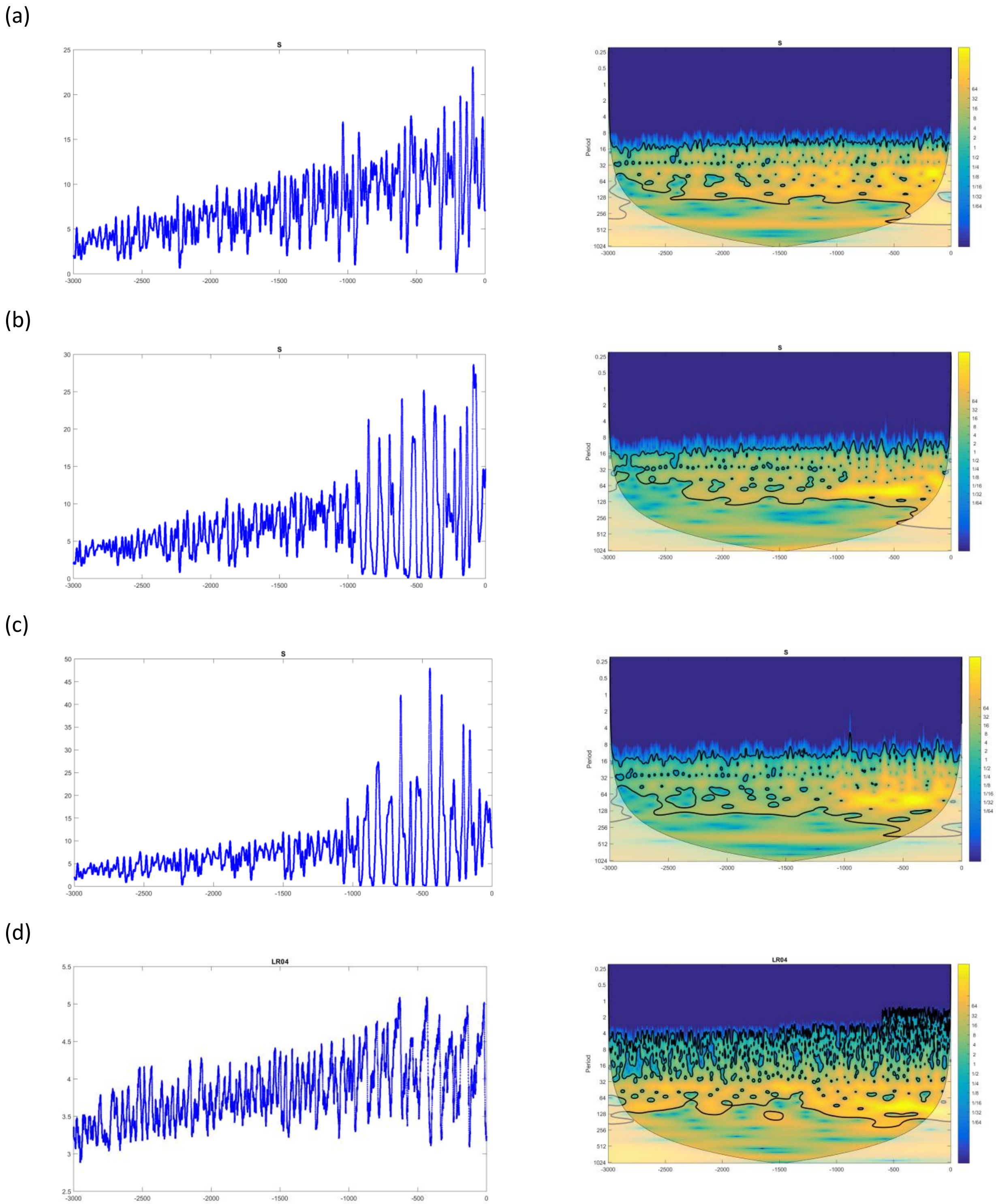

Figure AC2-1. Ice–climate system response to pure orbital (**a**) and to a combination of orbital and alkalinity (CO_2) forcing (**b** - additional alkalinity (CO_2) forcing contains periodicity transition from 41 kyr to 80 kyr**, c** - additional alkalinity (CO_2) forcing contains periodicity transition from 20 kyr to 42 kyr) presented as time series and evolutions of wavelet spectra over 3 Myr for calculated ice-sheet glaciation area *S* (10^6 km^2) (**a**, **b, c**) and for the Lisiecki and Raymo (2005) benthic $\delta^{18}O$ record (**d**). The vertical axis of wavelet spectra is the period (kyr); the horizontal axis is time (kyr before present).The color scale shows the continuous Morlet wavelet amplitude, the thick line indicates the peaks with 95 % confidence, and the shaded area indicates the cone of influence for wavelet transform.

On its own accord, VCV18 can produce a period shift if a positive feedback is sufficiently strong. We now set this positive feedback weaker to deprive VCV18 of this ability to produce an MPT-like event.

In **Figure AC2-1(a**), we show the weak-positive-feedback VCV18 evolution under the imposed cooling trend without additional alkalinity (CO_2) forcing. This time series does not exhibit MPT-like periodicity changes. When the additional alkalinity (CO_2) forcing containing periods shift from 41 kyr to 80 kyr is applied, the glaciation-climate system produces a 40-to-100 kyr glacial rhythmicity transition resembling the LR04 data (**Figure AC2-1 (b)** *vs* **(d)**). This is the case of the direct alkalinity-forced period transition that could probably be anticipated. Yet, it is quite remarkable and very unintuitive that the alkalinity forcing may entertain a more subtle interplay with the direct orbital forcing. This becomes evident in the experiment when we forced VCV18 model with an alkalinity (CO_2) forcing containing periodicity transitions from 20 kyr to 42 kyr. ***A non-linear interplay of the direct orbital forcing*** (i.e., mid-July insolation at 65^{0}N, Berger and Loutre, 1991) ***and of ~40-kyr periods of the alkalinity forcing may produce glaciation periods of ~100 kyr*** also consistent with the LR04 data (**Figure AC2-1 (c)** *vs* **(d)**).

As we have already mentioned in our response to Anonymous Reviewer 1, we do not aspire to precisely reproduce the empirical time series and by doing so to claim any specific attribution. Instead, with the above experiments, we simply demonstrate that the calcifier-alkalinity dynamics may have a profound effect on the climate system, and what we call an MPT event in terms of the period of ***alkalinity*** dynamics can be translated into an MPT event in terms of ***glacial*** rhythmicity.

Action: To the extent tolerable by the Rapid Communication format and/or with the Editor's permission, we will include the above discussion in the revised version of the paper.

Major comments:

Please comment on why it is justified to consider T, k0, I0, M and C(0) as constant. For me it is not obvious why P/τ should only depend on α and A(0)/F, but not on Mτ or C(0)/F

Response:

$T, k_0, I_0, M, C(0)$ have been set constant only for the purpose of the current study, as we wanted to focus on the impacts of the orbital forcing and the initial conditions, following, as we mentioned in lines 48-53, the motivation of Verbitsky and Volobuev (2024). Although the orbital period *T* may reasonably be considered constant, we do not have similar constraints on the other parameters.

Action: We will clarify this in the revised version of the paper.

In Fig. 3: is there any physical justification for the used parameter bounds? Can some of the areas in the parameter space be ruled out due to constraints from observations? Based on this analysis, the authors claim that "[...] the MPT could have been not just of the 40 – 80 kyr type, as we observe in the available data, but also of a 20 – 40, 80 – 100, 40 – 120, or even 80 – 40 kyr type" (L.18 f.). Especially the 80-40 kyr scenario, which

means a reduction in periodicity, seems to occur very rarely in the simulations. It mainly appears in the lower left and upper left parts of Fig 3a) and 3b), where the blue-coloured areas transition to green-coloured areas. Are these scenarios realistic?

Response: The range in A(0), which determines the vertical axis range in Fig. 3, was chosen based on the estimated total weathering input of $CaCO_3$ (Milliman et al., 1999), which could give rise to alkalinity variations of up to ~20% on ~100-kyr timescales (Omta et al., 2013). The lower and higher ends of the range, where the 80-to-40 kyr shifts occur, are probably a bit less likely than the middle part of the range. There is no obvious constraint on α (horizontal axis in Fig. 3), which is why we varied that parameter by two orders of magnitude.

Action: We will discuss this in the revised version of the manuscript.

Fig. AC1-1 shows that the time until equilibrium is reached is highly variable and for the three shown simulations ranges from ~2 - 6Myr. While this article mainly focuses on the periodicity, the timing until the asymptotic period is reached is important for a full view on the MPT and it would be interesting to include some insights on the mean equilibrium times of the ensemble simulations

Response: The main period shift typically occurs within a few Myr from the start of the simulation. Subsequently, there are usually still some minor fluctuations in the period damping out on a timescale of several Myr (see Figs. 2 and AC1-1).

Action: We will discuss this in the revised version of the manuscript.

Large parts in Fig.3 do not change in colour. Does this imply that a large quantity of the simulations reach the asymptotic period within the first 1Myr of simulations? Hence, the mentioned shifts in period are only occurring for very specific sets of initial values and forcing amplitudes?

Response: Indeed, most of the simulations reach their asymptotic periods within the first 1 Myr. A period shift after 1 Myr occurs in 3,217 out of the 12,798 simulations (about 25%) represented in Fig. 3. In our view, something that happens 25% of the time is not a particularly rare event. Moreover, the observed Pleistocene climate is essentially a single time series. Therefore, it is impossible to infer from the proxy data how common or rare a shift in the dominant period of the glacial-interglacial cycle actually is in the real World.

Action: We will discuss this in the revised version of the manuscript.

Minor comments

All minor comments have been gratefully accepted and will be taken care of.

References

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