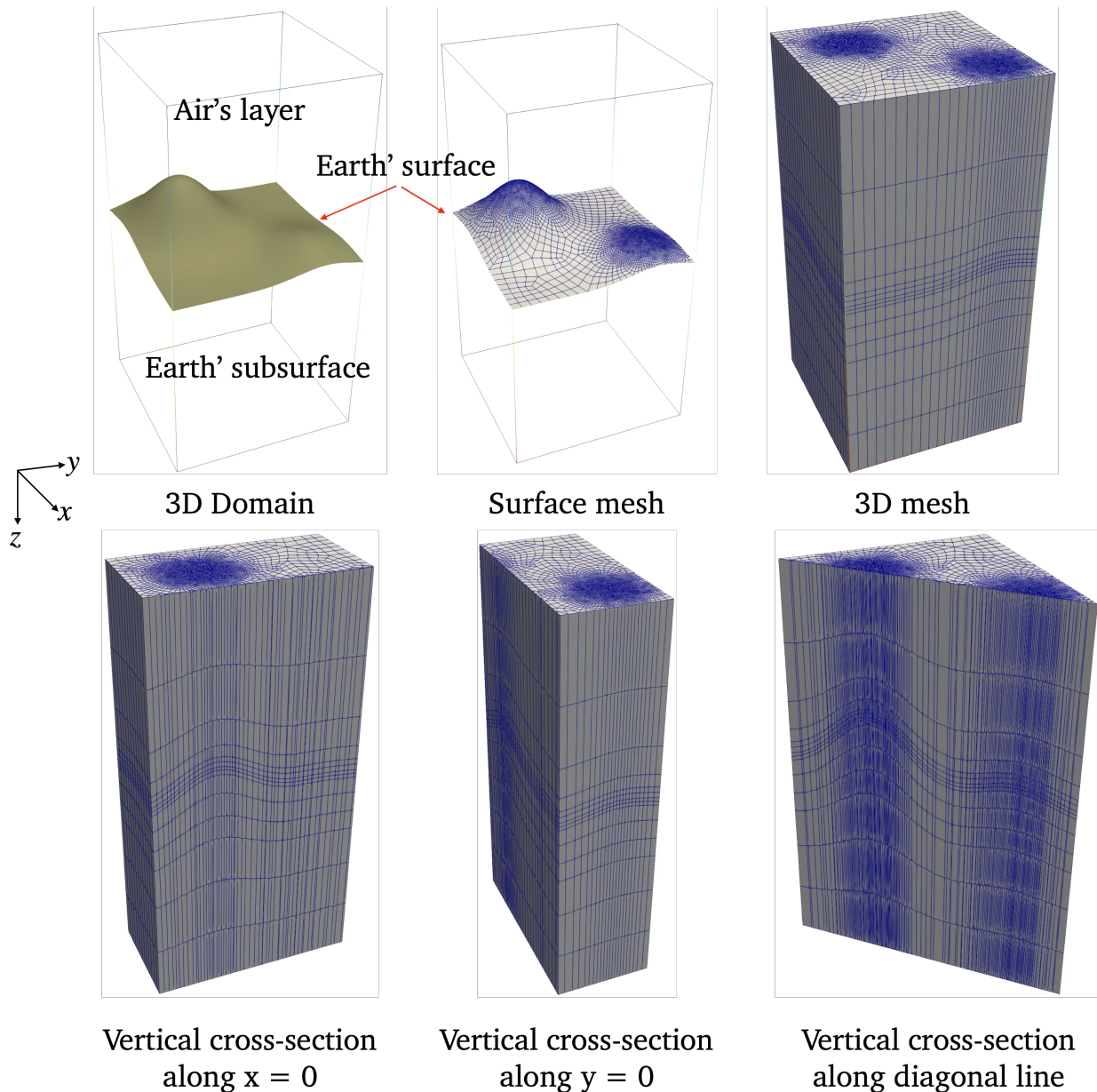


Autor's response to Reviewer 2

Referee's comment-1: From the presented material it is difficult to intuitively understand how the semi-unstructured horizontal mesh relates to the more conventional vertical mesh, especially in the first few model layers. I suggest that it would be very instructive for a reader to have an expanded version of Figure 11. The expanded figure would include some additional horizontal slices through the shallow parts of the model showing the mesh structure. Alternatively, a slice through the model under the topography feature with a depth extent of approx. 4 km would allow a fuller understanding of how the mesh functions in the shallow subsurface.

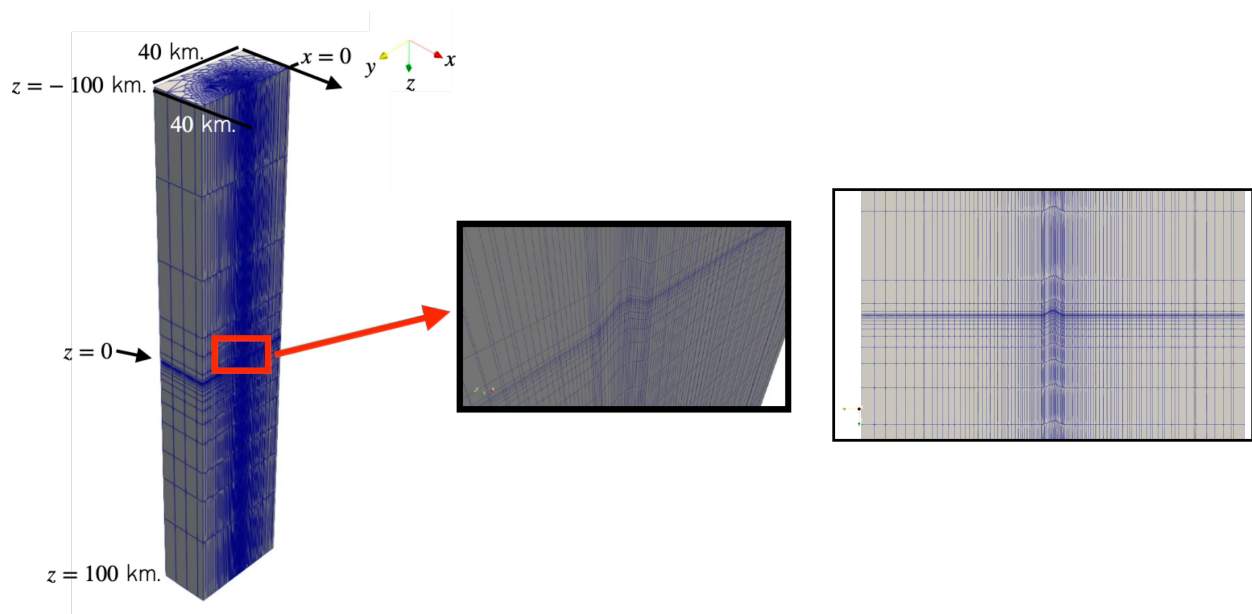
Author's response: For this comment, I believe it is the same as the one from referee-1. Due to unclear descriptions and visualization, I have provided and visualized more explicit descriptions here, as I responded to referee-1, but further clarified. I think the visualization in Figure 2 needs to be revised by representing the steps. The new representation of Figure 2 is shown below.



Starting with the computing domain that has an Air-Earth interface —a non-flat surface. An unstructured quadrilateral mesh is generated on this surface. Next, the extrusion process extends from this mesh surface to the top air layer and down to Earth's bottom. The extrusion may not be uniform for each mesh section. The z-coordinates of the mesh's nodes within the same section may vary. However, the z-coordinate in each mesh section tends to approach the same value at both the top and bottom of the domain. This feature can be achieved before imposing those constraints when the Earth is layered, if the sea's layer exists. The constraint at the top of the air is sea level (here, we set $z = 0$). In the revised version of the manuscript, the explanation of the mesh generation concept will be added in Section 3.1. The overall key steps are

1. Geometry Setup Phase
2. Refinement Strategy
3. Mesh Generation Phase
4. Surface Projection (or Topographic projection for real case)
5. Quality Control

Based on the description above, the expanded subfigure for Figure 11 is shown below.



Referee's comment-2: A similar additional section or couple of slices might also be instructive for the mesh structure in Figure 7.

Author's response: As illustrated above, an additional vertical slice mesh will be added, and the mesh representation will be revised in the revised version for clarity for this figure.

Additional minor suggestions -

There are a few overly long sentences in the manuscript that could be shortened for clarity. Specifically:

Referee's comment-3: Line 21. "In the early era, the IE method, based on applying Green's function to the scattering equation, and the SGFD, a variant of the SGFD method that

enforces the continuity of electric current along the block's edges and across its faces (Yee, 1966; Siripunvaraporn et al., 2002), are accurate and efficient for the simple 3D domain."

Author's response: For these sentences. The clearer the shorter these sentences are,

"In the early era, the IE method using Green's function and the SGFD, which enforces current continuity (Yee, 1966; Siripunvaraporn et al., 2002), was accurate and efficient for simple 3D domains."

Referee's comment-4: Line 94. "By combining the geometric flexibility of a paving algorithm for the horizontal plane with controlled, adaptable vertical layering, this approach is expected to offer new and improved performance for 3D MT modeling through the EBF approach, providing more accurate solutions and better computational efficiency by effectively addressing electromagnetic fields across highly contrast geophysical structures, which are crucial for correct simulation of MT responses."

Author's response: On line 94, there are no sentences above in our manuscript. These sentences appear at the start of line 104. Please see our response to these sentences in the following response.

The sentences that appeared on line 94 in our manuscript are

"Starting from this 2D mesh, the 3D hexahedral mesh is created by extruding these quadrilateral elements vertically. Importantly, to accurately capture the significant resistivity contrasts and rapidly changing electromagnetic fields near the highly irregular topography and bathymetry, this extrusion uses an automatic, non-uniform layer thickness controlled by a fixed geometric transition rate. This adaptive vertical layering focuses mesh resolution where it's most needed – near the complex air-earth interface and other regions of expected high field gradients and penetration depth effects (Simpson and Bahr, 2005)– without excessively increasing the total element count across the large-scale domain."

We can shorten the above for clarification as

"Starting from the surface mesh, the 3D volumetric mesh with hexahedral elements is generated by extruding these quadrilateral elements vertically. Importantly, to accurately capture the significant resistivity contrasts and rapidly changing electromagnetic fields near the highly irregular topography and bathymetry, this extrusion uses an automatic, non-uniform layer thickness controlled by a fixed geometric transition rate. This adaptive vertical layering concentrates mesh resolution where it is most needed—near the complex air-earth interface and other regions of expected high field gradients and penetration depth effects (Simpson and Bahr, 2005)—without excessively increasing the total element count across the large-scale domain."

Referee's comment-5: Line 104. "By combining the geometric flexibility of a paving algorithm for the horizontal plane with controlled, adaptable vertical layering, this approach is expected to offer new and improved performance for 3D MT modeling through the EBF approach, providing more accurate solutions and better computational efficiency by effectively addressing electromagnetic fields across highly contrast geophysical structures, which are crucial for correct simulation of MT responses."

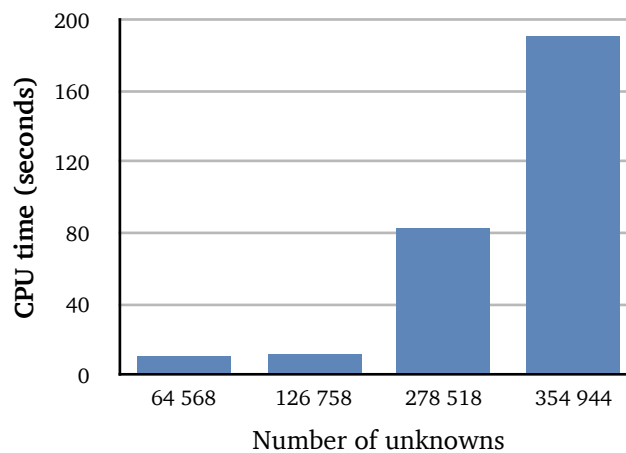
Author's response: These sentences are revised as follows to make it clearer

"Combining a flexible our algorithm for the horizontal plane with adaptive vertical layering enhances 3D MT modeling via the EBF approach. It improves accuracy and efficiency by better handling electromagnetic fields in highly contrasting geophysical structures, crucial for accurate MT response simulation."

Other comments:

Referee's comment-6: The sentence on line 160 needs to be edited to make the meaning clear.

Author's response: For lines 158-168, we will revise to make them clearer. At this time, we found that the Multifrontal Massively Parallel Sparse (MUMPS) Direct Solver (<https://mumps-solver.org/index.php>, related publications: <https://mumps-solver.org/index.php?page=doc>) provides a more efficient computation. However, the MUMPS requires a Coordinate (COO) format as input for the sparse matrix (represented by three arrays: one for row indices, one for column indices, and one for the corresponding matrix values). Therefore, our coefficient matrix in CSR format must be converted to COO format before implementation. Note that MUMPS is available for both Fortran 90/95 and Python. We have now updated our Fortran 90/95 code to implement the MUMPS solver. The results of our experiment in the submitted manuscript may need to be updated to account for CPU time and the computing tools used. However, the algorithms' accuracy remains unchanged. Note that the CPU time for converting from CSR to COO is very short and has no significant impact compared to the main Factorization and substitution process in the MUMPS solver (less than 0.05%). Initial tests reveal the relationship between the number of unknowns in the system of equations and approximate CPU time usage, as shown in the following figures.



Note that the tests are made on a MacBook Pro (M1) 14-inch with 16 GB RAM under macOS 15.6.1 (Sequoia). So, the lines 158-168 will be replaced with the following sentences in the revised version.

“The system of equations in Equation (16) can be solved using either Krylov subspace solvers with preconditioning (Nam et al., 2007; Smith, 1996; Mitsuhashi and Uchida, 2004; Liu et al., 2008) or direct solvers (Kordy et al., 2016). Currently, efficient direct solvers for large, sparse linear systems from electromagnetic modeling are well established (Kordy et al., 2016; Shantsev et al., 2017). Examples include the PARDISO (Schenk et al., 2001) and MUMPS (Amestoy et al., 2000; Shantsev et al., 2017) libraries. In this work, the MUMPS direct solver with an OpenMP interface is used instead of an iterative solver to prevent instability. Note that our coefficient matrix K in Equation (16), stored in CSR format, is converted to COO format, which is standard for the MUMPS solver. The CPU time for the CSR-to-COO conversion is minimal and has no significant impact compared to the main factorization and substitution steps in the MUMPS solver (less than 0.05%).

”

Referee’s comment-7: The sentence on line 229 needs to be edited to make the meaning clearer.

Author’s response: To make the meaning more straightforward, on line 227, continue to line 229. We will revise the explanation as

“The mesh design still follows the same concept as the previous model, meaning that mesh refinement is applied around stations and interfaces such as the Air-Earth interface, anomaly-background interface, etc. Additionally, local refinement is also implemented along the edges of the anomaly boundaries. For this case, the number of mesh points along the vertical z-direction is 39 for both structured and semi-unstructured hexahedral cases.”

Referee’s comment-8: Nam et al. is incorrectly marked as Name et al. in Figure 12.

Author’s response: This label will be corrected for the revised version.

Referee’s comment-9: Figure 5 uses both east-west/north-south and x/y nomenclature when describing the model in the bottom two panels. It would be better to only use one reference system to make the figure clearer.

Author’s response: The label “east-west/north-south” will be removed and replaced with x [km] / y [km] instead in the revised version.

Finally, the author would like to make additional improvements to the revised version to clarify the work's efficiency, accuracy, and usefulness to readers. All details will be included for the revised process.

Assistant Prof. Weerachai Sarakorn, Ph.D.