Editor's comments are in blue, our reply in black, quotes in the revised manuscript in red.

Dear Junshun Wang and co-authors,

Your manuscript has received two constructive reviews that reflect a wide range of opinions. This divergence highlights both the potential and the current limitations of the work. On one hand, the study introduces an approach that could provide new insights into geothermal heat flux variability in Antarctica by quantifying internal inconsistencies in ice sheet models. This is a potentially valuable contribution to the modeling community, especially in regions where direct observations are sparse.

On the other hand, several major issues were identified, most notably by Reviewer #2. A key concern is the clarity of the manuscript's novelty and its distinction from previous work, particularly Huang et al. (2024). While you have provided thoughtful and detailed responses, it remains important to explicitly clarify the unique contribution of this study, both in methodology and in application, relative to earlier work. I strongly encourage you to address this directly, perhaps through a dedicated paragraph or subsection within the revised manuscript.

In summary, while the manuscript shows promise, it requires substantial revision to address concerns related to novelty, clarity, and interpretation. I therefore encourage you to submit a thoroughly revised version. Once received, I will seek reevaluation by the reviewers to determine whether the revisions meet the standards for publication in The Cryosphere.

Best regards,
Cheng Gong

Reply: Thanks for the editor's comments. To address concerns related to novelty, clarity, and interpretation, we carefully improved the revision.

We clarity the inconsistencies in Section 2.1 as below:
The inconsistencies defined in this study are essentially between a sliding inversion and the temperature/rheology field used as an input to that inversion. More specifically, the inconsistencies are between modelled basal sliding (which is tuned to match the observed fast surface velocity during the inversion) and modelled frozen bed, and between observed slow surface velocity (which is most likely indicative of a non-slip basal condition) and modelled thawed bed.

We add two short sections to show the difference in methodology between this study and Huang et al. (2024), and clarify the novelty and the unique contribution of this study.

**2.2 Methodology in Huang et al. (2024)**

Huang et al. (2024) employed thermo-mechanical coupled simulations using eight GHF datasets to investigate the steady-state thermal regime of Totten Glacier. The methodology comprised two interconnected modeling components:

1. Forward Modeling: An enhanced shallow-ice approximation model integrated with a subglacial hydrology module was utilized to simulate englacial temperature profiles.
2. Inverse Problem: A full-Stokes ice flow model was applied to resolve basal friction coefficients through inverse analysis, to minimize the misfit between simulated and observed velocities while simultaneously generating velocity predictions.

A feedback loop was then established: the velocity outputs from the inverse model were used to refine key parameters in the forward model - specifically constraining the basal slip ratio, rheological properties, and shape functions. This bidirectional coupling process underwent multiple iterations to achieve convergent steady-state solutions.

Huang et al. (2024) utilized radar specularity content data to differentiate localized wet (thawed) versus dry (frozen) basal conditions and used this data as a two-sided constraint on the basal thermal state. They compared modeled basal thermal states derived from different GHFs to evaluate the reliability of the GHF datasets.

### 2.3 Distinction from Huang et al. (2024)

In Huang et al. (2024), modelled surface velocity velocities are compared with observations over the whole domain during the inversion for basal parameters for each GHF dataset. Here, surface velocities act as the observational constraints for the mechanical inversion.

Although the overheating metrics here use the surface velocities and can thus be considered a subset of the inversion residual, our overcooling metrics are based on the basal sliding velocity derived from the inversion, which is not part of the mechanical inversion's residual. A mechanical inversion does not take into account the physical plausibility of the sliding result it produces. Therefore, it is not circular reasoning to compare two different parts of a model to each other; rather, it is a check of internal consistency, or lack thereof. A mechanical inversion may fit the surface velocity observations equally well when forced with many different models of the ice sheet thermal structure and rheology; however, if some models require high sliding velocities in frozen-based regions, then they should be downweighted in comparison to models that show a good agreement between basal temperature and velocity.

The method here does not require any additional observations beyond the surface velocities used in the mechanical inversion. However, there are “independent constraints” in the method here, which are not observations, but rather the a priori physical understandings that: 1) rapid sliding requires warm basal temperatures and subglacial water; 2) reducing the basal slip coefficient cannot prevent the ice from flowing by internal shear deformation. The inconsistency metrics developed in this paper are an attempt to quantify and rank the extent to which these basic (and uncontroversial) physical understandings are violated.

We changed “warm bed” to “thawed bed”, and “cold bed” to “frozen bed” in the text according to the comment by Reviewer 2.

We also add a section as below according to the comment by Reviewer 1. We show the spatial fields of the inconsistencies metrics (Section 2.1) for the modelled result in Huang et al. (2024) with Martos et al. (2017) GHF as an example, and provide an interpretation, before conducting a comprehensive comparative analysis for the result with 8 GHF datasets.

### 3.2 Spatial Distribution of Inconsistencies with one GHF dataset

In this section, we show the spatial fields of the inconsistency metrics (Section 2.1) for the modelled result in Huang et al. (2024), using Martos et al. (2017) GHF as an example. This example illustrates the interpretation process before conducting a comprehensive comparative analysis for the result with 8 GHF datasets.

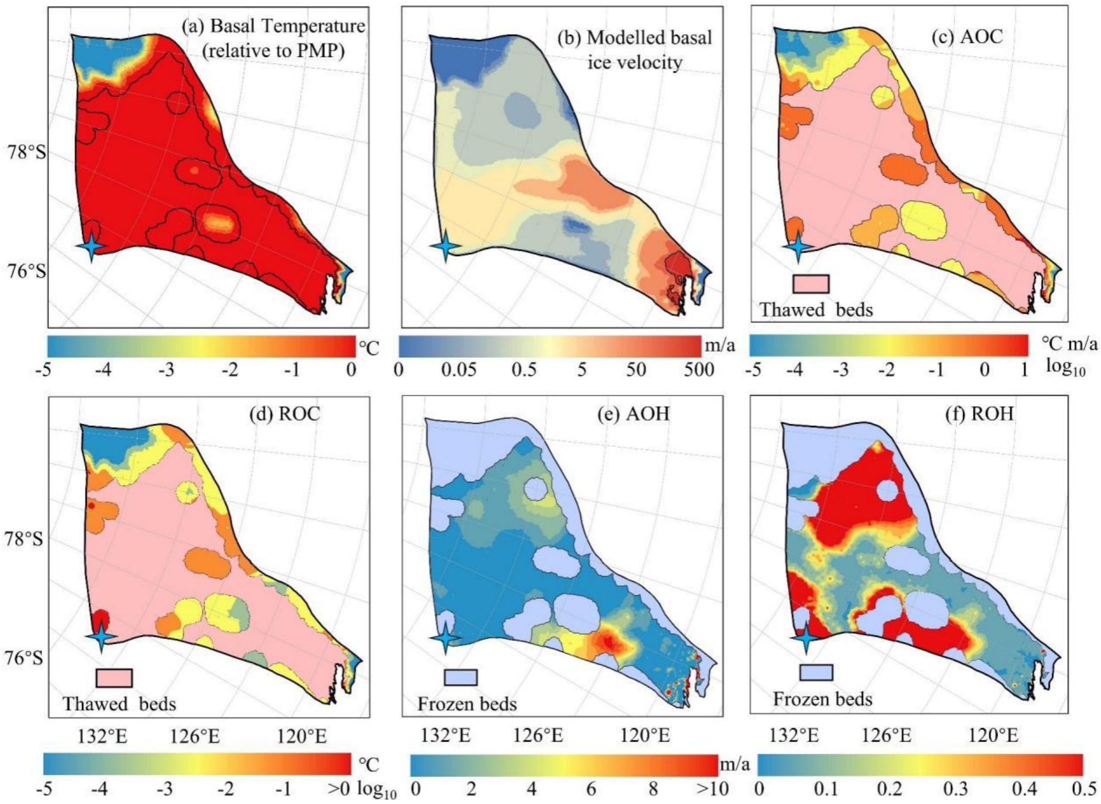


Figure 3. Spatial distribution of modelled basal ice temperature (a), modelled basal ice speed (b), *AOC* (c), *ROC* (d) inconsistencies in modelled frozen-bedded regions, and *AOH* (e) and *ROH* (f) inconsistencies in modelled thawed-bedded regions associated with Martos et al. (2017) GHF. The colormap in (c) and (d) is on logarithmic scale. The pink region in (c) and (d) represents modelled thawed bed, while the blue region in (e) and (f) indicates frozen-bedded areas.

The modelled result based on the Martos et al. (2017) GHF reveals extensive regions of thawed bed with limited areas of frozen bed. The frozen bed is predominantly located in the southern corner of the study domain, where the modelled basal ice speed approaches zero, consistent with cold basal ice temperature. Consequently, the AOC inconsistency at this marginal zone is negligible (Fig. 3). Along the western margin of Totten Glacier, basal ice temperature remains below the pressure melting point, albeit approaching it. However, localized regions exhibit high basal velocities of several tens of meters per year, contradicting the presence of a frozen bed and resulting in large AOC inconsistencies.

Conversely, large AOH values are observed between 69°S and 71°S in the eastern Totten Glacier region, where the simulated surface ice speed exceeds observational data by >5 m $a^{-1}$ (Fig. 3e). In this area, the modelled basal ice temperature reaches the pressure melting point, with the modelled basal ice speed at approximately 0.05 m $a^{-1}$. Basal friction inversion failed to reproduce observed surface ice speed due to the model's overestimation of ice temperature and softness. This pronounced velocity mismatch highlights a fundamental inconsistency in the eastern glacier region, likely originating from discrepancies in the input datasets. Regions of high ROH and ROC values coincide with areas of relatively high AOH and AOC, particularly where the observed surface velocities are slow, as per their formulations.

We also add a paragraph in the discussion to show the things one could check to isolate the causes of inconsistencies in application.

While evaluating inconsistencies highlights the spatial distribution of mismatches, it does not inherently elucidate their underlying causes. The primary factors to investigate are surface temperature, GHF, accumulation rate, and ice thickness, representing the most critical boundary conditions. Furthermore, integrating multiple sources of prior knowledge can help constrain model parameters:

1. High-resolution radar measurements: The availability of ice thickness data along flight lines should be assessed to validate geometric boundary conditions.
2. Paleoclimate context: Historical climate reconstructions indicate significantly colder surface temperatures during glacial periods compared to present-day conditions, with correspondingly lower accumulation rates. These paleo-temperature conditions likely induced a long-term thermal memory within the ice column, potentially contributing to observed discrepancies between modeled and measured basal properties.

Therefore, we recommend a systematic evaluation of: (1) The spatial distribution of radar-derived ice thickness measurements; (2) The temporal consistency of surface temperature boundary conditions; (3) The sensitivity of model results to GHF variations; (4) Accumulation rate reconstructions during key climatic periods. This multi-faceted approach helps isolate the causes of inconsistencies in ice sheet simulations.

We hope these edits can address concerns related to novelty, clarity, and interpretation of this study.