Correction Note — Section 2.8

Equation (42) in the originally submitted manuscript was incorrect. The corrected form is provided here for consistency and clarity.

Previous incorrect equation (old Eq. 42):

$$C_{s_v} = \left[C_{s_{om}}\ f_{v_{om}} + C_{s_{ms}}\ (1 - f_{v_{om}})\right]\ (1 - w_{sat})$$

Corrected equation (new Eq. 46):

$$C_{s_v} = \left[C_{s_{om}}\ f^s_{v_{om}} + C_{s_{ms}}\ (1 - f^s_{v_{om}})\right]\ (1 - w_{sat})$$

Here, $f^s_{v_{om}}$ denotes the volumetric fraction of SOM within the soil solid phase (Balland and Arp, 2005; Cuynet et al., 2025). This correction aligns the notation with the derivation presented in the revised text and supports the equation actually used in the manuscript (old Eq. 41, now Eq. 45):

$$C_{s_v} = c_{om}\ \rho_{b_{om}}\ f_{v_{om}} + c_{ms}\ \rho_{b_{ms}}\ (1 - f_{v_{om}})$$

This change has no effect on numerical results, figures, or conclusions. More details can be found in the new Section 2.8. Finally, on line 554 of the previous manuscript, the text will be updated to read: "Assuming C_{s_v} ($\mathrm{J \cdot m^{-3} \cdot K^{-1}}$) can be independently estimated from Equation (45)" instead of "Equation (41)".

The new Section 2.8 can be found on the next page.

References

References

Balland, V. and Arp, P. A.: Modeling soil thermal conductivities over a wide range of conditions, *Journal of Environmental Engineering and Science*, **4**, 549–558, doi:10.1139/s05-007, 2005.

Cuynet, A., Salmon, E., Lopéz-Blanco, E., Goeckede, M., Ikawa, H., Kobayashi, H., Lohila, A., and Ottlé, C.: Enhanced prescription of soil organic and mineral content in the ORCHIDEE LSM to better simulate soil temperatures: application at nine high-latitude GEM and FLUXNET sites, doi:10.22541/au.173748260.01626603/v1, 2025.

2.2.8 SOM thermal properties

Although the impact of soil organic matter on thermal processes is not the primary focus of this study, we briefly review a physically consistent approach to represent it within LSMs, as it complements the broader treatment of SOM hydrodynamics presented here. In LSMs, soil heat transport is typically described by the one-dimensional heat diffusion equation, derived from Fourier's law:

$$C_{soil}\frac{\partial T}{\partial t}=\frac{\partial}{\partial z}\left(\lambda_{soil}\frac{\partial T}{\partial z}\right) \tag{37}$$

where C_{soil} is the volumetric heat capacity of the soil (J.m^{-3}.K^{-1}), λ_{soil} the soil thermal conductivity (W.m^{-1}.K^{-1}), T the soil temperature (K), t the time (s), and z the soil depth (m).

In many LSMs, the thermal conductivity λ_{soil} is computed using a combination of dry and saturated soil conductivities, weighted by the Kersten number, which reflects the degree of saturation of each soil layer (Johansen, 1977; Farouki, 1981; Peters-Lidard et al., 1998). Within this framework, the saturated thermal conductivity is calculated as a volumetric-weighted geometric mean of the thermal conductivities of the solid phase, liquid water, and ice. Lawrence and Slater (2008) or Decharme et al. (2016) proposed that the dry conductivity (λ_{dry}) and the solid-phase conductivity (λ_s) be calculated using arithmetic or geometric mixing, respectively, based on the volumetric fractions of the organic and mineral components. The corresponding conductivity values for the organic domain can be taken from Table S1 in the Supplement, while mineral values can be taken from Peters-Lidard et al. (1998) as mentioned in section 2.1.2. In the present study, we focus exclusively on λ_{dry} and show that the geo-harmonic mean also provides a suitable alternative, while the computation of λ_s is addressed in the companion paper to this work.

The total volumetric heat capacity of soil, C_{soil}, can be derived from its fundamental physical definition, expressed as the heat capacity per unit volume of bulk soil:

$$C_{soil}=\frac{1}{v_b}\sum_k m_k c_k \tag{38}$$

where v_b is the total soil volume, and m_k (kg) and c_k (J.kg^{-1}.K^{-1}) denote the mass and specific heat capacity of each soil constituent k, including the solid matrix, liquid water, ice, and air. Assuming that air has negligible heat capacity compared to other phases, its contribution can be ignored. Grounding into the soil mixture theory and substituting the contributions of each relevant phase (organic matter, mineral matter, liquid water, and ice) into Equation (38), the total volumetric heat capacity becomes:

$$C_{soil}=\frac{1}{v_b}\left(m_{om}\,c_{om}+m_{ms}\,c_{ms}+m_w\,c_w+m_i\,c_i\right) \tag{39}$$

where m_{om} and m_{ms} (kg) are the masses of organic and mineral solids, m_w and m_i (kg) the masses of liquid water and ice, c_{om}, c_{ms}, c_w, and c_i (J.kg^{-1}.K^{-1}) their respective specific heat capacities. Each term in the summation of Equation (39) can be decomposed as the product of a mass concentration (mass per unit volume of the constituent's domain) and the corresponding volume fraction within the bulk soil. This leads to the following equivalent formulation:

$$C_{soil} = c_{om}\,\frac{m_{om}}{v_{om}}\,\frac{v_{om}}{v_b} + c_{ms}\,\frac{m_{ms}}{v_{ms}}\,\frac{v_{ms}}{v_b} + c_w\,\frac{m_w}{v_w}\,\frac{v_w}{v_b} + c_i\,\frac{m_i}{v_i}\,\frac{v_i}{v_b} \tag{40}$$

where v_k (m^3) denotes the volume of each soil constituent, with liquid water and ice occupying the pore space, and organic matter and mineral substrate forming the soil solid. For liquid water and ice, the ratios m_w/v_w and m_i/v_i are simply their specific densities, ρ_w and ρ_i (kg·m^{-3}), and the ratios v_w/v_b and v_i/v_b correspond to their volumetric contents, w_w and w_i (m^3·m^{-3}), respectively. For the solid constituents, v_{om} and v_{ms} correspond to the volumes of organic matter and mineral substrate within the soil solid, i.e. $v_{om} = v_{s_{om}}$ and $v_{ms} = v_{s_{ms}}$. Consequently, in Equation (40), $m_{om}/v_{om} = m_{om}/v_{s_{om}}$ and $m_{ms}/v_{ms} = m_{ms}/v_{s_{ms}}$ are simply their solid densities, $\rho_{s_{om}}$ and $\rho_{s_{ms}}$, as defined by Equation (13b). The computation of the ratios $v_{s_{om}}/v_b$ and $v_{s_{ms}}/v_b$ is less trivial, but they can be expressed as the product of their respective solid-phase volumetric fractions, $v_{s_{om}}/v_s = f^s_{v_{om}}$ and $v_{s_{ms}}/v_s = (1 - f^s_{v_{om}})$ (infered via Equation 12b), and the fraction of solids in the bulk soil, $v_s/v_b = (1 - w_{sat})$ (infered via Equation 10). Substituting these relationships into Equation (40) leads to the following formulation for the total volumetric heat capacity C_{soil} expressed as an arithmetic mean of the product of density and specific heat capacity of each soil constituent :

$$C_{soil} = C_{s_v} + C_w + C_i \quad \text{with} \quad \begin{cases} C_{s_v} = \left[c_{om}\;\rho_{s_{om}}\;f^s_{v_{om}} + c_{ms}\;\rho_{s_{ms}}\;(1 - f^s_{v_{om}})\right]\;(1 - w_{sat}) \\ C_w = c_w\;\rho_w\;w_l \\ C_i = c_i\;\rho_i\;w_i \end{cases} \tag{41}$$

Here, C_{s_v} (J.m^{-3}.K^{-1}) is the volumetric heat capacity of the dry solid matrix, while C_w and C_i represent the contributions of the liquid water and ice phases, respectively. As with porosity, Equation (41) supports the use of an arithmetic mixing formulation, as adopted by Lawrence and Slater (2008) and most LSMs, to compute the volumetric heat capacity of the dry soil matrix.

In Equation (41), the appropriate quantity to use is the volumetric fraction of SOM within the soil solid phase ($f^s_{v_{om}}$), consistent with previous findings (Balland and Arp, 2005; Cuynet et al., 2025), rather than the volumetric fraction of SOM in the bulk soil ($f_{v_{om}}$) as commonly assumed in earlier LSM parameterizations. Consistently with Equation (22), $f^s_{v_{om}}$ (m^3.m^{-3}) is defined as the ratio between the SOM density in the soil solid, expressed as the SOM mass fraction ($f_{m_{om}}$) relative to the soil particle density (ρ_s), and the SOM particle density ($\rho_{s_{om}}$):

$$f^s_{v_{om}} = \frac{f_{m_{om}}\;\rho_s}{\rho_{s_{om}}} \tag{42}$$

$\rho_{s_{om}}$ can be related to its bulk density $\rho_{b_{om}}$ and porosity $w_{sat_{om}}$ via Equation (19):

$$\rho_{s_{om}} = \frac{\rho_{b_{om}}}{1 - w_{sat_{om}}} \tag{43}$$

As ρ_s is related to ρ_b through the soil porosity (w_{sat}), replacing ρ_s and $\rho_{s_{om}}$ using Equations (11) and (43) in Equation (42) leads to the following relationship between $f^s_{v_{om}}$ and $f_{v_{om}}$:

$$f^s_{v_{om}} = f_{v_{om}}\,\frac{1 - w_{sat_{om}}}{1 - w_{sat}} \tag{44}$$

This relationship shows that $f^s_{v_{om}}$ can be readily obtained from $f_{v_{om}}$. Furthermore, substituting Equations (42) and (44) into Equation (41) and applying the complement of Equation (18) with respect to unity (i.e. $1 - w_{sat} = f_{v_{om}}(1 - w_{sat_{om}}) + (1 - f_{v_{om}})(1 - w_{sat_{ms}})$, the volumetric heat capacity of the dry soil matrix can be expressed directly in terms of the bulk densities and volumetric fractions of the organic and mineral domains:

$$C_{s_v} = c_{om}\,\rho_{b_{om}}\,f_{v_{om}} + c_{ms}\,\rho_{b_{ms}}\,(1 - f_{v_{om}}) \tag{45}$$

Equation (45) may, in some cases, be more straightforward to apply than Equation (41), provided that the bulk densities of each constituent are known.

A more usual form of C_{s_v} currently used in LSMs can be derived from Equation (41) using solid heat capacities for each soil component :

$$C_{s_v} = \left[C_{s_{om}}\,f^s_{v_{om}} + C_{s_{ms}}\,(1 - f^s_{v_{om}})\right]\,(1 - w_{sat}) \quad \text{with} \quad \begin{cases} C_{s_{om}} = c_{om}\,\rho_{s_{om}} \\ C_{s_{ms}} = c_{ms}\,\rho_{s_{ms}} \end{cases} \tag{46}$$

where $C_{s_{om}}$ and $C_{s_{ms}}$ (J.m^{-3}.K^{-1}) are the soil solid heat capacity for organic matter and mineral substance, which can be specified as in Lawrence and Slater (2008) from a lookup table (see Table S1 in the Supplement) or as in the proposed framework from their specific heat capacities and particle densities. Following Peters-Lidard et al. (1998), and based on the average values reported in Farouki (1981), we adopt $c_{ms} = 733$ J.kg^{-1}.K^{-1} for mineral matter and $c_{om} = 1972$ J.kg^{-1}.K^{-1} for organic matter. The particle density of mineral solids ($\rho_{s_{ms}}$) can be estimated using the PTF from Ruehlmann (2020) (Equation 25). For organic matter, the particle density $\rho_{s_{om}}$ can be computed using Equation (43). Finally, the total soil porosity, w_{sat}, can be computed via arithmetic mixing using the PTF from Liu and Lennartz (2019) for the organic matter domain (Equation 30), and, for instance, the PTF from Cosby et al. (1984) for the mineral soil. This approach ensures physical consistency between the volumetric heat capacity of the dry soil matrix, total soil porosity, and the particle densities of each solid soil component.

Table 1. Summary of the proposed framework, including the steps to derive the "true" soil organic volumetric fraction, the PTFs used to compute SOM hydraulic properties for the Brooks and Corey (1964) model, and the mixing rules applied to combine organic and mineral contributions into bulk soil properties. The framework relies on a limited set of input data commonly available in all current regional and global soil databases: the soil organic carbon mass fraction ($f_{m_{oc}}$), the dry bulk density of the fine earth, and the mass fractions of clay ($f_{m_{clay}}$), sand ($f_{m_{sand}}$), and silt ($f_{m_{silt}}$). In this study, the hydraulic properties of the mineral domain ($w_{sat_{ms}}$, b_{ms}, $\psi_{sat_{ms}}$, $k_{sat_{ms}}$) are taken from the texture-based pedotransfer functions of Cosby et al. (1984). The dry thermal conductivity of the mineral domain can be estimated using the approach proposed by Peters-Lidard et al. (1998).

Process	**Compute**	**Formula / Method**	**Ref.**
Soil organic volumetric fraction	1) SOM mass fraction from SOC	$f_{m_{om}} = \min[1, 1.848\ f_{m_{oc}}^{0.967}]$ (Ruehlmann, 2020)	Eq. (28)
	2) Particle density of mineral domain	$\rho_{s_{ms}} = \left(\frac{f_{m_{clay}}}{\rho_{clay}^{(a)}} + \frac{f_{m_{silt}}}{\rho_{silt}^{(b)}} + \frac{f_{m_{sand}}}{\rho_{sand}^{(c)}} \right)^{-1}$ (Ruehlmann, 2020)	Eq. (25)
	3) Bulk density of mineral domain	$\rho_{b_{ms}} = (1 - w_{sat_{ms}})\ \rho_{s_{ms}}$	Eq. (24)
	4) Bulk density of organic domain	$\rho_{b_{om}} = f_{m_{om}} \left(\frac{1}{\rho_b} - \frac{1 - f_{m_{om}}}{\rho_{b_{ms}}} \right)^{-1}$	Eq. (23)
	5) Bulk SOM volumetric fraction	$f_{v_{om}} = f_{m_{om}}\ \rho_b / \rho_{b_{om}}$	Eq. (22)
	6) Specific SOM volumetric fraction	$f_{v_{om}}^{s} = f_{m_{om}}\ \rho_s / \rho_{s_{om}} = f_{v_{om}}\ (1 - w_{sat_{om}})/(1 - w_{sat})$	Eq. (44)
Soil hydrology of SOM domain	7) SOM porosity	$w_{sat_{om}} = 0.95 - 0.437\ r_{b_{om}}^{(d)}$ (Liu and Lennartz, 2019)	Eq. (30)
	8) SOM particle density	$\rho_{s_{om}} = \rho_{b_{om}} / (1 - w_{sat_{om}})$	Eq. (43)
	9) SOM pore-size index	$b_{om} = 2.933 + 0.442\ r_{b_{om}}^{0.463} + e^{(1.321\ r_{b_{om}})}$	Eq. (34a)
	10) SOM air-entry potential	$\psi_{sat_{om}} = (101.663\ r_{b_{om}}^{4} - 46.913\ r_{b_{om}}^{5} - 61.625\ r_{b_{om}}^{2.635})\ 0.0168^{r_{b_{om}}}$	Eq. (34b)
	11) SOM saturated conductivity	$\log_{10}(k_{sat_{om}}) = -7.955 - 1.89 \log_{10}(z^{*} + 0.068) - 2.96 \log_{10}(r_{b_{om}}^{*} + 0.045)$	Eq. (36)
Total soil	12) Soil porosity	$w_{sat} = f_{v_{om}}\ w_{sat_{om}} + (1 - f_{v_{om}})\ w_{sat_{ms}}$	Eq. (18)
	13) Soil pore-size index	$b = f_{v_{om}}\ b_{om} + (1 - f_{v_{om}})\ b_{ms}$	Eq. (4)
	14) Soil air-entry potential	$\psi_{sat} = f_{v_{om}}\ \psi_{sat_{om}} + (1 - f_{v_{om}})\ \psi_{sat_{ms}}$	Eq. (4)
	15) Soil saturated conductivity	$k_{sat} = (k_{sat_{om}})^{f_{v_{om}}}\ (k_{sat_{ms}})^{1 - f_{v_{om}}}$	Eq. (5)
	16) Soil volumetric heat capacity	$C_{s_v} = [c_{om}^{(e)}\ \rho_{s_{om}}\ f_{v_{om}}^{s} + c_{ms}^{(f)}\ \rho_{s_{ms}}\ (1 - f_{v_{om}}^{s})]\ (1 - w_{sat})$	Eq. (41)
	17) Dry thermal conductivity	$\lambda_{dry} = \left[\frac{\sqrt{\lambda_{dry_{om}}^{(g)}\ \lambda_{dry_{ms}}}}{(1 - f_{v_{om}})\ \sqrt{\lambda_{dry_{om}}} + f_{v_{om}}\ \sqrt{\lambda_{dry_{ms}}}} \right]^{2}$ (Nielson and Rogers, 1982)	Eq. (7)

$^{(a)}\rho_{clay} = 2761$ kg.m^{-3}, $^{(b)}\rho_{sand} = 2656$ kg.m^{-3}, $^{(c)}\rho_{silt} = 2692$ kg.m^{-3}, $^{(d)}r_{b_{om}} = \rho_{b_{om}}/1000$, $^{(e)}c_{om} = 1972$ J.kg^{-1}.K^{-1}, $^{(f)}c_{ms} = 733$ J.kg^{-1}.K^{-1}, $^{(g)}\lambda_{dry_{om}} = 0.05$ W.m^{-1}.K^{-1}

2.2.8 SOM thermal properties

Although the impact of soil organic matter on thermal processes is not the primary focus of this study, we briefly review a physically consistent approach to represent it within LSMs, as it complements the broader treatment of SOM hydrodynamics presented here. In LSMs, soil heat transport is typically described by the one-dimensional heat diffusion equation, derived from Fourier's law:

$$C_{soil}\frac{\partial T}{\partial t}=\frac{\partial}{\partial z}\left(\lambda_{soil}\frac{\partial T}{\partial z}\right) \tag{37}$$

where C_{soil} is the volumetric heat capacity of the soil (J.m^{-3}.K^{-1}), λ_{soil} the soil thermal conductivity (W.m^{-1}.K^{-1}), T the soil temperature (K), t the time (s), and z the soil depth (m).

In many LSMs, the thermal conductivity λ_{soil} is computed using a combination of dry and saturated soil conductivities, weighted by the Kersten number, which reflects the degree of saturation of each soil layer (Johansen, 1977; Farouki, 1981; Peters-Lidard et al., 1998). Within this framework, the saturated thermal conductivity is calculated as a volumetric-weighted geometric mean of the thermal conductivities of the solid phase, liquid water, and ice. ~~Following~~ Lawrence and Slater (2008) or Decharme et al. (2016) ~~, both~~ proposed that the dry conductivity (λ_{dry}) and the solid-phase conductivity (~~λ_s) can be computed~~ λ_s) be calculated using arithmetic or geometric mixing, respectively, based on the volumetric fractions of the organic and mineral components. The corresponding conductivity values for the organic domain can be taken from Table S1 in the Supplement, while mineral values can be taken from Peters-Lidard et al. (1998) as mentioned in section 2.1.2. In ~~this~~ the present study, we ~~also~~ focus exclusively on λ_{dry} and show that the geo-harmonic mean ~~can be~~ also provides a suitable alternative, while the computation of λ_s is addressed in the companion paper to this work.

The total volumetric heat capacity of soil, C_{soil}, can be derived from its fundamental physical definition, expressed as the heat capacity per unit volume of bulk soil:

$$C_{soil}=\frac{1}{v_b}\sum_k m_k c_k \tag{38}$$

where v_b is the total soil volume, and m_k (kg) and c_k (J.kg^{-1}.K^{-1}) denote the mass and specific heat capacity of each soil constituent k, including the solid matrix, liquid water, ice, and air. Assuming that air has negligible heat capacity compared to other phases, its contribution can be ignored. Grounding into the soil mixture theory and substituting the contributions of each relevant phase (organic matter, mineral matter, liquid water, and ice) into Equation (38), the total volumetric heat capacity becomes:

$$C_{soil}=\frac{1}{v_b}\left(m_{om}\,c_{om}+m_{ms}\,c_{ms}+m_w\,c_w+m_i\,c_i\right) \tag{39}$$

where m_{om} and m_{ms} (kg) are the masses of organic and mineral solids, m_w and m_i (kg) the masses of liquid water and ice, c_{om}, c_{ms}, c_w, and c_i (J.kg^{-1}.K^{-1}) their respective specific heat capacities. Each term in the summation of Equation (39) can be decomposed as the product of a mass concentration (mass per unit volume of the constituent's domain) and the corresponding volume fraction within the bulk soil. This leads to the following equivalent formulation:

$$C_{soil} = c_{om}\,\frac{m_{om}}{v_{b_{om}}}\frac{m_{om}}{v_{om}}\,\frac{v_{b_{om}}}{v_b}\frac{v_{om}}{v_b} + c_{ms}\,\frac{m_{ms}}{v_{b_{ms}}}\frac{m_{ms}}{v_{ms}}\,\frac{v_{b_{ms}}}{v_b}\frac{v_{ms}}{v_b} + c_w\,\frac{m_w}{v_{b_w}}\frac{m_w}{v_w}\,\frac{v_{b_w}}{v_b}\frac{v_w}{v_b} + c_i\,\frac{m_i}{v_{b_i}}\frac{m_i}{v_i}\,\frac{v_{b_i}}{v_b}\frac{v_i}{v_b} \tag{40}$$

where ~~v_{b_w} and v_{b_i}~~ v_k (m^3) ~~are the bulk volumes occupied by the~~ denotes the volume of each soil constituent, with liquid water and ice ~~in the soil . Each component's contribution is thus expressed as the product of its phase-specific density, its volumetric proportion within the soil,~~ occupying the pore space, and organic matter and mineral substrate forming the soil solid. For liquid water and ice, the ratios m_w/v_w and ~~its specific heat capacity. This formulation makes explicit the structure of the soil mixture and reflects the mass–volume consistency underlying the mixture theory. In line with this theory, and recognizing for each soil constituent~~ m_i/v_i are simply their specific densities, ρ_w and ρ_i (kg·m^{-3}), and the ratios v_w/v_b and v_i/v_b correspond to their volumetric contents, w_w and w_i (m^3·m^{-3}), respectively. For the solid constituents, v_{om} and v_{ms} correspond to the volumes of organic matter and mineral substrate within the soil solid, i.e. $v_{om} = v_{s_{om}}$ and $v_{ms} = v_{s_{ms}}$. Consequently, in Equation (40)~~the definitions of the phase-specific densities ($\rho_{b_k} = m_k/v_{b_k}$) and their volumetric fractions ($f_{v_k} = v_{b_k}/v_b$)or content ($w_k = v_{b_k}/v_b$)~~, $m_{om}/v_{om} = m_{om}/v_{s_{om}}$ and $m_{ms}/v_{ms} = m_{ms}/v_{s_{ms}}$ are simply their solid densities, $\rho_{s_{om}}$ and $\rho_{s_{ms}}$, as defined by Equation (13b). The computation of the ratios $v_{s_{om}}/v_b$ and $v_{s_{ms}}/v_b$ is less trivial, but they can be expressed as the product of their respective solid-phase volumetric fractions, $v_{s_{om}}/v_s = f^s_{v_{om}}$ and $v_{s_{ms}}/v_s = (1 - f^s_{v_{om}})$ (infered via Equation 12b), and the fraction of solids in the bulk soil, $v_s/v_b = (1 - w_{sat})$ (infered via Equation 10). Substituting these relationships into Equation (40) leads to the following formulation for the total volumetric heat capacity C_{soil} ~~can be~~ expressed as an arithmetic mean of the product of density and specific heat capacity of each soil constituent ~~, as proposed by Farouki (1981):~~ :

$$C_{soil} = C_{s_v} + C_w + C_i \quad \text{with} \quad \begin{cases} C_{s_v} = \left[c_{om}\;\rho_{s_{om}}\;f^s_{v_{om}} + c_{ms}\;\rho_{s_{ms}}\;(1 - f^s_{v_{om}})\right]\;(1 - w_{sat}) \\ C_w = c_w\;\rho_w\;w_l \\ C_i = c_i\;\rho_i\;w_i \end{cases} \tag{41}$$

Here, C_{s_v} (J.m^{-3}.K^{-1}) is the volumetric heat capacity of the dry solid matrix, while C_w and C_i represent the contributions of the liquid water and ice phases, respectively. ~~The variables w_l and w_i (m^3.m^{-3}) are the volumetric contents of liquid water and ice.~~ As with porosity, Equation (41) supports the use of an arithmetic mixing formulation, as adopted by Lawrence and Slater (2008) and most ~~land surface models (LSMs)~~LSMs, to compute the volumetric heat capacity of the dry soil matrix.

~~A more usual form of C_{s_v} can be derived from Equation (41) using~~ In Equation (41), the appropriate quantity to use is the volumetric fraction of SOM within the soil solid phase ($f^s_{v_{om}}$), consistent with previous findings (Balland and Arp, 2005; Cuynet et al., 202 , rather than the volumetric fraction of SOM in the bulk soil ($f_{v_{om}}$) as commonly assumed in earlier LSM parameterizations. Consistently with Equation (22), $f^s_{v_{om}}$ (m^3.m^{-3}) is defined as the ratio between the SOM density in the soil solid, expressed as the SOM mass fraction ($f_{m_{om}}$) relative to the soil particle density (ρ_s), and the SOM particle density ($\rho_{s_{om}}$):

$$f^s_{v_{om}} = \frac{f_{m_{om}}\;\rho_s}{\rho_{s_{om}}} \tag{42}$$

$\rho_{s_{om}}$ can be related to its bulk density $\rho_{b_{om}}$ and porosity $w_{sat_{om}}$ via Equation (19):

$$\rho_{s_{om}} = \frac{\rho_{b_{om}}}{1 - w_{sat_{om}}} \tag{43}$$

As ρ_s is related to ρ_b through the soil porosity (w_{sat}), replacing ρ_s and $\rho_{s_{om}}$ using Equations (11) and (43) in Equation (42) leads to the following relationship between $f^s_{v_{om}}$ and $f_{v_{om}}$:

$$f^s_{v_{om}} = f_{v_{om}} \frac{1 - w_{sat_{om}}}{1 - w_{sat}} \tag{44}$$

This relationship shows that $f^s_{v_{om}}$ can be readily obtained from $f_{v_{om}}$. Furthermore, substituting Equations (42) and (44) into Equation (41) and applying the complement of Equation (18) with respect to unity (i.e. $1 - w_{sat} = f_{v_{om}}(1 - w_{sat_{om}}) + (1 - f_{v_{om}})(1 - w_{sa}$ the volumetric heat capacity of the dry soil matrix can be expressed directly in terms of the bulk densities and volumetric fractions of the organic and mineral ~~particle densities (Equation 19), which is~~ domains:

$$C_{s_v} = c_{om}\,\rho_{b_{om}}\,f_{v_{om}} + c_{ms}\,\rho_{b_{ms}}\,(1 - f_{v_{om}}) \tag{45}$$

Equation (45) may, in some cases, be more straightforward to apply than Equation (41), provided that the bulk densities of each constituent are known.

A more usual form of C_{s_v} currently used in LSMs ~~:~~ can be derived from Equation (41) using solid heat capacities for each soil component :

$$C_{s_v} = \left[C_{s_{om}}\,{f_{v_{om}}}^{s} + C_{s_{ms}}\,(1 - {f_{v_{om}}}^{s})\right]\,(1 - w_{sat}) \quad \text{with} \quad \begin{cases} C_{s_{om}} = c_{om}\,\rho_{s_{om}} \\ C_{s_{ms}} = c_{ms}\,\rho_{s_{ms}} \end{cases} \tag{46}$$

where $C_{s_{om}}$ and $C_{s_{ms}}$ (J.m^{-3}.K^{-1}) are the soil solid heat capacity for organic matter and mineral substance, which can be specified as in Lawrence and Slater (2008) from a lookup table (see Table S1 in the Supplement) or as in the proposed framework from their specific heat capacities and particle densities. Following Peters-Lidard et al. (1998), and based on the average values reported in Farouki (1981), we adopt $c_{ms} = 733$ J.kg^{-1}.K^{-1} for mineral matter and $c_{om} = 1972$ J.kg^{-1}.K^{-1} for organic matter. The particle density of mineral solids ($\rho_{s_{ms}}$) can be estimated using the PTF from Ruehlmann (2020) (Equation 25). For organic matter, the particle density $\rho_{s_{om}}$ can be computed ~~from $\rho_{b_{om}}$ and $w_{sat_{om}}$ using Equation (19):~~

~~$\rho_{s_{om}} = \frac{\rho_{b_{om}}}{(1 - w_{sat_{om}})}$~~

using Equation (43). Finally, the total soil porosity, w_{sat}, can be computed via arithmetic mixing using the PTF from Liu and Lennartz (2019) for the organic matter domain (Equation 30), and, for instance, the PTF from Cosby et al. (1984) for the mineral soil. This approach ensures physical consistency between the volumetric heat capacity of the dry soil matrix, total soil porosity, and the particle densities of each solid soil component.

Table 1. Summary of the proposed framework, including the steps to derive the "true" soil organic volumetric fraction, the PTFs used to compute SOM hydraulic properties for the Brooks and Corey (1964) model, and the mixing rules applied to combine organic and mineral contributions into bulk soil properties. The framework relies on a limited set of input data commonly available in all current regional and global soil databases: the soil organic carbon mass fraction ($f_{m_{oc}}$), the dry bulk density of the fine earth, and the mass fractions of clay ($f_{m_{clay}}$), sand ($f_{m_{sand}}$), and silt ($f_{m_{silt}}$). In this study, the hydraulic properties of the mineral domain ($w_{sat_{ms}}$, b_{ms}, $\psi_{sat_{ms}}$, $k_{sat_{ms}}$) are taken from the texture-based pedotransfer functions of Cosby et al. (1984). ~~Thermal conductivities~~ The dry thermal conductivity of the mineral domain can be estimated using the approach proposed by Peters-Lidard et al. (1998).

Process	**Compute**	**Formula / Method**	**Ref.**
Soil organic volumetric fraction	1) SOM mass fraction from SOC	$f_{m_{om}} = \min[1, 1.848\ f_{m_{oc}}^{0.967}]$ (Ruehlmann, 2020)	Eq. (28)
	2) Particle density of mineral domain	$\rho_{s_{ms}} = \left(\frac{f_{m_{clay}}}{\rho_{clay}^{(a)}} + \frac{f_{m_{silt}}}{\rho_{silt}^{(b)}} + \frac{f_{m_{sand}}}{\rho_{sand}^{(c)}} \right)^{-1}$ (Ruehlmann, 2020)	Eq. (25)
	3) Bulk density of mineral domain	$\rho_{b_{ms}} = (1 - w_{sat_{ms}})\ \rho_{s_{ms}}$	Eq. (24)
	4) Bulk density of organic domain	$\rho_{b_{om}} = f_{m_{om}} \left(\frac{1}{\rho_b} - \frac{1 - f_{mom}}{\rho_{b_{ms}}} \right)^{-1}$	Eq. (23)
	5) ~~Soil organic~~ Bulk SOM volumetric fraction	$f_{v_{om}} = f_{m_{om}}\ \rho_b / \rho_{b_{om}}$	Eq. (22)
	6) Specific SOM volumetric fraction	$f_{v_{om}}^{s} = f_{m_{om}}\ \rho_s / \rho_{s_{om}} = f_{v_{om}}\ (1 - w_{sat_{om}})/(1 - w_{sat})$	Eq. (44)
Soil hydrology of SOM domain	~~6~~7) SOM porosity	$w_{sat_{om}} = 0.95 - 0.437\ r_{b_{om}}^{(d)}$ (Liu and Lennartz, 2019)	Eq. (30)
	~~7~~8) SOM particle density	$\rho_{s_{om}} = \rho_{b_{om}} / (1 - w_{sat_{om}})$	Eq. (43)
	~~8~~9) SOM pore-size index	$b_{om} = 2.933 + 0.442\ r_{b_{om}}^{0.463} + e^{(1.321\ r_{b_{om}})}$	Eq. (34a)
	~~9~~10) SOM air-entry potential	$\psi_{sat_{om}} = (101.663\ r_{b_{om}}^{4} - 46.913\ r_{b_{om}}^{5} - 61.625\ r_{b_{om}}^{2.635})\ 0.0168^{r_{b_{om}}}$	Eq. (34b)
	~~10~~11) SOM saturated conductivity	$\log_{10}(k_{sat_{om}}) = -7.955 - 1.89 \log_{10}(z^{*} + 0.068) - 2.96 \log_{10}(r_{b_{om}}^{*} + 0.045)$	Eq. (36)
Total soil	~~11~~12) Soil porosity	$w_{sat} = f_{v_{om}}\ w_{sat_{om}} + (1 - f_{v_{om}})\ w_{sat_{ms}}$	Eq. (18)
	~~12~~13) Soil pore-size index	$b = f_{v_{om}}\ b_{om} + (1 - f_{v_{om}})\ b_{ms}$	Eq. (4)
	~~13~~14) Soil air-entry potential	$\psi_{sat} = f_{v_{om}}\ \psi_{sat_{om}} + (1 - f_{v_{om}})\ \psi_{sat_{ms}}$	Eq. (4)
	~~14~~15) Soil saturated conductivity	$k_{sat} = (k_{sat_{om}})^{f_{v_{om}}}\ (k_{sat_{ms}})^{1 - f_{v_{om}}}$	Eq. (5)
	~~15~~16) Soil volumetric heat capacity	~~$C_{s_v} = [c_{om}^{(e)}\ \rho_{s_{om}}\ f_{v_{om}} + c_{ms}^{(f)}\ \rho_{s_{ms}}\ (1 - f_{v_{om}})]\ (1 - w_{sat})$~~ $C_{s_v} = [c_{om}^{(e)}\ \rho_{s_{om}}\ f_{v_{om}}^{s} + c_{ms}^{(f)}\ \rho_{s_{ms}}\ (1 - f_{v_{om}}^{s})]\ (1 - w_{sat})$	Eq. ~~(46)~~ (41)
	~~16~~17) Dry thermal conductivity	$\lambda_{dry} = \left[\frac{\sqrt{\lambda_{dry_{om}}^{(g)}\ \lambda_{dry_{ms}}}}{(1 - f_{v_{om}})\ \sqrt{\lambda_{dry_{om}}} + f_{v_{om}}\ \sqrt{\lambda_{dry_{ms}}}} \right]^{2}$ (Nielson and Rogers, 1982)	Eq. (7)

$^{(a)}\rho_{clay} = 2761$ kg.m^{-3}, $^{(b)}\rho_{sand} = 2656$ kg.m^{-3}, $^{(c)}\rho_{silt} = 2692$ kg.m^{-3}, $^{(d)}r_{b_{om}} = \rho_{b_{om}}/1000$, $^{(e)}c_{om} = 1972$ J.kg^{-1}.K^{-1}, $^{(f)}c_{ms} = 733$ J.kg^{-1}.K^{-1}, $^{(g)}\lambda_{dry_{om}} = 0.05$ W.m^{-1}.K^{-1} ~~, $^{(h)}\lambda_{s_{om}} = 0.25$ W. m^{-1}. K^{-1}~~