



1 **Interaction mechanism between the karst aquifer and stream under**
2 **precipitation infiltration recharge**

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14 **Abstract:** The variation in seasonal precipitation intensity impacts the dynamic
15 interaction between the karst aquifer and stream. However, the interaction mechanism
16 between the karst aquifer and stream is currently still unclear, and characterizing the
17 impact of dynamic saturation process of groundwater in karst media on the interaction
18 process remains a challenge. To delve into the impacts of varying precipitation
19 intensities, different water retention models, multi-stage conduit arrangements, and
20 multiple precipitation events on the interaction process between the karst aquifer and
21 stream, this study employs the multiphase Darcy-Brinkman-Stokes equation to analyze
22 the interaction process between the karst aquifer and stream. Additionally, the Phase
23 Indicator Function is used to capture the dynamic changes in saturation levels of various
24 media, and the Brooks-Corey (BC) and van Genuchten-Mualem (VGM) equations are



25 employed to characterize the soil-properties of porous media. The results show that as
26 the intensity of precipitation increases, the interaction process between the karst aquifer
27 and stream becomes more complex, involving the synergistic recharge of multi-media
28 and dynamic interactions with the karst aquifer. Discharges in both upper (PM I) and
29 lower (PM II) porous media rise with precipitation intensity, but PM II shows a more
30 significant increase and earlier peak discharge. Secondly, during the middle to late
31 stages of precipitation, the discharge predicted by the BC model exceeds that of the
32 VGM model. The multi-stage conduit arrangement significantly affects stream and
33 karst conduit hydrology during heavy precipitation but has less impact on other media.
34 Finally, multiple precipitation events with different intensities could affect the ease of
35 recharge from media in different strata of the karst aquifer. The Darcy-Brinkman-
36 Stokes model can effectively simulate the interaction process between the karst aquifer
37 and stream under the influence of precipitation. It can accurately depict the two-phase
38 interactive flow between various media controlled by the dynamic saturation process,
39 and reveal the dynamic interaction process between karst aquifers affected by the
40 epikarst, sinkholes, and conduits under infiltration recharge and stream. Meanwhile, it
41 can precisely explain the processes of infiltration, overflow, and recession.

42 **Keywords:** the karst aquifer and stream; precipitation recharge; two-phase flow;
43 Darcy-Brinkman-Stokes equation; interaction mechanism

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46 **1. Introduction**

47 Karst aquifer is not only a repository of substantial freshwater resources (Li et al., 2017;
48 Ford & Williams, 2007; Sivellet et al., 2021), but also provides drinking water for 10%
49 to 25% of the global population (Longenecker et al., 2017; Goldscheider et al., 2020;
50 Mahler et al., 2021). However, karst-developed areas feature intricate pore structures
51 and fractures (Kuniansky, 2016), leading to pronounced heterogeneity and anisotropy
52 in the movement and storage of water within them (Zhang et al., 2020). In particular,
53 the complex coupled flow involving various flow paths such as karst conduits,
54 sinkholes, and epikarst, along with porous media, further intensifies the nonlinear
55 recharge and discharge processes and the formation of preferential flow paths in the
56 karst aquifer. With seasonal variations in precipitation intensity, the heterogeneity of
57 the groundwater flow field is further exacerbated, and water levels in the karst aquifer
58 and stream fluctuate, leading to complex interactions between the aquifer and stream
59 (Bonacci, 2015). Unveiling the interaction mechanism between the karst aquifer and
60 stream under varying precipitation intensities is crucial for assessing the storage of
61 water resources in karst regions (Gao et al., 2021; Guo and Jiang, 2020).

62 The interaction process between the karst aquifer and stream is significantly
63 influenced by karst media. In epikarst where the soil layer is shallow and dissolution
64 weathering is pronounced, most precipitation can directly recharge the karst aquifer
65 (Lee and Krothe, 2001; O'Leppo et al., 2018). Karst conduits and sinkholes are important
66 media involved in karst hydrological cycle. As rapid discharge channels, the size,



connectivity, and distribution of karst conduits have a significant impact on karst hydrological processes (Duran et al., 2020; Bittner et al., 2020). Surface water collected into sinkholes can directly recharge the karst aquifer (Bianchini et al., 2022), thereby regulating the water level of the aquifer and the discharge volume to the stream, which is influenced by precipitation intensity, size and distribution of sinkhole. The permeability of sinkholes and conduits typically exhibits multilevel characteristics and varies with scale (Halihan et al., 1999), meaning there are strata structures with different permeabilities, which complicates the flow of water within the karst aquifer and increases the catchment area. Therefore, the recharge items to the stream adjacent to the karst aquifer usually include direct precipitation recharge, lateral runoff from the epikarst, discharge from karst springs, and recharge through porous media base flow.

The interaction process between the karst aquifer and stream is also regulated by the dynamic saturation process within the aquifer. The degree of dynamic saturation in different media determines the path and velocity of water flow. Unsaturated aquifers gradually saturate the underlying aquifers under the influence of gravity, while saturated underlying aquifers can cause water to overflow (Worthington, 1991; Huang et al., 2024). In addition, the dynamic saturation processes within the karst aquifer are regulated by factors such as seasonal water level fluctuations, the infiltration and flow of groundwater, and the periodic filling and draining of karst conduits (Huang et al., 2024).

Numerical methods are commonly employed as effective means to accurately



88 simulate karst groundwater movement and assess karst groundwater resources.
89 Shoemaker et al. (2008) proposed a method that discretely embeds conduits, connected
90 by nodes, into the porous media grid (MODFLOW-CFP). This method not only
91 evaluates the water resources of the entire karst aquifer but also considers the geometric
92 shape and distribution of karst conduits on the hydrological processes. Estimations of
93 karst groundwater movement and storage worldwide rely on this method (Kavousi et
94 al., 2020; Qiu et al., 2019; Gao et al., 2020, 2024). Although MODFLOW-CFP is
95 relatively comprehensive for regional karst groundwater simulation studies, it is
96 necessary to couple seepage (porous media) with free flow (conduits and stream) and
97 to describe the dynamic saturation process of the karst aquifer. This can be achieved
98 by using the Navier-Stokes (N-S) equations to couple free flow with seepage through
99 additional source terms (Soulaine and Tchelepi, 2016 ;Carrillo et al., 2020). The Phase
100 Indicator Function for two-phase flow, combined with the phase transition method, can
101 effectively describe the variable saturation process within the karst aquifer (Huang et
102 al., 2024; Zhai et al., 2024). The Darcy-Brinkman-Stokes equations have been utilized
103 to couple seepage flow and free flow (Huang et al., 2024; Nillama et al., 2022; Carrillo
104 et al., 2020). Lu et al. (2023) analyzed a model that integrates fast discharge channels
105 in fractures and conduits with slow seepage in porous media . The results demonstrate
106 that the Darcy-Brinkman-Stokes equations can effectively describe two-phase flow in
107 karst aquifers, and Soulaine (2024) proposed that mixed-scale models based on the
108 Darcy-Brinkman-Stokes equations have strong potential for simulating coupled



109 processes in porous systems. The karst aquifer are typically accompanied by turbulent
110 flow. Reimann et al. (2011) conducted thorough research on turbulent flow in the karst
111 aquifer. To reflect the dissipation of turbulent processes throughout the system, the N-
112 S and Darcy-Brinkman-Stokes equations can be studied using the Reynolds Averaged
113 Network System (RANS) method, where the $k-\varepsilon$ turbulence model effectively
114 characterizes turbulent flows in porous media, as demonstrated by del Jesus et al. (2012).
115 The RANS method has been progressively refined for evaluating turbulent flow in both
116 free-flow regions and porous media (Huang et al., 2024; Zhai et al., 2024; Higuera et
117 al., 2014).

118 Currently, the interaction mechanism between the karst aquifer and stream during
119 precipitation infiltration remains unclear, particularly how varying saturation levels in
120 different karst media affect this interaction. Additionally, existing numerical methods
121 fall short in accurately depicting the combined recharge processes across these diverse
122 media within the karst aquifer. To better understand the interaction mechanisms
123 between the karst aquifer and stream during precipitation infiltration, this study
124 employs the Darcy-Brinkman-Stokes equations to model the coupled processes of
125 seepage in porous media and free flow in karst conduit and stream. The Brooks-Corey
126 (BC) and van Genuchten-Mualem (VGM) models are used to characterize the
127 unsaturated seepage in karst media. The Volume of Fluid (VOF) method is applied to
128 monitor the dynamic changes in aquifer saturation. This research elucidates how
129 saturation dynamics in different karst media impact the coordinated recharge among



media during precipitation infiltration, and examines the evolving interaction between the karst aquifer and stream under such recharge conditions. Given the complexity of the interaction mechanism between the karst aquifer and stream, this study specifically investigates the impact of four factors on the interaction mechanism: (1) changes in precipitation intensity, (2) different water retention models, (3) multi-stage conduit arrangements, and (4) multiple precipitation events. This study elucidates the interaction mechanisms between the karst aquifer and the stream under variable precipitation intensity recharge with seasonal changes, offering a scientific basis for the precise assessment of karst groundwater movement and storage.

2. Materials and methods

The study examines the interaction between karst aquifer and stream, as well as groundwater flow through various karst media, involving the coupling of seepage and free flow processes. Therefore, the Darcy-Brinkman-Stokes equations are adopted as the governing equations to describe the groundwater flow between the karst aquifer and stream, as well as within the karst media. The VOF phase transition method is applied to depict the two-phase flow of water and air in the media, and different water retention models are employed to characterize the unsaturated flow process of karst groundwater.

2.1 Mathematical model for simulating interaction process

The Darcy-Brinkman-Stokes equations are utilized to couple seepage flow in porous media with free flow in conduit and stream (Carrillo et al., 2020; Huang et al., 2024; Soulaïne, 2024; Lu et al., 2023). The two-phase flow is captured using a phase



151 indicator function. Additionally, the k-ε turbulence model is employed to characterize
 152 the turbulent flow features in both porous media seepage within the aquifer and free
 153 flow in conduit and stream.

$$154 \quad \nabla \cdot v_t = 0 \quad (1)$$

$$155 \quad \frac{\partial \rho \alpha_l}{\partial t} + \nabla \cdot (\alpha_l v_t) + \nabla \cdot (\varphi \alpha_l \alpha_g v_{rt}) = 0 \quad (2)$$

$$156 \quad \frac{1}{\varphi} \left((1 + c) \frac{\partial \rho v_t}{\partial t} + \nabla \cdot \left(\frac{\rho}{\varphi} v_t v_t \right) \right) =$$

$$- \nabla p^* + \rho g \cdot X + \nabla \cdot (\mu_{eff} (\nabla v_t + \nabla v_t^T)) - \mu_{eff} k^{-1} v_t + F_c. \quad (3)$$

157 where t denotes the calculation time [s] and φ the porosity; $\alpha_l = \frac{V_l}{V_g + V_l}$ is the aqueous-
 158 phase saturation, $\alpha_g = \frac{V_g}{V_g + V_l}$ is the gas-phase saturation, and V_g and V_l are the gas-
 159 phase and the aqueous-phase volumes, respectively; v_t is the fluid flow rate [m/s]; v_{rt}
 160 is the relative velocity between groundwater and air [m/s]; μ_{eff} is the effective
 161 viscosity that It can be defined as μ_{eff} , μ is the viscosity [m² /s], $\mu = \alpha_g \mu_g + \alpha_l \mu_l$,
 162 and μ_g and μ_l are the viscosity of the gas and liquid phases, respectively; v_{turb} is the
 163 turbulent kinetic viscosity; ρ represents the average density of groundwater and air ; p^*
 164 is pressure [pa]; g is the gravitational acceleration, 9.81 [m² /s]; k is the permeability
 165 [m²]; and F_c is the surface tension.

166 The eddy viscosity is expressed as:

$$167 \quad \mu_t = \rho C_\mu \frac{k^2}{\varepsilon} \quad (4)$$

168 where k represents the turbulent kinetic energy, ε is the dissipation rate of turbulent
 169 kinetic energy, and C_μ is a constant , equal to 0.09.

170 Phase Indicator Function can be expressed as:



$$\alpha_l = \begin{cases} 1 & \text{water} \\ 0 < \alpha < 1 & \text{two-phase zone} \\ 0 & \text{air} \end{cases} \quad (5)$$

where α_l represents the saturation of groundwater. Relative permeability is key to describing the migration of groundwater and gas (Kuang and Jiao, 2011). In relative permeability model for two-phase flow, the effective saturation of the aqueous phase, $\alpha_{l,e}$, is given by:

$$\alpha_{l,e} = \frac{\alpha_l - \alpha_{l,r}}{1 - \alpha_{g,r} - \alpha_{l,r}} \quad (6)$$

where, $\alpha_{l,r}$ and $\alpha_{g,r}$ are the residual saturations of water and air, respectively. In the Brooks and Corey (BC) model, the expression for the relative permeability k_r is given by (Brooks and Corey et al., 1964):

$$k_{r,g} = (1 - \alpha_{l,e})^{0.5} (1 - \alpha_{l,e}^{1/m})^{2m} \quad (7)$$

$$k_{r,l} = \alpha_{l,e}^{0.5} \left(1 - (1 - \alpha_{l,e}^{1/m})^m \right)^2 \quad (8)$$

where, m is a dimensionless parameter that is determined based on the characteristics of the porous media within the karst aquifer. The expression for relative permeability in the van Genuchten–Mualem (VGM) model (Parker et al., 1987) is defined as follows:

$$k_{r,g} = (1 - \alpha_{l,e})^{0.5} (1 - \alpha_{l,e}^{1/m})^{2m} \quad (9)$$

$$k_{r,l} = \alpha_{l,e}^{0.5} \left(1 - (1 - \alpha_{l,e}^{1/m})^m \right)^2 \quad (10)$$

In the free and porous regions, the source term μk^{-1} in the Darcy-Brinkman-Stokes equation varies in form and can be expressed as (Soulaire, 2024; Huang et al., 2024):

$$\mu_{eff} k^{-1} = \rho \mathbf{v}_{turb} k^{-1} + \begin{cases} 0, & \text{free region} \\ k_0^{-1} \left(\frac{k_{r,l}}{\mu_l} + \frac{k_{r,g}}{\mu_g} \right)^{-1}, & \text{porous region} \end{cases} \quad (11)$$



191 The permeability coefficient k_0 , which is determined by the geometric structure
 192 of the porous medium, controls both free flow and seepage. When permeability is high,
 193 the governing equation (Equation 3) simplifies to the Navier-Stokes equation (Equation
 194 12). Conversely, when permeability is low, the equation reduces to Darcy's law
 195 (Equation 13).

$$196 \quad (1+c) \frac{\partial \rho v_t}{\partial t} + \nabla \cdot (\rho v_t v_t) = -\nabla p^* + \rho g \cdot X + \nabla \cdot (\mu_{eff} (\nabla v_t + \nabla v_t^T)) + F_c, \text{ if } \varphi = 1. \quad (12)$$

$$197 \quad 0 = -\nabla p^* + \rho g \cdot X - \mu_{eff} k^{-1} v_t + F_c, \text{ if } \varphi \in]0,1[. \quad (13)$$

198 Meanwhile, the surface tension F_c and density ρ in the free-flow and porous media
 199 regions are as follows (Huang et al., 2024):

$$200 \quad F_c = \begin{cases} -\frac{\sigma}{\varphi} \nabla \cdot \left(\frac{\nabla \alpha_l}{|\nabla \alpha_l|} \right) \nabla \alpha_l, & \text{free region} \\ k_0 \frac{\left(\frac{k_{r,l}}{\mu_l} \alpha_g - \frac{k_{r,g}}{\mu_g} \alpha_l \right) \left(\frac{\partial p_c}{\partial \alpha_l} \right) - p_c}{\frac{k_{r,l}}{\mu_l} + \frac{k_{r,g}}{\mu_g}} \nabla \alpha_l, & \text{porous region} \end{cases} \quad (14)$$

$$201 \quad \rho = \begin{cases} \rho_l \alpha_l + \rho_g \alpha_g, & \text{free regions} \\ k_0 \frac{\left(\rho_g \frac{k_{r,g}}{\mu_g} + \rho_l \frac{k_{r,l}}{\mu_l} \right)}{\frac{k_{r,l}}{\mu_l} + \frac{k_{r,g}}{\mu_g}}, & \text{porous regions} \end{cases} \quad (15)$$

203 where σ is the interfacial tension [N/m], p_c is capillary pressure [pa].

$$204 \quad k_{r,g} = (1 - \alpha_{l,e})^{0.5} \left(1 - \alpha_{l,e}^{\frac{1}{m}} \right)^{2m} \quad (16)$$

205 Numerical modeling

206 2.2 Numerical modelling



207 The numerical model is developed according to the conceptual model of the karst
208 aquifer adjacent to a stream, as depicted in Fig. 1. The model incorporates the
209 distinctive features of karst regions, including sinkholes, epikarst, and karst conduit
210 (Fig. 1). The karst conduit is connected to the epikarst through a sinkhole. The outcrop
211 of the karst spring is located at the end of the karst conduit, directly leading to the stream.
212 In the karst aquifer, the saturation levels within the porous media are dynamically
213 altered by precipitation, and the water levels in both the karst conduit and the stream
214 experience substantial fluctuations. As a result, the interaction between the porous
215 media and the stream displays a clear multi-scale characteristic. From a hydrological
216 perspective of the watershed, the recharge and discharge processes of karst conduit are
217 controlled by the saturation degree of the surrounding porous media and the water level
218 within the conduit themselves. Based on spatial relationships, the area between the karst
219 conduit and the epikarst is divided into Porous Medium I (PM I) above the conduit,
220 Porous Medium II (PM II) on the sides, and Porous Medium III (PM III) directly below
221 the conduit. During a single precipitation event, some of the rainfall directly
222 replenishes the stream, while the remainder percolates down to recharge the karst
223 aquifer. The infiltration recharge consists of two processes: (1) infiltration recharge to
224 the epikarst, and (2) downward infiltration recharge to the aquifer through sinkhole and
225 porous media. These two processes sequentially recharge the stream: groundwater
226 discharges laterally through the epikarst to the stream; precipitation rapidly recharges
227 the connected karst conduit through the sinkhole and recharges to the stream through



the karst spring, while groundwater in the aquifer also discharges laterally to the stream. Compared to PM I, groundwater in the epikarst recharges the stream at a faster rate, causing the water level of stream to rise and subsequently recharging PM I and PM II. As the water level of stream gradually rises, the stream will recharge the karst aquifer. Due to the rapid flow velocity of the stream, the water level drops, allowing groundwater in the lower porous media to discharge back into the stream. This study constructs a numerical model based on the dynamic interaction process between the karst aquifer and stream, revealing the interaction mechanism between the karst aquifer and stream under the influence of precipitation intensity changes, different water retention models, multi-level permeability arrangements, and multiple precipitation events.

The upper boundary of the strata is a transient natural precipitation boundary condition. The boundary condition for precipitation infiltration recharge is adopted using the following equation (Huang et al., 2024):

$$I(t) = \frac{b}{\sqrt{2\pi\sigma^2}} \sum e^{-\frac{(t_i - \mu)^2}{2\sigma^2}} \quad (17)$$

where t_i represents the time of the i th precipitation event, and $I(t)$ represents the total precipitation at that moment. According to Chang et al. (2015), μ , σ^2 and a are set as constants (90, 1.5 and 20, respectively). During precipitation infiltration recharge, changes in precipitation intensity are adjusted via the dimensionless parameter b .

Some researchers have positioned the karst conduit at the bottom of the model grid (Kavousi et al., 2020; Li et al., 2023). This study employs a programming approach to



249 dynamically generate the sinkhole and conduit grids, enabling the creation of conduit
250 and sinkhole of varying diameters at any position within the model by adjusting
251 parameters such as conduit radius and center coordinates. This enhances the
252 adaptability and practicality of the model.

253 Figure 2 illustrates the discretization scheme adopted by this study, clearly
254 showing the division and distribution of grids in each region. Based on the thickness
255 of the epikarst layer and the position of the stream, except for the stream, sinkhole,
256 epikarst, and karst conduit, and the remaining areas are divided into porous media
257 regions, and the grids in the free-flow regions are further refined. Given that the flow
258 in the conduit, sinkhole, and stream is free-flowing, fine grids are required to capture
259 the microscopic changes in water levels and interfaces, so the grids in these regions are
260 locally refined. In the porous media, groundwater flows in a seepage manner, with its
261 velocity decreasing with the increase in distance from the discharge end, forming a
262 funnel-shaped pressure drop flow characteristic. Thus, a grid spacing approach with
263 gradual increments in the porous media regions is employed. The edge grids are
264 designed to be twice as long as those near the conduit. This method ensures precise
265 simulation of flow near the discharge end while significantly reducing computational
266 resource usage in distant areas. Given the dissolution effects near the sinkholes and
267 epikarst, the permeability of the porous media in the karst aquifer decreases from the
268 interior to the exterior, and it is assigned in a graded manner. The values of the model
269 parameters are listed in Table 1.



270 **3. Results**

271 **3.1 Interaction process between the karst aquifer and stream under precipitation**

272 **infiltration recharge**

273 The changes in hydrological process curves, water level fluctuations, and their
274 differences during the interaction between karst media and stream under different
275 precipitation intensities are shown in Fig. 3. In the early stage of precipitation, the flow
276 in the stream primarily originates from direct precipitation recharge and lateral
277 groundwater recharge from epikarst (Fig. 3(a)). As the water level in the stream
278 gradually rises, the flow not only continues downstream but also begins to recharge the
279 karst aquifer, particularly the PM II. The peak recharge to PM II coincides with the
280 peaks of epikarst recharge to the stream (Epikarst in Fig. 3) and direct precipitation
281 recharge (P-River in Fig. 3). Therefore, the interaction process between the karst
282 aquifer and stream during the early precipitation stage is significantly influenced by
283 lateral groundwater discharge from the epikarst and the direct precipitation recharge.
284 As groundwater recharge from epikarst to the stream declines (Fig. 3 (a)), groundwater
285 moves downward through the epikarst to PM I, and begins to gradually recharge the
286 stream. Due to the low permeability of the epikarst, lateral discharge from PM I to the
287 stream will be delayed. During this process, the discharge volume of PM I exhibits two
288 distinct peaks. The first peak is due to the recharge of groundwater from the epikarst,
289 while the second peak is caused by the gradual saturation of PM II and the karst conduit,
290 with a proportion of groundwater overflowing from PM I and discharging laterally to



291 the stream. After the end of precipitation recharge, the hydrological process curve of
292 PM I rapidly declined, and the discharge volume of the karst conduit, PM III and PM II
293 gradually increase, causing the water level in the stream to rise (Fig. 3 (d)). When the
294 water level in the stream gradually exceeds that of PM I, the stream begins to gradually
295 recharge PM I. The karst conduit, PM II and PM III continue to discharge to the stream
296 during this stage due to higher internal water pressure, forming a local hydrological
297 cycle with the upper layer. In the late stage of precipitation, the hydrological process
298 of the stream primarily shows a gradual decline in baseflow.

299 As depicted in Figs. 3b and 3c, the recharge and discharge dynamics between the
300 karst aquifer and stream across different media shift notably with escalating
301 precipitation intensity. The recharge volumes from the stream to PM I and PM II both
302 decrease. The reduction in the recharge to PM II from the stream is primarily due to
303 the acceleration of groundwater movement downward as precipitation intensity
304 increases, causing groundwater to move more rapidly to the bottom of the karst aquifer,
305 thereby recharging PM II. Consequently, part of pore space that should have been
306 recharged by the stream is instead recharged from PM I downward. The decrease in
307 the recharge to PM I can be attributed to its high internal saturation level and the rise in
308 water level. On the other hand, the water level in the stream does not significantly
309 exceed that of the upper aquifer, making it difficult for the stream to effectively
310 recharge the aquifer. Due to the reduced recharge volume to the aquifer, the discharge
311 from the stream is partially lower than the discharge from the epikarst during the early



312 stage of the hydrological process.

313 With changes in precipitation intensity ($b = 3, 5$, and 7), the water level
314 variations and their differences between the karst aquifer and stream exhibit complex
315 dynamic characteristics (Figs. 3d, 3e and 3f). During the early stage of precipitation,
316 despite the increasing water level difference, the discharge from the stream to the
317 aquifer is gradually decreasing (as shown by the negative values for PM I and PM II in
318 Fig. 3a, 3b and 3c). This phenomenon indicates that water level is not the only factor
319 controlling the interaction between the karst aquifer and stream; changes in the degree
320 of saturation also play a significant role. As shown in Fig. 3d, under low precipitation
321 intensity, the water level difference between the karst aquifer and stream is often greater
322 than the water level of the stream during the middle and later stages of precipitation.
323 However, as precipitation intensity increases, the water level difference tends to
324 decrease (Fig. 3b and 3c). This change is primarily due to the increased precipitation
325 intensity leading to a faster saturation of the karst aquifer, thereby limiting the ability
326 of the stream to recharge the aquifer. After the middle stage of precipitation, the
327 interaction between the stream and the upper part of the aquifer gradually intensifies,
328 while the lower part of the aquifer discharges to the stream (Fig. 3a). Due to the gradual
329 decrease in water level difference, it is difficult for the stream to effectively recharge
330 the aquifer. In this process, the interaction between the aquifer and stream is controlled
331 by the dynamic changes in saturation.

332 It is self-evident that changes in precipitation intensity significantly affect the



333 recharge and discharge processes between the karst aquifer and stream. The water
334 levels and saturation degrees of the respective media act as core controlling factors that
335 jointly influence the interactive dynamics between the aquifer and stream. To gain a
336 deeper understanding of these influencing factors and their interaction mechanisms, and
337 to further elucidate the interaction process mechanisms between the karst aquifer and
338 stream, this study focuses on the hydrological interaction process between the two
339 during the early stage of precipitation.

340 **3.2 Interaction process between the karst aquifer and stream during early stage of** 341 **precipitation**

342 Figure 4 illustrates how the interaction volume between the epikarst, porous media,
343 and stream varies under different precipitation intensities. As shown in Fig. 4a, at a
344 precipitation intensity $b = 3$, the contribution ratios of the epikarst, PM I, and PM II to
345 the recharge of the stream are similar. This indicates that during the early stage of
346 precipitation, the recharge effects of each medium on the stream are relatively balanced.
347 Since groundwater vertically recharges the underlying aquifer through the epikarst, the
348 discharge peak of PM II is relatively delayed compared to the epikarst and PM I.

349 As the precipitation intensity increases ($b = 5$), the contribution ratios of the
350 epikarst, PM I, and PM II to the recharge of stream experience significant changes (Fig.
351 4b). Upon comparing Fig. 4a and 4b, it is evident that an increase in precipitation
352 intensity leads to higher discharge volumes for both PM I and PM II, with PM II
353 experiencing a more pronounced rise. Additionally, the peaks of their discharges occur



354 earlier. The first peak of PM I is primarily caused by infiltration recharge from
355 precipitation. With the increase in precipitation intensity, the infiltration velocity
356 accelerates and the recharge volume increases, leading to a larger discharge volume and
357 an earlier peak for PM I (vertical recharge peak). Groundwater continues to move
358 downward from PM I, and the saturation of PM II rises, allowing more groundwater to
359 overflow and discharge through PM I, thereby generating the second peak (overflow
360 peak). For PM II, as discussed in Section 3.1, increase in saturation reduces the
361 recharge from stream, but the discharge volume increases gradually after the middle
362 stage of precipitation, and its contribution to the recharge of the stream becomes
363 dominant among the three. This is due to the increased precipitation intensity, which
364 allows PM II to receive more vertical recharge, enhancing its discharge capacity. When
365 the precipitation intensity continues to increase ($b = 7$, Fig. 4c), PM II gradually
366 reaches saturation. According to the analyses in Section 3.1, the ability of PM II to
367 receive recharge is limited by its own saturation level, making it difficult to receive
368 vertical recharge. Therefore, despite the increased precipitation intensity, the discharge
369 volume of PM II does not increase significantly. Conversely, due to the influence of
370 the saturation state of the underlying aquifer medium, the second peak (overflow peak)
371 of PM I is more pronounced, indicating a more evident overflow phenomenon. Under
372 higher precipitation intensity, the recharge contribution of PM I to the stream dominates.

373 Thus, variations in precipitation intensity notably influence the interaction volume
374 between the karst media and stream. As precipitation intensity increases, the discharge



375 volume and peak values of each medium are altered. Specifically, the two peaks of PM
376 I show sequential changes in intensity, which are modulated by the saturation levels
377 of the adjacent media.

378 **3.3 Impact of different water retention characteristics on the interaction process** 379 **between the karst aquifer and stream**

380 Figure 5 illustrates the changes in saturation of water retention curves based on
381 two different retention equation for karst aquifer: the BC model (equations (12)-(13))
382 and the VGM model (equations (14)-(15)). For the same infiltration periods, the water
383 content predicted by BC model is generally higher than that predicted by the VGM
384 model. The BC model may focus more on the static retention of groundwater in the
385 medium, while the VGM model may emphasize the dynamic transport and distribution
386 of groundwater within the medium. Moreover, the VGM model predicts a greater
387 distance of groundwater movement compared to the BC model, indicating that the
388 VGM model may have higher sensitivity in simulating infiltration processes of
389 groundwater in the medium. This difference is of significant importance for the
390 dynamic process of unsaturated two-phase flow in the karst aquifer and for accurately
391 predicting the advancement path of groundwater.

392 In addition, there are differences between the BC model and the VGM model in
393 simulating the saturation changes of the water retention curve (Fig. 5). Not only do
394 they differ in the degree of saturation change at the same time and location, but their
395 simulation results for the distance of groundwater movement also vary. Therefore, it is



396 crucial to select the appropriate model based on specific lithological conditions, as this
397 can more accurately describe and predict the two-phase flow of karst groundwater.

398 The impact of different water retention models on the interaction process between
399 the karst aquifer and stream is shown in Fig. 6. Compared to the BC model, the VGM
400 model generally calculates lower discharge volumes from the stream. Therefore, under
401 the simulation conditions of the VGM model, more groundwater is retained in the
402 porous medium rather than being discharged through the stream. This reflects the
403 differences between the two water retention models in simulating the movement and
404 storage mechanisms of groundwater in the porous medium. During the early stage of
405 precipitation recharge, the VGM model results show that the stream is more likely to
406 recharge the karst conduit (Fig. 6b). Although the karst conduit receives more recharge
407 from stream, their discharge to stream is relatively low. This indicates that the karst
408 conduit derived from the VGM model receive relatively lower recharge intensities from
409 the porous medium and sinkhole. As shown in Fig. 6c, due to the shorter distance of
410 groundwater movement derived from the VGM model within the same time, the vertical
411 infiltration capacity of the epikarst is reduced, thereby increasing the discharge volume
412 of the epikarst to the stream. This indicates that the interaction process between stream
413 and the epikarst is significantly influenced by the water retention characteristics. In the
414 VGM model, the difficulty of groundwater moving vertically through the epikarst
415 increases, leading to a decrease in the discharge volume of PM I (Fig. 6d). Since PM I
416 receives limited recharge from the epikarst, its saturation remains at a low level, making



417 it more favorable to receive recharge from stream (see Section 3.1).

418 The VGM model results suggest that the stream predominantly recharges PM II
419 (as seen in Figs. 6e and 6c). However, during the middle and later stages, the stream
420 recharge volume calculated by the BC model surpasses that of the VGM model. Figure
421 6f illustrates that the groundwater in the porous medium beneath the karst conduit
422 primarily originates from conduit recharge. Meanwhile, Fig. 6b shows an increase in
423 the discharge volume from the karst conduit, as estimated by the BC model. This
424 increase subsequently affects the discharge volume of the porous medium below the
425 karst conduit.

426 Therefore, different water retention models have a significant impact on the
427 interaction process between the karst aquifer and stream. These impacts are not only
428 reflected in the changes of discharge and recharge volumes but also involve the
429 movement and storage mechanisms of groundwater in different media. In practical
430 applications, selecting an appropriate water retention model based on the corresponding
431 lithology can more accurately simulate and predict the interaction process between the
432 karst aquifer and stream.

433 **3.4 Impact of multi-stage permeability and porosity arrangement on the** 434 **interaction process between the karst aquifer and stream**

435 By comparing the effects of multi-level and single conduit arrangements on the
436 interaction process, it is found that using multi-level and single conduit arrangements
437 in underlying media does not cause significant changes in the hydrological processes



438 of the epikarst and porous media (PM I and PM II, Fig. 7). As shown in Fig. 7a, when
439 multi-level conduit arrangements are adopted, the peak of stream hydrological process
440 increases, indicating that multi-level conduit arrangements enhance the recharge
441 volume of stream. However, during the recession phase, the flow under multi-level
442 conduit arrangements is relatively low. This is because multi-level conduit collects a
443 proportion of the flow that should have been contributed by the later stage matrix
444 recession and discharge it to stream, thereby affecting the peak of the recession process.
445 As shown in Fig. 7b, under multi-level conduit arrangements, sinkhole can absorb more
446 water and discharge it through karst conduit. This indicates that multi-level conduit
447 arrangements can more effectively play their roles in water absorption and discharge
448 during heavy precipitation events. However, in the case of lower precipitation intensity
449 in the early stage, the water absorption priority of multi-level conduit is not fully
450 manifested. By comparing Figs. 7c, 7d, and 7e, it is found that multi-level conduit
451 arrangements have no significant impact on the hydrological processes of the epikarst
452 and porous media (PM I and PM II). This suggests that multi-level conduit
453 arrangements mainly affect the interaction between the karst conduit and stream, with
454 relatively little impact on other media. The hydrological responses of the karst conduit
455 and PM II under multi-level conduit arrangements are shown in Figs. 7f and 7b. Under
456 multi-level conduit arrangements, the discharge volume of the karst conduit
457 significantly increases. At the same time, due to the increase in karst conduit flow, PM
458 II also receives more recharge, leading to a corresponding increase in the discharge



459 volume of this portion of porous media to stream. This further indicates that multi-
460 level conduit configurations can notably influence the hydrological processes of stream
461 and karst conduit under specific precipitation intensities, with minimal effects on other
462 media.

463 **3.5 Impact of multiple precipitation events on the interaction process between the** 464 **karst aquifer and stream**

465 Rainy seasons typically experience multiple precipitation events, during which
466 differences in precipitation peaks, durations, and cumulative precipitation events can
467 all impact the interaction process between the karst aquifer and stream. Based on
468 understanding the interaction mechanism of a single precipitation event, this study
469 further analyzes the impact of multiple precipitation events on the interaction process.
470 Figure 8 shows the changes in water level of stream under continuous precipitation
471 events. When the intensities of two consecutive precipitation events remain constant,
472 the water level of stream reaches both the highest and the lowest points, indicating that
473 the water level is related to the total precipitation intensity. Even with different
474 intensities of the first precipitation event ($b_1 = 3$ and $b_1 = 5$), the trend of the water level
475 changes in stream is consistent (Fig. 8① and ④). After the first precipitation event,
476 the karst aquifer receives infiltration recharge from the precipitation and can store part
477 of the water, so the water level of stream will be higher during the second precipitation
478 event, and the greater the intensity of the second precipitation event, the higher the
479 water level of stream (Fig. 8① and ②, or ③ and ④). This indicates that the intensity



480 of the second precipitation event determines the amount of recharge from each medium
481 to stream. Therefore, when the intensity of the first precipitation event is the same, the
482 amplitude of the water level change in stream during the second precipitation event is
483 only related to the intensity of the second precipitation event. When the intensity of the
484 second precipitation event is the same, the storage capacity of the karst aquifer during
485 the first precipitation event determines the amplitude of the water level change in stream
486 during the second precipitation event. When the total precipitation intensity is the same
487 (Fig. 8 ② and ③), if the intensity of the first precipitation event is lower than that of
488 the second one, the amplitude of the water level change in stream is higher, and vice
489 versa. This is because, in the case of two consecutive precipitation events, part of the
490 precipitation infiltrates and recharge the storage during the first event, and the other part
491 is discharged to stream through the aquifer. Combining Fig. 3d and 3e, during the first
492 precipitation event, the water level in the porous medium rises and stores a proportion
493 of water, but the discharge volume to stream is greater when the precipitation intensity
494 is higher ($b_1 = 5$) compared to when it is lower ($b_1 = 3$, Fig. 3a and b). When the
495 second precipitation event occurs, due to the similar saturation levels of the karst aquifer,
496 the greater the intensity of the second precipitation event, the larger the amount of
497 groundwater recharged to stream through the aquifer, and the more pronounced the
498 amplitude of the water level in stream.

499 Figure 9 illustrates the hydrological process curves of the stream during two
500 consecutive precipitation events, as well as the interaction processes between the



501 various media of the karst aquifer and stream. Under different precipitation intensities,
502 the various media of the karst aquifer recharge the stream with varying intensities,
503 resulting in significant fluctuations in the water level of stream. Combining Fig. 9a and
504 Fig. 8 (② and ④), it can be concluded that during two consecutive precipitation events,
505 when the intensity of the second precipitation is greater than or equal to that of the first
506 one, the amplitude of the hydrological process of stream is larger. As shown in Fig. 9b,
507 the epikarst discharges quickly and is not easily affected by multiple precipitation
508 events. However, when the intensity of the first precipitation is high and the intensity
509 of the second precipitation is the same (① and ③), the discharge volume of the epikarst
510 to stream during the second precipitation period is slightly larger. When the intensity
511 of the first precipitation is different and the intensity of the second precipitation is the
512 same (Fig. 9c ② and ④), the discharge volume of groundwater through karst conduit
513 to stream during the second precipitation period is almost the same. This is because
514 karst conduit discharge quickly, and the storage volume of the conduit during the first
515 precipitation period has little impact on the storage volume during the second
516 precipitation period. Therefore, combining with Fig. 8, it is known that the storage
517 effect of the karst aquifer mainly occurs in the porous medium, and it also indicates that
518 relying solely on changes in the water level of stream makes it difficult to clearly
519 determine the storage volume of the porous medium and conduit during the first
520 precipitation event, and their respective impacts on the second precipitation period (Fig.
521 8). When the intensity of the second precipitation is higher (Fig. 9c ②, ③ and ④), the



522 discharge volume of the porous medium (PM II) to stream does not increase
523 significantly. This is because the intensity of the second precipitation is larger, causing
524 the water level of stream to rise (Fig. 8), making it difficult for the porous medium (PM
525 II) to recharge stream.

526 Therefore, under the influence of two consecutive precipitation events, the greater
527 the total precipitation intensity, the larger the discharge volume of the karst aquifer to
528 stream. The storage effect of the karst aquifer occurs in the porous medium and affects
529 subsequent precipitation processes. The lower-level porous medium (PM II), due to
530 the high water level and large fluctuations of stream, is more difficult to recharge stream,
531 and the recharge from stream mostly comes from overflow supply from the media in
532 other layers.

533 **4. Discussion**

534 **4.1 Comparison with the simulation results of MODFLOW-CFP**

535 To better assess the applicability of the Darcy-Brinkman-Stokes model in
536 simulating the interaction between the karst aquifer and stream, this study compares its
537 simulation outcomes with those from MODFLOW-CFP. As depicted in Fig. 10(a.1),
538 the study contrasts the coupling approaches of conduits and porous media in both
539 MODFLOW-CFP and the Darcy-Brinkman-Stokes model, focusing on control
540 equations and grid discretization. In the MODFLOW-CFP model, the groundwater
541 flow during the interaction process is determined by the stable water levels between the
542 conduit-porous media and stream-porous media interfaces (Fig. 10(a.2)). In the Darcy-



543 Brinkman-Stokes model, however, the groundwater interaction among the conduit, the
544 stream, and the porous media depends on the saturation and pressure differences
545 between adjacent grid points. It allows the groundwater interaction among the conduit,
546 the stream, and the porous media to be recharged or discharged simultaneously at
547 different positions. However, this also requires calculating the changes in all grid fluxes
548 (Fig. 10(a.3)).

549 This study further compares the interaction modes between the stream and porous
550 media in MODFLOW-CFP and Darcy-Brinkman-Stokes. In MODFLOW-CFP, the
551 stream is discretized among single grid cells. The interaction volume between the
552 stream and porous media depends on the water level difference between them. The
553 fluctuating water surface of the stream is generalized to a unified water level value, and
554 the “dry area” cannot be simulated in the porous media area (as shown in Fig. 10(a.4)).
555 In Darcy-Brinkman-Stokes, the modeling of each medium is completed by specifying
556 the specific porosity and permeability at each grid node. At the interface, the values are
557 interpolated to the average grid cell value based on the values between nodes. Therefore,
558 the interaction interface where the conduit is directly connected to the stream does not
559 need the porous media as an intermediate. On this basis, the shape of the stream can be
560 established as a regular rectangle or an irregular channel. The Darcy-Brinkman-Stokes
561 simulates the free flow by reconstructing the water-vapor interface tracer through the
562 VOF (Volume of Fluid) and Front-tracking methods. Therefore, when the grid is fine
563 enough, it can simulate the fluctuating water-vapor interface (as shown in Fig. 10(a.5)).



564 This study further reveals the interaction mode of groundwater between the
 565 conduit and stream in the Darcy-Brinkman-Stokes model and the differences from the
 566 simulation results of MODFLOW-CFP through a generalized karst aquifer. As shown
 567 in Fig. 11, the karst conduit is surrounded by porous media and are directly connected
 568 to the stream. The aquifer is 200 m long, 200 m wide, and 30 m thick. As shown in
 569 Fig. 11(a.1), groundwater is replenished from the porous media to the conduit and
 570 discharged into the stream. The model parameters are as follows: assume that the
 571 porous media is a homogeneous medium with a hydraulic conductivity of 0.65 m/s.
 572 The interaction parameter of the conduit wall is 25 m/s, the conduit diameter is 1 m, the
 573 conduit roughness is 0.01 m, and the conduit outlet and the stream are in the same grid
 574 cell, and the interaction is simulated through the porous media. The initial spring flow
 575 is set to zero, the initial head of the porous media is also set to 10 m, and the vertical
 576 height of the conduit node is 1 m. The groundwater temperature in the conduit is set to
 577 20 °C, and the surrounding boundaries are no-flow boundaries. The upper boundary is
 578 a rainfall boundary (Equation 16), where $b = 5$. According to Huang et al. (2024), μ ,
 579 σ^2 , and a are set as constants (90, 1.5, and 20) respectively. The total simulation period
 580 is 25000 s. In the MODFLOW-CFP model, each stress period is set to 1 minute, and in
 581 the Darcy-Brinkman-Stokes model, due to the Courant number limitation, each time
 582 step is less than 0.1 s.

583 Through a comparative analysis of the simulations of MODFLOW-CFP and
 584 Darcy-Brinkman-Stokes (the simulated hydrograph of the stream in the Darcy-



585 Brinkman-Stokes model in Fig. 3(b)), as shown in Fig. 11(a.2), there are three
586 differences in the simulation results between the MODFLOW-CFP model and the
587 Darcy-Brinkman-Stokes model: (1) The hydrograph of the stream in the MODFLOW-
588 CFP model lags behind the Darcy-Brinkman-Stokes model in terms of rising. This is
589 because in the Darcy-Brinkman-Stokes model, the rapid lateral runoff of epikarst causes
590 the hydrograph of the stream to rise rapidly. (2) The peak discharge of the Darcy-
591 Brinkman-Stokes model is slightly lower. This is because a part of the storage capacity
592 of the porous medium has been drained by the rapid lateral runoff of epikarst. By using
593 the rainfall directly infiltrating into the stream (P -River) to represent the time nodes of
594 the rainfall peak and comparing it with the time nodes of the peak discharge of the
595 stream simulated by MODFLOW-CFP and Darcy-Brinkman-Stokes, it is found that
596 both MODFLOW-CFP and Darcy-Brinkman-Stokes exhibit a lag of 2000 s in the
597 rainfall peak. (3) In the Darcy-Brinkman-Stokes model, the rapid lateral runoff of
598 epikarst reduces the storage capacity of the upper porous medium. Therefore, during
599 the initial base flow recession stage, the discharge of the stream in the Darcy-Brinkman-
600 Stokes model is lower than the simulated value of MODFLOW-CFP. As the storage
601 capacity of the porous medium gradually decreases, the influence of epikarst gradually
602 weakens, and the recession amounts of the two tend to be the same.

603 **4.2 Dynamic interaction processes among various media**

604 Through a comparative analysis of the hydrographs of the stream simulated by
605 MODFLOW-CFP model and the Darcy-Brinkman-Stokes model, it is found that the



606 two models have similar effects in simulating the interaction between the karst aquifer
607 and stream under rainfall infiltration recharge. However, with its fine grid and two-
608 phase flow simulation capabilities, the Darcy-Brinkman-Stokes model can accurately
609 capture the interaction processes among various media, such as between the saturated
610 and unsaturated zones at various stages under the influence of the dynamic saturation
611 process, and between the conduit and the stream, under rainfall infiltration recharge.

612 As shown in Fig. 12, the Darcy-Brinkman-Stokes model clearly demonstrates the
613 changes in the saturation levels of epikarst, porous media, and the karst spring; the
614 saturation fields and the interaction between various media at 4000 s, 6105 s, and 7363
615 s; the interaction amounts between epikarst, porous media I, II, III, and the stream.
616 From Fig. 12 (a.1), it can be seen that the saturation level of epikarst rises and declines
617 earliest, but the saturation level is relatively low, and it is in a completely unsaturated
618 flow state. Porous media I and III rise synchronously before 5000 s, while porous media
619 II and the karst spring rise rapidly at 4611 s. At 7409 s, the karst spring and porous
620 media I successively enter the decline stage. Due to the rapid drainage of the conduit,
621 the saturation level decreases. The saturation level of the karst spring decreases faster
622 than that of porous media I and intersects with porous media I at 9670 s.

623 Combining Fig. 12 (a.2) with other sub-figures, the stages with obvious
624 interactions among porous media can be divided into the infiltration stage (green), the
625 overflow stage (red), and the recession stage (blue). During the infiltration stage from
626 4000 s to 4611 s, as shown in Fig. 12 (a.2.1), epikarst vertically replenishes porous



627 medium I and infiltrates downward. However, the infiltrating water does not reach the
628 lower media. Meanwhile, the saturation levels of porous media II, III, and the conduit
629 gradually increase (see Fig. 12 (a.1)). Combining with Fig. 12 (a.3), it can be seen that
630 epikarst laterally replenishes the stream, and quickly drops to the bottom of the riverbed
631 due to gravity. At this time, the lower aquifer system (porous media II, III, and the
632 conduit) is in a dry state, so the stream replenishes the lower aquifer. The amount of
633 recharge received by porous medium III and the conduit is less than that of porous
634 medium II (analyzed by combining Fig. 12 (a.3) and (a.4)), but their saturation levels
635 increase faster. There are two reasons for this situation: First, the bottom elevation of
636 the conduit is 1 m, and the water level of the stream needs to submerge the 1 m water
637 level before it can recharge the conduit. Second, porous medium III is not only
638 replenished by the stream, but also the sinkhole diverts the groundwater in epikarst and
639 porous medium I to the conduit (the sinkhole flow velocity and saturation as shown in
640 Fig. 12 (a.2.1)), and then replenishes porous medium III. As the lower aquifer media
641 gradually tends to be saturated with rainfall recharge, as shown in Fig. 12 (a.2.2), porous
642 media II and III tend to be saturated (see Fig. 12 (a.2.1)). Due to the weak
643 compressibility of water, after the upper part infiltrates and replenishes porous medium
644 I, it tends to laterally replenish the stream from the interface between porous medium
645 II and stream. As the saturation level of porous medium I gets higher, the lateral
646 recharge to the stream becomes more significant, showing an obvious overflow state.
647 The depression between the two peaks is caused by the rapid rise of the stream water



648 level. During the flood peak stage, the discharge from porous media to stream decreases.
649 At the same time, the rise of the stream water level makes it difficult for the lower
650 porous media to replenish the stream, and porous medium II tends to be saturated,
651 making it difficult to replenish porous medium I. During this stage, the flow between
652 porous media I and II is in a dynamic equilibrium state. As shown in Fig. 12 (a.2.3),
653 during the recession stage, the rainfall infiltration intensity decreases rapidly. Under
654 the action of gravity, the groundwater vertically replenishes porous medium I, the
655 conduit, and porous medium II successively recedes. And the water level of the stream
656 drops rapidly (see Fig. 3 (e)). The groundwater tends to be discharged to the stream
657 through porous medium I and the karst spring. Porous medium I is replenished by
658 porous medium II on the one hand and discharges to the stream on the other hand.
659 Therefore, during a single rainfall event, during the infiltration stage, part of the amount
660 of water replenished from epikarst to the stream is discharged, and other part is
661 redirected to replenish the lower porous media; during the overflow stage, the stream is
662 mainly replenished through the karst conduit and porous medium II. Porous medium I
663 and the stream are in a dynamic equilibrium state. During the recession stage, the
664 porous media act as the main medium to replenish the stream.

665 As shown in Fig. 12 (a.4), the karst spring reaches its peak at 7409 s. This is due
666 to the rainfall infiltration, the recharge from porous medium I, and the subsequent
667 discharge to the stream. As the storage volume decreases, the amount of recharge from
668 the karst spring to the stream decreases. A trough appears at 11642 s. This is because



669 as the water level of the stream drops, groundwater is more easily discharged into the
670 stream. However, as the overall storage volume continues to decline, after a peak
671 appears at 13057 s, it enters a complete recession stage. Affected by the decline of the
672 stream water level, the discharge from porous medium III to the stream gradually
673 increases during the recession stage. Combining with Fig. 12 (a.1), it can be seen that
674 while porous medium III is discharging, its saturation remains at level I continuously,
675 indicating that the conduit continuously supplies water vertically to porous medium III.

676 Under the recharge of rainfall infiltration, the interaction process between the karst
677 aquifer affected by epikarst, sinkholes, conduit and the stream shows dynamic changes
678 in terms of staged characteristics, main interaction media, and the dynamic equilibrium
679 process among different media. The accurate simulation of the above complex
680 processes depends on the support of a three-dimensional two-phase numerical model
681 (Darcy-Brinkman-Stokes model).

682 **5. Conclusions**

683 This study employed the Darcy-Brinkman-Stokes equation to characterize
684 groundwater flow in the karst aquifer and stream, as well as within the karst media. The
685 VOF phase change method was used to illustrate the two-phase flow of water and air
686 in porous media, while various water retention models were applied to describe the
687 unsaturated flow processes in the karst aquifer. The results indicate that changes in
688 precipitation intensity have a significant impact on the interaction between the karst
689 aquifer and stream. As the precipitation intensity increases, the interaction process



690 between the two becomes more complex, involving multi-media synergistic recharge
691 and dynamic interaction with the karst aquifer. The contribution ratios of the epikarst,
692 upper layer, and PM II to the stream change with increasing precipitation intensity. In
693 the early stages of precipitation, the recharge effects of each medium on the stream are
694 relatively balanced; as the precipitation intensity increases, the discharge volumes of
695 PM I and PM II both increase, especially the increase in PM II is more significant, and
696 the timing of its discharge peak advances; when the precipitation intensity further
697 increases, PM II gradually reaches saturation, limiting its discharge capacity; and
698 during this process, the double peak intensity of PM I changes with the precipitation
699 intensity; at the same time, due to the saturation of PM II, a more pronounced overflow
700 phenomenon occurs in PM I, which dominates the contribution of recharge volume to
701 the stream. Therefore, the change in precipitation intensity not only affects the
702 discharge volume and discharge peak of each medium in the karst aquifer but also is
703 influenced by the dynamic saturation process of adjacent media.

704 The impact of different water retention models on the hydrological processes of
705 the stream and various media is also quite significant. The VGM model leads to more
706 water being retained in the porous media, thereby reducing the discharge volume of the
707 stream. In the early stage of precipitation, the VGM model enhances the recharge effect
708 of the stream on the karst conduit, but the discharge volume of the karst conduit to the
709 stream is relatively low in the middle and later stages. Additionally, the VGM model
710 predicts shorter movement distances of karst groundwater, weakening the vertical



711 infiltration capacity of the epikarst, resulting in an increase in the discharge volume of
712 the epikarst to the stream. At the same time, based on the VGM model, the discharge
713 volume of PM I decreases, but due to the smaller recharge volume, lower saturation
714 level, and water level, the stream is more likely to recharge it. The porous media located
715 below the karst conduit mainly rely on the recharge from the conduit, and based on the
716 BC model, the discharge volume of the karst conduit is larger. Ultimately, based on the
717 VGM model, it is found that the stream is more likely to recharge PM II in the early
718 stage of precipitation; in the middle to late stages of precipitation, the discharge volume
719 predicted by the BC model exceeds that of the VGM model.

720 During heavy precipitation events, multi-level conduit arrangements can
721 significantly impact the interaction process between the karst aquifer and stream, and
722 exhibit higher discharge efficiency. However, this arrangement has relatively little
723 impact on other media, indicating that multi-level conduit arrangements primarily
724 influence the hydrological process by regulating the interaction between the karst
725 conduit and stream.

726 For two consecutive precipitation events, the total precipitation intensity directly
727 affects the changes in water level of stream. Different precipitation intensities result in
728 different trends of water level changes in stream. The porous media of the karst aquifer
729 store a proportion of groundwater during the first precipitation period, which affects the
730 water level changes of stream during the second precipitation period. Due to the rapid
731 discharge characteristics of the karst conduit, the storage volume of the conduit during



732 the first precipitation period has little impact on the storage volume during the second
733 precipitation period. When the intensity of the first precipitation is higher than that of
734 the second, the amplitude of water level changes in stream is smaller, and vice versa.
735 The changes in water level of stream can affect the ease of recharge from different layer
736 media of the karst aquifer to the stream.

737 Through the generalized karst aquifer, the interaction mode of groundwater
738 between the conduit and stream in the Darcy-Brinkman-Stokes model and the
739 differences from the simulation results of MODFLOW-CFP were further revealed.
740 Moreover, a comparative analysis of the two types of models was carried out through
741 the hydrograph of the stream. The results show that the Darcy-Brinkman-Stokes model
742 can effectively simulate the interaction process between the karst aquifer and stream
743 under the influence of rainfall, and refine the two-phase interactive flow among various
744 media affected by the dynamic saturation process. At the same time, the Darcy-
745 Brinkman-Stokes model can represent the dynamic interaction process affected by
746 epikarst, sinkholes, conduit and the stream under the recharge of rainfall infiltration,
747 and refine and explain the significant infiltration process, overflow process and
748 recession process.

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753

754 **Data availability.** All raw data can be provided by the corresponding author upon
 755 request.

756

757 **Author contributions.** **FH:** conceptualization, methodology, formal analysis,
 758 visualization, writing original draft. **YG:** conceptualization, methodology, formal
 759 analysis, visualization, review and editing. **XH:** visualization, review and editing. **XW:**
 760 methodology, review and editing. **SP:** writing original draft and review and editing.

761

762 **Competing interests.** The authors declare that they have no conflict of interest.

763

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Table 1: Different parameter used in the models

Parameters	Units	Value
Conduit radius r_c	m	0.5
Secondary Conduit radius r_{c2}	m	1
Tertiary Conduit radius r_{c3}	m	1.5
Sinkhole radius r_s	m	0.5
Secondary Sinkhole radius r_{s2}	m	1
Tertiary Sinkhole radius r_{s3}	m	2
Level IV Sinkhole radius r_{s4}	m	5
Conduit height h_s	m	2
River width L_r	m	2
EpiKarst thickness	m	4
Porous medium I thickness	m	13
Porous medium II thickness	m	3
Porous medium III thickness	m	1
Porous medium length L_{py}	m	200
Porous media width L_{px}	m	200
Gravity g	m/s^2	9.81
Porous medium Porosity φ	/	0.4
Secondary permeability coefficient k_{02}	m^2	10^{-6}
Tertiary permeability coefficient k_{03}	m^2	10^{-7}
Level IV permeability coefficient k_{04}	m^2	10^{-8}
Porous medium Permeability coefficient k_0	m^2	10^{-9}
Gas phase viscosity μ_a	m^2/s	$1.48 \cdot 10^{-5}$
Gas phase density ρ_a	Kg/m^3	1.29
Liquid phase viscosity μ_w	m^2/s	10^{-6}
Liquid phase density ρ_w	Kg/m^3	10^3
m (Van Genuchten)	/	0.75
m (Brooks and Corey)	/	3
Precipitation Intensity b	m	3-7

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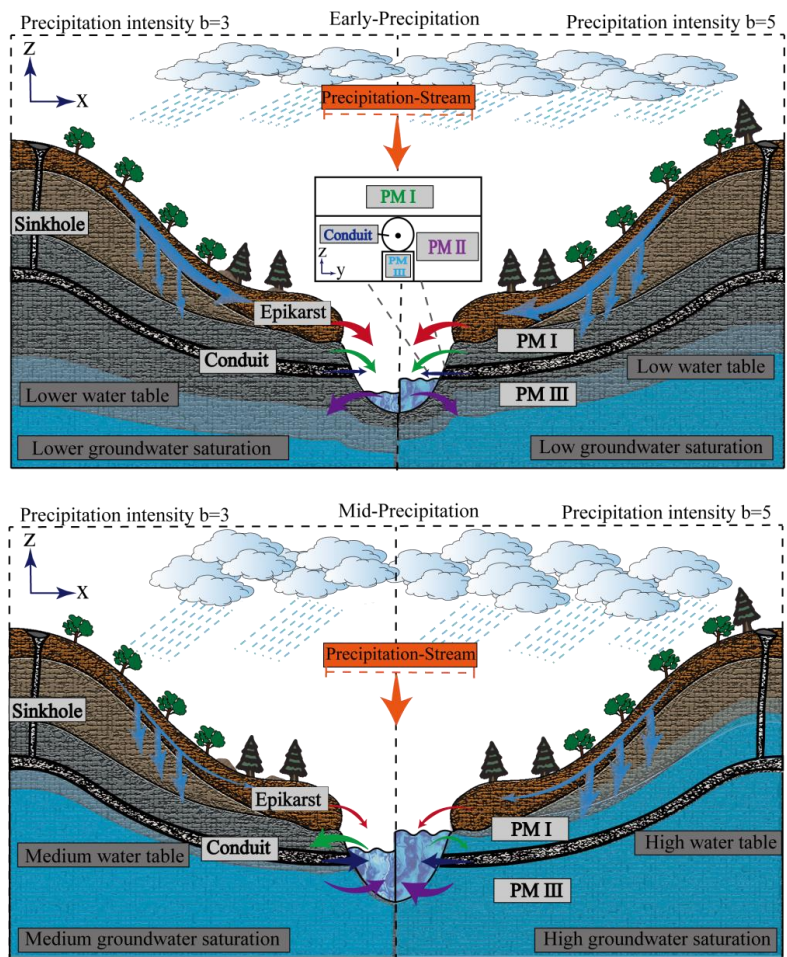


Figure 1. Schematic diagrams of the modelling of the interaction between the karst aquifer (epikarst, sinkhole, karst conduit, PM I, PM II, and PM III) and stream under dimensionless precipitation intensities ($b = 3$ and $b = 5$). (a) Schematic diagram of the interaction flow between each medium and stream in the early stage of a precipitation event; (b) Schematic diagram of the interaction flow between each medium and stream in the middle stage of a precipitation event. The size of the arrows represents the magnitude of the flow rate, and the direction of the arrows represents the direction of interaction between the two.

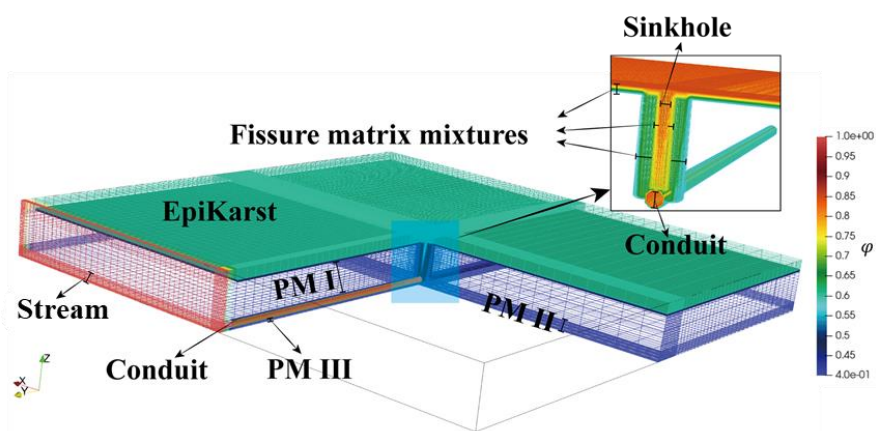


Figure 2. Discrete scheme for the karst aquifer and stream model.

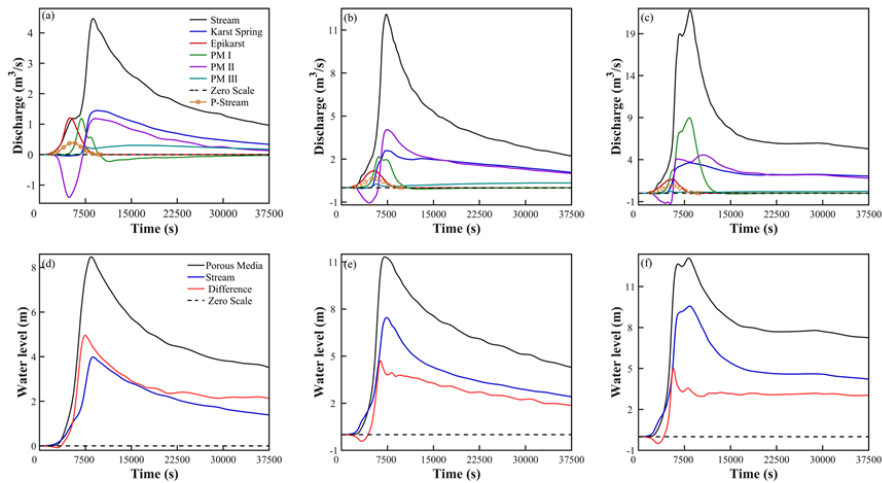


Figure 3. Hydrological process curves of each medium in the karst aquifer and stream for different precipitation intensities: (a) $b = 3$, (b) $b = 5$, (c) $b = 7$. Water level changes and differences in water levels in the karst aquifer and stream for different precipitation intensities: (d) $b = 3$, (e) $b = 5$, (f) $b = 7$.

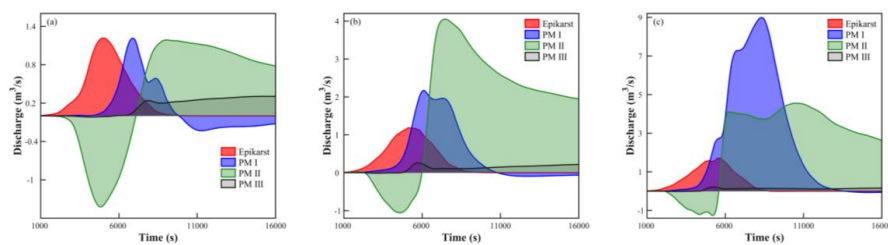


Figure 4. Interaction process of epikarst, porous media, and stream for different precipitation intensities: (a) $b = 3$, (b) $b = 5$, (c) $b = 7$.

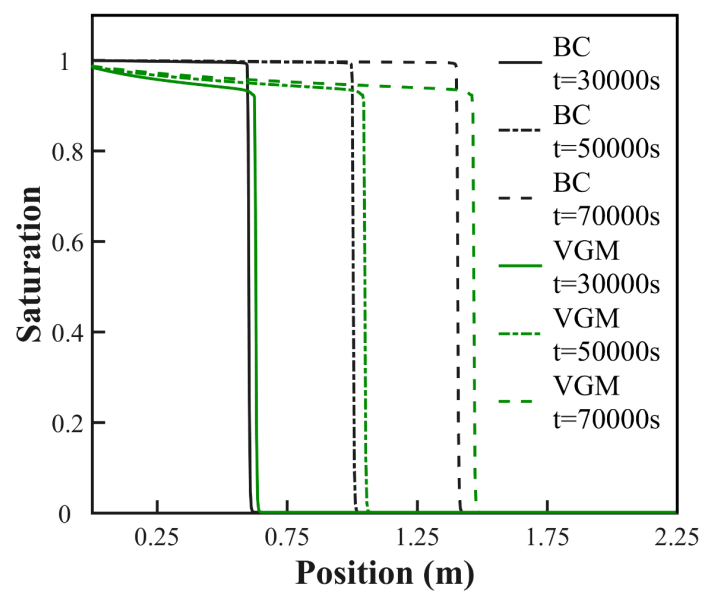


Figure 5. Water retention curves obtained using BC and VGMM water retention equations.

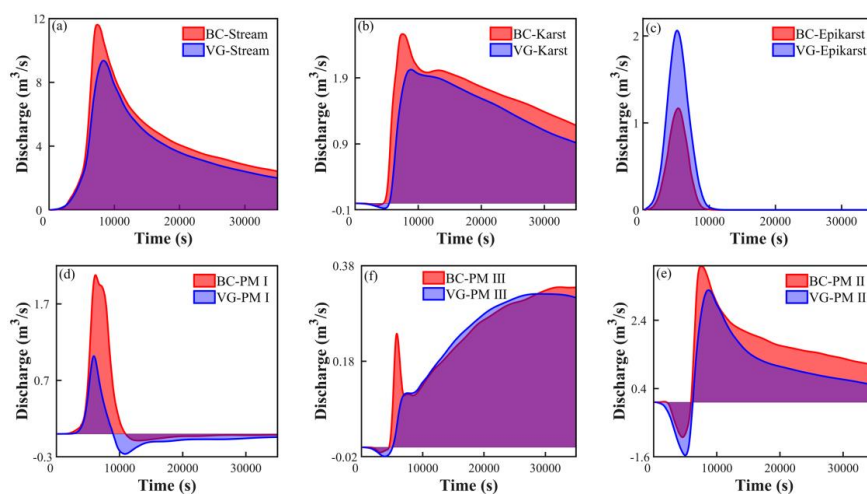


Figure 6. Changes in hydrological processes in the media of the karst aquifer derived from BC and VGMM water retention equations for a precipitation intensity $b = 5$.

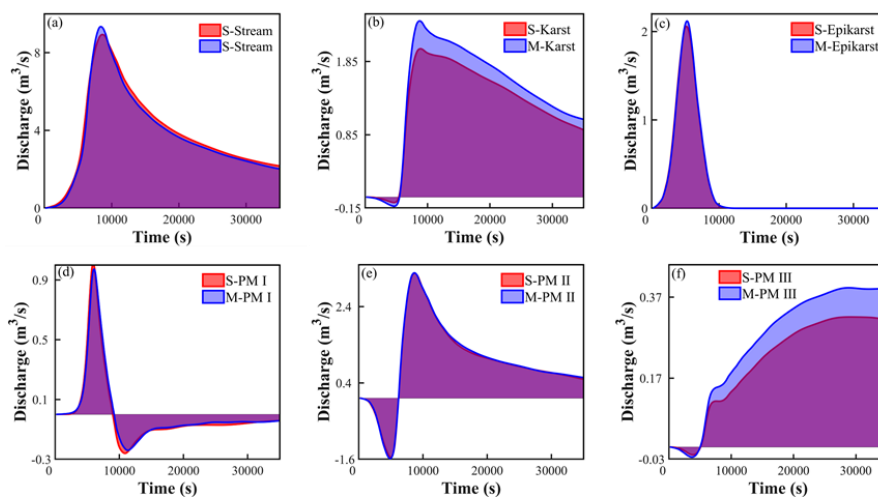
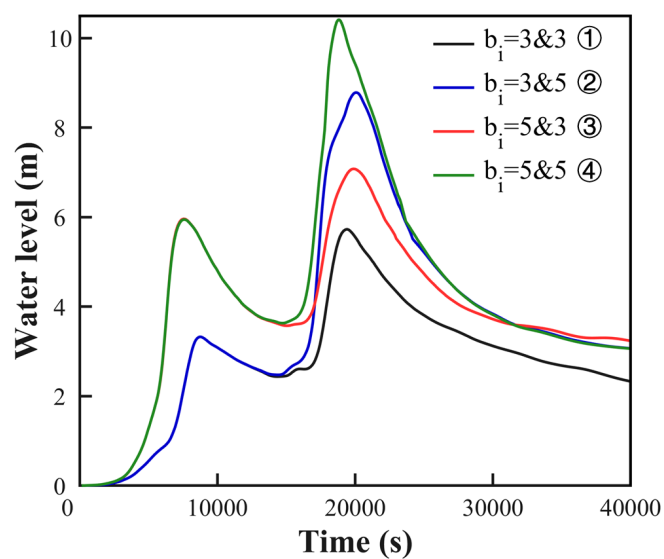
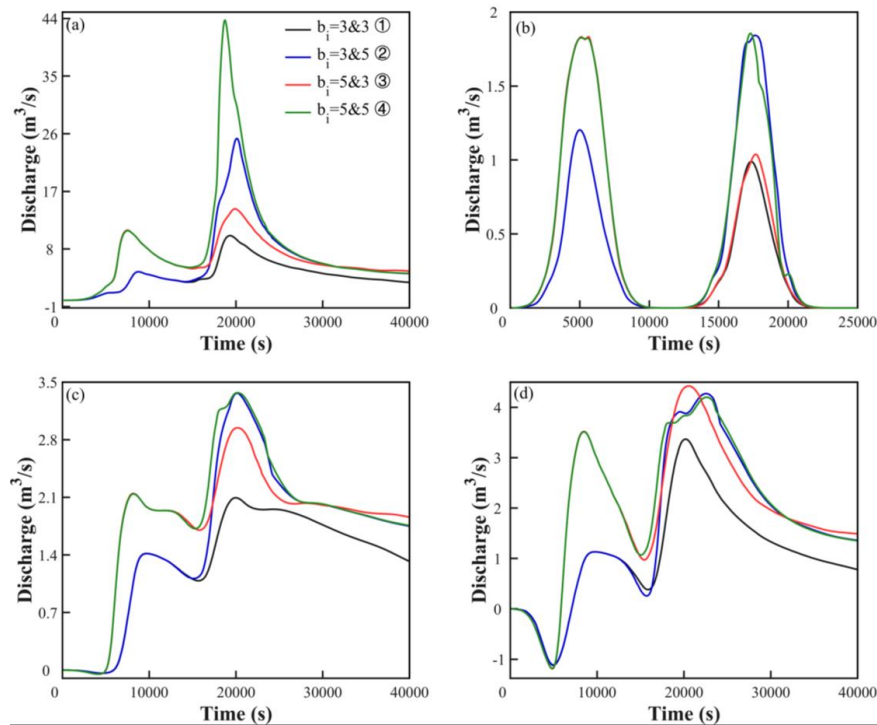


Figure 7. Impacts of single-stage and multi-stage conduit hydrological process changes in various media of the karst aquifer for a precipitation intensity $b = 5$.



917
 918 Figure 8. Water levels in stream for two consecutive precipitation events with first and second
 919 precipitation intensities ① $b_1 = 3$ and $b_2 = 3$; ② $b_1 = 3$ and $b_2 = 5$; ③ $b_1 = 5$ and $b_2 = 3$; ④ $b_1 =$
 920 5 and $b_2 = 5$, respectively.



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 922 Figure 9. (a) Hydrological process curves of the stream; (b) Discharge process of groundwater through
 923 the epikarst to the stream; (c) Discharge process of groundwater through the karst conduit to the
 924 stream; (d) Discharge process of porous media (PM II) to the stream, for two consecutive precipitation
 925 events with first and second precipitation intensities ① $b_1 = 3$ and $b_2 = 3$; ② $b_1 = 3$ and $b_2 = 5$; ③
 926 $b_1 = 5$ and $b_2 = 3$; ④ $b_1 = 5$ and $b_2 = 5$, respectively.
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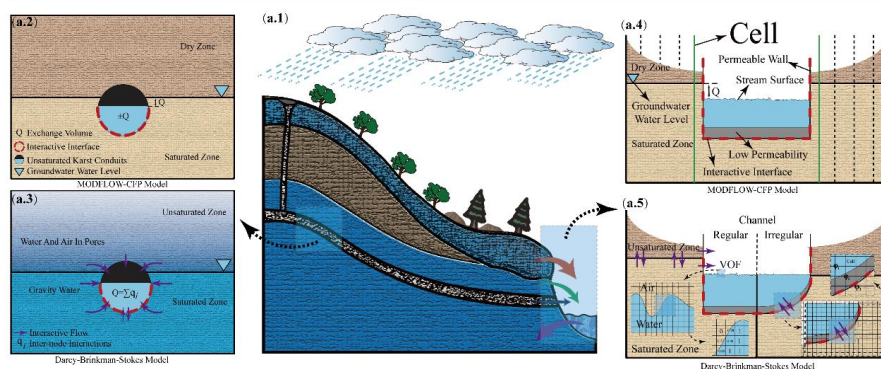


Figure 10. The differences in the interaction patterns of groundwater between conduit and porous media in (a) the MODFLOW-CFP model and (b) the Darcy-Brinkman-Stokes model.

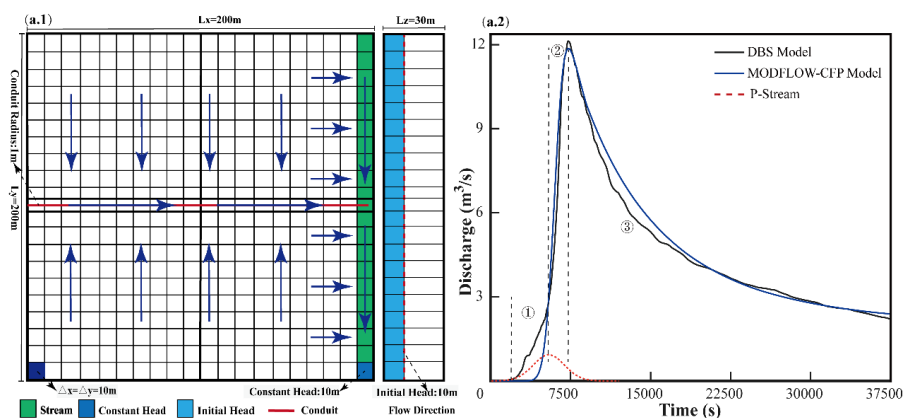


Figure 11. (a.1) Mesh division of the x-y plane and y-z plane of the MODFLOW-CFP two-dimensional model. (b.1) Comparison of the hydrographs of the stream for the MODFLOW-CFP model and the Darcy-Brinkman-Stokes model under the rainfall boundary condition when $b = 5$.

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