Response to the Reviewer Comments on Manuscript "Scale invariance in kilometer-scale sea ice deformation" "Reviewer #1"

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We sincerely thank the reviewer for their important and constructive comments regarding the potential impact of radar noise on our observation on sea ice deformation ceasing to exhibit scale invariance below $\sim 10^2$ m. We fully agree that any new observation challenging the status quo requires thorough investigation on any possible errors, and we appreciate the opportunity to highlight the checks we performed to rule out noise as the source of the observed lower-limit cutoff.

General comments

According to our understanding, ship radar data includes two different kind of noise which impact on the quality of the derived results. Firstly, in some situations, high frequency ($<$ 30 sec) speckle can be detected from raw images. Those are probably due to waves in pack ice which cause vertical motion of ridges and edges of leads. Consequently, location and intensity of the strong backscattering objects are changing. Motion related to those events is not detectable with the radar system and in this analysis, we have been filtered that noise and consider only meter scale displacements. Moreover, wave induced speckle is assumed to be very small during the these selected periods.

The other sources of noise are due vibration of the radar antenna, shaking of the entire ship and interference with other radars. These effects were particular evident during the March deformation case and thus we paid attention to improve quality to data.

We note that our initial suspicion was indeed that the high-frequency noise in the MOSAiC radar data might introduce some spurious effects below some threshold scale. Consequently, a long time was devoted to attempting to invalidate this lower-limit scaling collapse. We, thus, examined our results from multiple angles. In (1), we measured the accuracy of our deep-learning-based optical flow by applying it to synthetic sea ice motion fields. Because these fields have known ground-truth displacements, we could quantify the end-point error over a range of motion types and magnitudes. The neural network had been trained originally on data sets with comparatively larger and more dynamic displacements, so our error estimates are assumed conservative when evaluating fine-scale motion in sea ice cover. In these artificial tests, no cutoff similar to the $\sim 10^2$ m could be observed. The algorithm works well when tracking small pixel displacements.

To provide further robustness against the unique high-frequency radar noise of the MOSAiC data, trained the neural network first with Gaussian noise over the training data and then specifically finetuned the model by applying noise maps sampled during stationary conditions in the MOSAiC data to apply them on top of the training set. This step is assumed to provide additional robustness against the radar noise in the MOSAiC data.

In the Supporting Information of (1), we evaluated the signal-to-noise ratio by following (2) and found that even at the finest scales used in our analysis, the signal remains sufficiently above the estimated noise. The strain fields over active leads yield a signal-to-noise ratio well above unity with no spatial or temporal averaging applied. We note that in the scaling analysis in the current article, we only considered highly active periods where the signal-to-noise ratio was ensured to be constantly sufficiently high.

To consider other data sets with very different noise maps, we analyzed data from R/V Kronprins Haakon, which lacked the same high-frequency noise artifacts. Despite those data being limited to a few time windows lasting less than 12 hours, the scaling analysis for pack ice in summer surprisingly displayed behavior that seems to support the MOSAiC record (Fig 1).

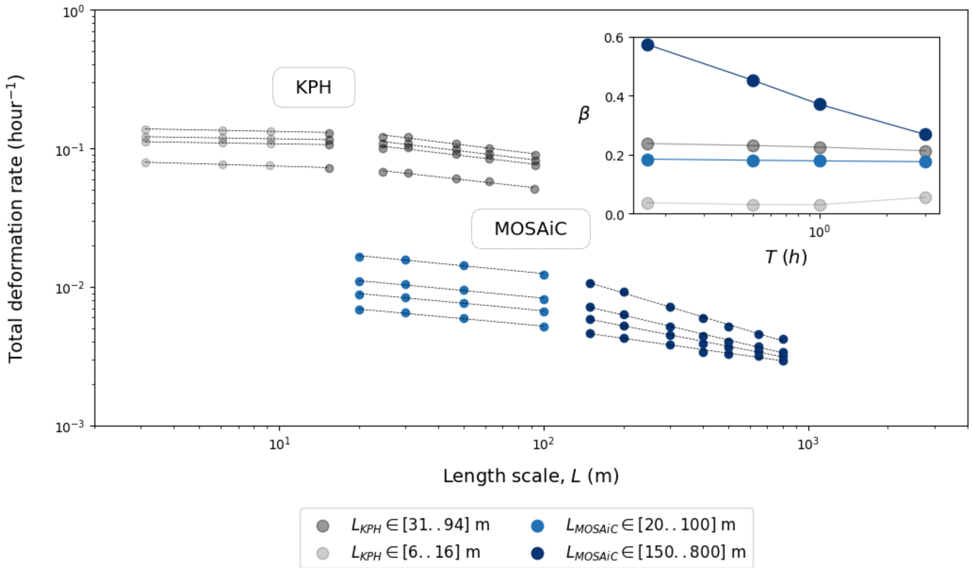

Fig. 1. Comparison between the MOSAiC data and ship radar data from R/V Kronprins Haakon. Both data sets are from pack ice conditions, but the R/V Kronprins Haakon data was gathered during August 2023, which resulted in the higher deformation rates. The overlap between the results suggests some possible physical connection between the ice behavior. An example of the deformation map can be found in (1).

One piece of evidence is that the July case, which has radar noise levels comparable to the winter period, does not exhibit the scale collapse. If noise alone were driving the cutoff, we would expect to see a similar breakdown in scaling throughout the MOSAiC data set. Instead, we find the cutoff consistently in winter pack ice and consistently absent in July conditions, which suggests that the scaling collapse would be tied to some mechanical processes rather than measurement artifacts. In the winter cases, we furthermore observe that the typical width of fracture zones or leads is on the order of hundred to few hundred meters, which overlaps with the magnitude of the scaling collapse.

We also note that a single-day analysis, focusing on either entirely quiescent or entirely active phases, shows a scaling fit down to smaller scales, whereas combining multiple episodes of intermittent deformation causes the collapse near 10^2 m also supports the view that the cutoff is not merely random noise. In that sense, the bulk statistics over an active two-week period detect transitions in activity that accumulate into a breakdown in the single power-law form below the size of the dominant deformation features.

We tested this with alternative approaches to scaling, such as coarse graining to confirm that the same cutoff behavior appears, albeit with differing values of the spatial scaling exponent.

We also note that we have ongoing work that expands this view of high-resolution ice dynamics observations to the multifractal aspects. In the multifractal analysis, we provide some support for these results, but also see that the behavior between the cases are highly different. Those results will, though, be published separately.

We agree with the reviewer that our observation on the scaling collapse requires further independent analysis. We hope our findings encourage additional campaigns, alternative methods, and fresh data to investigate whether a similar scale break appears in other regions or ice types. For the time being, our work with the synthetic-data tests, finetuning the neural network with the MOSAiC radar noise, signal-to-noise ratios, the disappearance of the scale collapse in the summer, the overlap with the R/V Kronprins Haakon ship radar data, and the physical consistency with the length scale of major fracture features gives us enough confidence in the result on the scaling collapse that we find it to be worth publishing. We will, however, revise the manuscript text to emphasize these points noise impacts. We hope that these clarifications address the reviewer's primary concern and suggests robustness of our conclusion.

Specific comments

1 *The paper in many places has sentences with grammar that I found hard to follow. A copy editor should be able to help here. I honestly stopped trying to correct them in my read. I will happily do this on a second draft of the paper.*

We will go through the article sentence by sentence to try to resolve the grammatical erros and confusing sentences.

2 *The gray lines in figure 3 were hard to see in my print out.*

We will increase the color intensity for the grey lines in Figure 3.

3 *The units of each term in figure A2 do not match, or is it that I am rusty in Einstein notation (I admit I prefer reading vector notation).*

The terms in Eq. A2 for the Green-Lagrangian strain tensor should be correct. Following the Einstein notation, the part of $\sum_k$ is typically omitted from the full form of

$$E_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \sum_k \frac{\partial u_k}{\partial x_i}\frac{\partial u_k}{\partial x_j}\right). \quad (1)$$

References

[1] M. Uusinoka, J. Haapala, and A. Polojärvi, "Deep learning-based optical flow in fine-scale deformation mapping of sea ice dynamics," *Geophysical Research Letters*, vol. 52, no. 2, p. e2024GL112000, 2025.

[2] A. Bouchat and B. Tremblay, "Reassessing the quality of sea-ice deformation estimates derived from the radarsat geophysical processor system and its impact on the spatiotemporal scaling statistics," *Journal of Geophysical Research: Oceans*, vol. 125, no. 8, p. e2019JC015944, 2020.