Response to Editor:

Liu et al.

Dear Editor T.J. Fudge,

Thank you very much for your positive evaluation of our manuscript. We take your comments regarding the writing very seriously.

During this revision, all authors have carefully reviewed and edited the entire manuscript to ensure it is human-written. Additionally, we have updated the manuscript to meet the formatting requirements noted by MS. Daria Karpachova. We have also corrected the numbering and captions for the supplement figures and added the mandatory "Competing interests" section.

Thank you again for your constructive guidance, which have undoubtedly improved the form.

Sincerely,

Xueyuan Tang

Response to Referee #1(Dr. Tobias Stål)

We sincerely thank Dr. Tobias Stål for your comments. Below are our point-by-point responses to the comments. The response is shown in red.

As per the previous round, it has been very interesting to read the study and accompanying discussion. There are several aspects that I do not entirely agree with, particularly the choice of observables; however, I'm often wrong, and there is significant value in presenting and examining alternative approaches. The assumptions and limitations are clearly acknowledged and thoughtfully discussed.

In my view, the main contribution lies in the method development and the integration of Particle Swarm Optimisation (PSO). This is one of the few studies in the field I have encountered that use this approach, and it is

commendable that the authors have applied it so effectively and described it in sufficient detail. Optimisation within machine learning frameworks often lags behind and leads to more confusion than improvement; Liu and colleagues here exemplify how it should be done.

The contribution also made me think about the importance of interdisciplinary collaboration. We mathematical geophysicists benefit greatly from constructive dialogue with observational glaciologists and geologists.

I would very much have loved to see the code shared. Software packages, datasets, and funding frameworks increasingly shape the direction of research (with only a few contextualising preprints and extended abstracts in between). I think the role of traditional papers (in PDF format) as vehicles for scientific narrative is fading, except for reviews and discussion papers.

With my apologies to the authors and my gratitude for their patience, I would still like to see some improvements, particularly in the introduction, to meet the journal's high standards. These should fall under minor revisions. A few statements are not entirely accurate; the authors are likely aware of this, possibly due to translation issues. Moreover, at times the text reads a little awkwardly.

We sincerely thank you for your interest in our study and your highly positive evaluation of our method development. We also deeply appreciate your open-mindedness regarding our approach and your insightful comments on interdisciplinary collaboration. We completely agree with your perspective on the critical role of software packages and datasets in shaping modern research. We have now made our code and datasets publicly available on Zenodo.

Regarding your general comments, please find our responses below:

Here are a few indicative suggestions:

Abstract: I would recommend outlining Particle Swarm Optimisation more, and perhaps reducing the amount of result interpretation.

Done. We have revised the abstract to provide more details on the PSO method and condensed the interpretation of the results accordingly.

Introduction: Needs one more round of proofreading. It is clearer now, but it lacks some disciplinary precision and is a bit awkward sometimes.

We have carefully proofread and manually rewritten several parts of the Introduction.

L. 28 “critical heat source” doesn’t really make sense. Maybe “a heat source which can have a critical impact”?

L. 28–30 “not only” – “, but also” is only applicable when one would expect a single application, but this is not implied anywhere in the text.

L. 29 No space before comma.

L. 34 “Unfortunately” is a rather dramatic opening for a paragraph.

L. 35 I believe Talalay’s 2020 review would be a more suitable reference than Fisher et al., 2015 here, or cite both.

L. 80 Add the year of the NGHF release.

These detailed suggestions for revision are excellent and helpful. We have made all the corresponding modifications exactly as suggested.

Table 1. You mention using ETOPO2022; what do you use in Antarctica—surface elevation or bed elevation? (Sorry if I overlooked this in the previous round!)

Thank you for checking this. As indicated by the term "Global Bedrock Topography" in Table 1, we utilized the bedrock elevation rather than the ice surface elevation in Antarctica. Specifically, we extracted the 15 arc-second bedrock elevation netCDF tiles from the ETOPO 2022 Global Relief Model.

L. 119 A clear and helpful explainer. Thank you.

Thanks for the positive feedback.

L. 469–L. 474 This is a strong presentation, but how does it actually affect your model? A corresponding map would have been very illustrative.

To clarify how this framework affects our model: in our architecture, Particle Swarm Optimization (PSO) is used to optimize the parameters of the Deep Neural Network (DNN) for the actual heat flow predictions, while the Bayesian framework serves as the output layer. Therefore, rather than altering the deterministic prediction values, the Bayesian layer "affects" the model by providing a rigorous quantification of confidence. It allows us to explicitly separate regions where predictive uncertainty is driven by inherent data noise (aleatoric) from those limited by model capacity (epistemic).

Furthermore, we have generated a new figure that visualizes the spatial distribution of the decomposed aleatoric and epistemic uncertainties across Antarctica. This map, Figure S4.2, has been provided in the supplementary material.

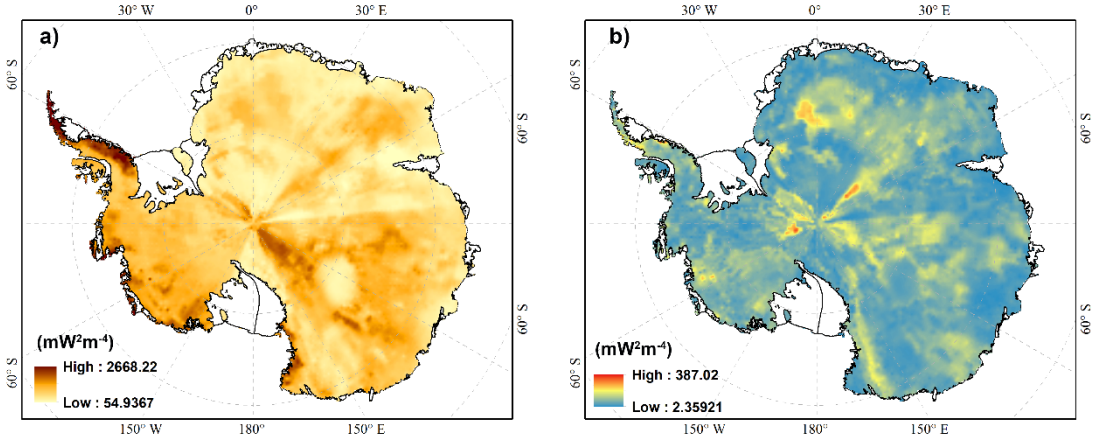

Figure S4.2. Spatial distribution of the decomposed predictive uncertainty variances across Antarctica. (a) Aleatoric uncertainty variance (mW^2m^{-4}). (b) Epistemic uncertainty variance (mW^2m^{-4}).

The relevant sentences have been revised appropriately to adapt to this:

"Our Bayesian framework decomposes total predictive uncertainty into aleatoric and epistemic components, revealing that aleatoric uncertainty, arising from inherent observational variability and unresolved geological heterogeneity, constitutes the primary source of total uncertainty across Antarctica (Fig.8a). This finding suggests that while our model architecture has sufficient capacity to capture underlying patterns, irreducible noise in heat flow observations and small-scale geological complexity impose fundamental limits on prediction accuracy (Fig.S4.2)."

L. 543 It is very good that the results are shared on Zenodo; however, would x, y coordinates in EPSG:3031 be more appropriate?

We agree. We have updated the dataset on Zenodo to include the x, y coordinates in EPSG:3031 (https://zenodo.org/records/18790294).

Supplementary material:

Please cite the actual studies when listing the observables. A short caption for each is advisable. Here is also a good place to explain how target observables in Antarctica differ from the reference observables used.

Done. We have added the proper citations and short captions for each observable in the supplementary material.