

Response to Reviewer #2 and Changes to the Manuscript

Reviewer: The Authors have attempted to refute the concerns of all three reviewers about potential bias in the retrieved values of the asymmetry parameter g . However, their arguments are insufficient, or in some cases plainly wrong. For instance, they claim in the response to Reviewer 2 that "the circular-aperture approximation introduces only negligible error into the asymmetry parameter". This is simply not true. To show this, I have calculated the asymmetry parameter for synthetic phase functions (broadly resembling the analytic phase function due to Baran et al. (2001)) constructed using diffraction on distributions of randomly oriented ellipses with mean equal area-equivalent size parameter of 101 and aspect ratios of 4 and 1 i.e. the latter were circular. The resulting g values are 0.735 and 0.754, respectively. Therefore in this case the circular aperture assumption introduces a discrepancy of +0.019, see Fig. 1. For size parameter 300, well into the "geometric optics" regime, the difference is still +0.01. In general, non-circular apertures shift the phase function towards larger angles, reducing g . Note that these errors alone are an order of magnitude or more larger than the value claimed in the original manuscript (0.001). Yet accordingly to the manuscript the diffraction calculation contributes one half of the retrieved asymmetry parameter (since PHIPS does not measure in this angular region), hence this is a serious weakness. For this reason I fail to see why "the circular-aperture approximation introduces only negligible error into the asymmetry parameter" because it concerns only the low scattering angle region that PHIPS does not measure: on the contrary, the lack of measured data in this region means that the retrieval relies at low angles solely on the diffraction calculation!

We thank the reviewer for carefully highlighting the potential bias introduced by assuming a circular diffraction aperture. Motivated by the reviewer's comment, we have now revisited the diffraction parameterisation and computed diffraction for non-spherical ice crystal geometries. In particular, we performed vector Fraunhofer diffraction calculations for hexagonal ice crystal shapes with realistic aspect ratios matching the observational range (please see new Figure 5 in the manuscript).

The diffraction calculations were carried out using a vector Fraunhofer integral. The external diffraction is computed by replacing the illuminated particle surface with a set of complementary apertures, in accordance with Babinet's principle. Three levels of geometric representation were considered: (i) a full three-dimensional hull, in which each illuminated facet of the crystal is treated as a separate polygonal aperture. The diffraction from each facet is computed individually, taking into account its relative position and orientation. The final scattered field is obtained as a coherent sum of the diffracted contributions from all facets. (ii) A two-dimensional projection, in which the illuminated facets are projected onto a plane perpendicular to the incident light. For convex particles, this reduces the diffraction problem to a single effective aperture. The diffracted field is then computed for this projected aperture, providing an efficient approximation of the total scattered intensity. (iii) The third geometry is an area-equivalent circle, in which the total area of the 2D projection is represented by a circle with the same area. All of these geometries were randomly oriented (100 orientations) and the average 1D scattering intensity as a function of scattering angle was calculated, from which we calculated the diffraction asymmetry parameter (g_D)

Figure 2 presents the results of the diffraction calculations alongside the parameterisation applied in the retrieval. The reviewer is correct, when pointing out that a circular aperture approximation tends to produce the highest g_D and the further the true particle deviates from circle, the more the circle approximation overestimates g_D . However, the observed maximum difference in g_D between the shapes is below 0.002 for sizes above ($15 \mu\text{m}$) and aspect ratios below 1.7 considered in our study. This difference rapidly decreases with increasing particle size and becomes negligible for the larger crystals that are most relevant for this study. This confirms our earlier assumption that for

orientation averaged particles shape has only a minor influence on g_D .

The parameterisation used in our retrieval (black line in Fig. 2) systematically underestimates g_D , most likely because the use of a fixed 2D spherical aperture neglects phase-coherent contributions associated with particle curvature or orientation, which shift diffracted energy toward smaller scattering angles. The deviation between this parameterisation and the mean of all diffraction models is largest in the smallest size bin (15–30 μm), where it ranges between 0.002 and 0.004.

Motivated by this systematic bias, we have revised the diffraction parameterisation used in the retrieval. Instead of relying on the spherical-aperture approximation, we fitted a fourth-degree polynomial to the mean of the results shown in Fig. 2. The revised parameterisation reproduces the ensemble-mean diffraction behaviour to within 0.002 over the particle size range relevant to this study and has been implemented in the retrieval algorithm in place of the previous spherical-aperture-based formulation. As a result, the systematic low bias in g_D at small particle sizes is removed, while the impact on larger particles remains negligible. Since diffraction contributes one half of the total asymmetry parameter, the remaining uncertainty associated with the diffraction treatment corresponds to an uncertainty of approximately 0.001 in the retrieved total g .

Changes made to the manuscript

The new parameterisation for the diffraction asymmetry parameter was now used in the analysis leading to revised g values in the manuscript. However, the changes in g are minor and only +0.001 at maximum.

We added a Figure 5 showing aspect ratio of ML and Arctic ice crystals derived from PHIPS stereo-microscopic images. This addition supports our claim to restrict our simulations to aspect ratios between 0.25 and 1.7.

Lines 245 - 251 and Figure 5

Figure 1 shows the ice crystal aspect ratios derived from the stereo-microscopic imager for different crystal sizes. The majority of the observed ice crystals exhibit median aspect ratios of approximately 0.8 for sub-100 μm particles and around 0.7 for larger crystals. Mid-latitude cirrus consistently shows lower aspect ratios than Arctic cirrus across all size bins. It should be noted, however, that the aspect ratios derived from the stereo-microscopic images represent the overall crystal geometry. Individual components of aggregates or polycrystals, such as the bullets forming bullet rosettes, can exhibit substantially higher aspect ratios. For example, ? reported aspect ratios between 2 and 6 for individual bullets within bullet rosettes.

Appendix A was revised as follows:

Lines 505 - 532 and Figure A2

In earlier implementation by Xu et al. (2022), the diffraction contribution was approximated using scalar diffraction theory for circular apertures, which provides a computationally efficient first-order description. To further improve the physical consistency of the diffraction treatment, here we update the diffraction parameterisation to account for non-spherical particle shapes.

The diffraction calculations were carried out using a vector Fraunhofer integral. The external diffraction is computed by replacing the illuminated particle surface with a set of complementary apertures, in accordance with Babinet’s principle. Three levels of geometric representation were considered: (i) a full three-dimensional hull, in which each illuminated facet is treated as an individual aperture and its diffracted field is computed accounting for relative position and orientation, with the total scattered field obtained from the coherent sum of all facets; (ii) a two-dimensional projection, where the illuminated facets are projected onto a plane normal to the incident light, re-

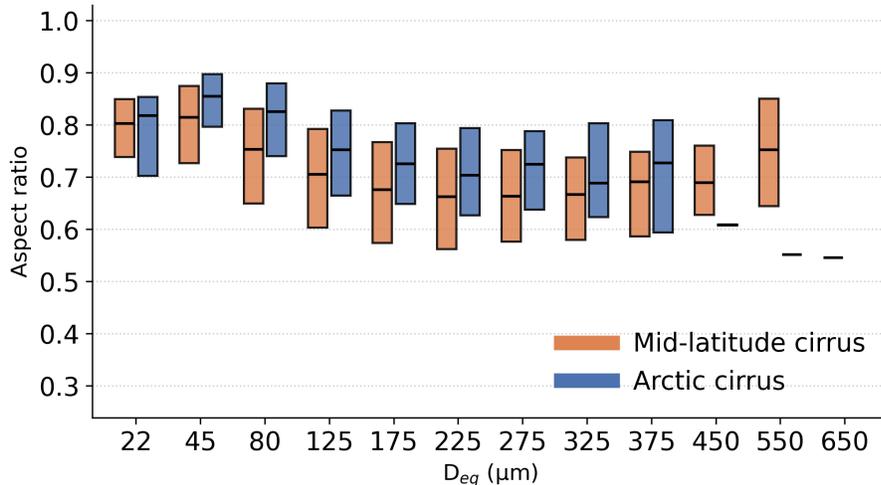


Figure 1: Ice crystal aspect ratio for mid-latitude and Arctic cirrus using a latitude threshold of 65° . Aspect ratio is defined as the minimum of the two orthogonal aspect ratios derived from particle imaging. Boxes represent the interquartile range and the central line denotes the median.

ducing the problem for convex particles to a single effective aperture; and (iii) an area-equivalent circle, in which the 2D projected particle cross section is represented by a circular aperture of cross section-equivalent area. For all geometries, 100 random orientations were simulated, and the orientation-averaged one-dimensional scattering intensity as a function of angle was used to derive the diffraction asymmetry parameter g_D .

The diffraction calculations confirm that, for orientation-averaged ice crystals, aperture shape and aspect ratio have only a minor influence on the diffraction phase function and the resulting diffraction asymmetry parameter g_D . For particle sizes ($d \geq 15 \mu\text{m}$) and aspect ratios ($0.25 < AR < 1.7$) relevant to this study, the spread in g_D across the different geometric representations remains below 0.002 and decreases further with increasing size. On the other hand, the diffraction parameterisation adopted by Xu et al. (2022), which is based on a fixed circular aperture, yields systematically lower values of $g_D(d)$.

Motivated by this systematic bias, we derived an updated diffraction parameterisation by fitting a fourth-degree polynomial to the ensemble-mean g_D obtained from all diffraction models. The underlying simulations span hexagonal ice crystals with aspect ratios between 0.25 and 1.7, consistent with the range observed by PHIPS. On a logarithmic size scale, the diffraction asymmetry parameter $g_D(d)$ is approximated by

$$g_D(d) = -3.31551145 \times 10^{-4} \ln^4(d) + 5.44285886 \times 10^{-3} \ln^3(d) - 3.36880804 \times 10^{-2} \ln^2(d) + 9.35390828 \times 10^{-2} \ln(d) + 9.01145674 \times 10^{-1}, \quad (1)$$

where d denotes the particle area-equivalent diameter in micrometres.

The revised parameterisation reproduces the ensemble-mean diffraction behaviour to within 0.002 over the particle size and aspect ratio range relevant to this study and replaces the previous spherical-aperture-based formulation in the retrieval algorithm. Since diffraction contributes one half of the total asymmetry parameter, the remaining uncertainty associated with the diffraction treatment corresponds to a maximum uncertainty of approximately 0.001 in the retrieved total g .

Reviewer: Some other responses by the Authors show characteristics of circular argument. It is claimed for example that using truncated Legendre polynomial expansions is justified because

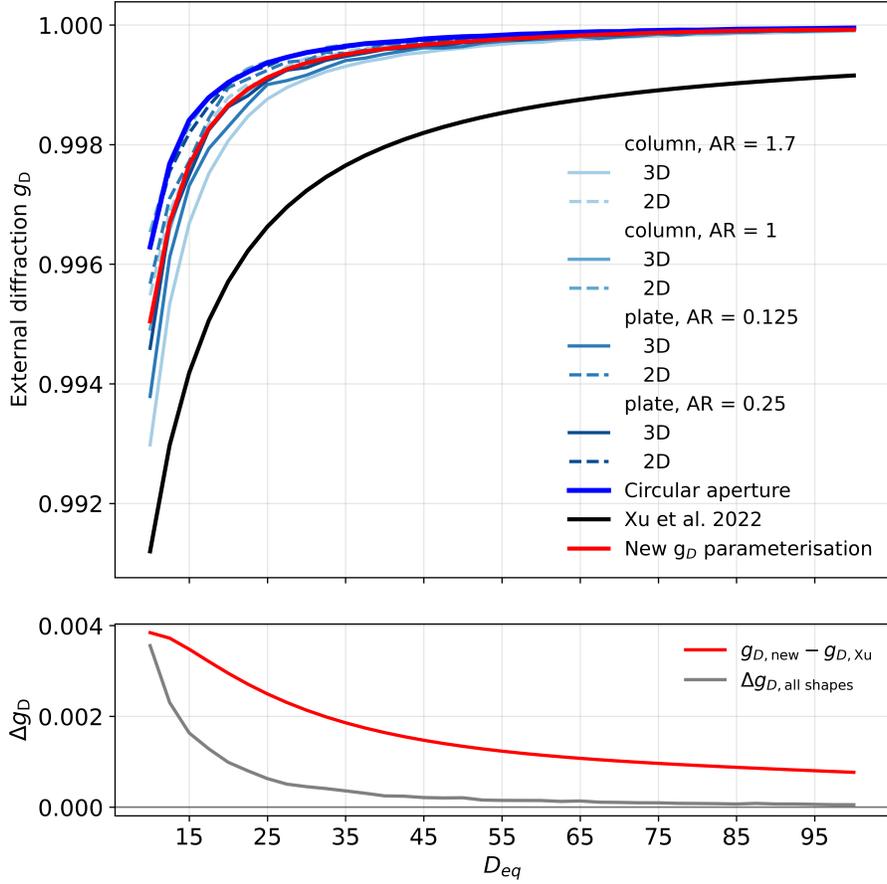


Figure 2: Asymmetry parameter from external diffraction calculated for orientation averaged hexagonal columns with different aspect ratios (AR). Both 3D geometry and 2D projection were considered. The blue line shows calculation for orientation averaged circular aperture with the same projected area. The black line shows the parameterisation used in Xu et al. (2022) and red line the updated parameterisation.

”our measurements do not contain smooth planar crystals”. This by implications means that the retrievals rely on the absence of ”smooth” crystals, and as a consequence cannot produce high g values. Yet in situ measurements in cirrus find a variety of crystal geometries, including smooth ones.

We do not see a circular argument here. The use of truncated Legendre polynomials is *necessary* due to the limited angular range of the observations. The *limitation* of using truncated Legendre polynomials is that this retrieval approach cannot be applied to phase functions with distinct features that cannot be represented by such low degree Legendre polynomials. Phase functions with distinct halo features are typical for optically pristine ice particles. In our measurements, however, we did not find any ensemble angular scattering function containing a halo feature, i.e., angular functions dominated by optically pristine particles.

Reviewer: The Authors justify the choice of old techniques (pure GO) by their widespread use in the past and the existence of a large body of literature. Yet using such techniques after they have been supplanted by improved, more accurate and more physically grounded techniques (e.g. IGO, see Yang et al. (2019) for a summary) is unwise. Is applying Newtonian mechanics to orbital calculations justified now because it was in wide use before the development of general relativity?

The IGOM code referenced by the reviewer is not publicly available and could therefore not be applied in this study. We emphasize that the choice of the geometric-optics-based method is fully justified for the particle size parameters relevant to our measurements.

We welcome independent comparisons using IGOM or other more advanced optical particle models. Our measurement dataset, according to standard scientific practice, will be made publicly available for this purpose. Nevertheless, it is important to note that even the most sophisticated theoretical models always require some degree of simplification of the reality. Therefore, all models require experimental validation, such as provided by our study. And, by the way, all advances in theoretical physics were either initiated by experimental observations or became accepted after an experimental validation.

Reviewer: Yet at the same time I note that the revised details provided in the response to Reviewer 3, the g uncertainty is 0.02, contradicting the Authors’ vehement denial of potentially large g measurement error in their response to Reviewer 1, where they insisted that the errors were below 0.001. It is worth considering here the impact of a 0.02 bias in g . Using the Authors’ Eq. 8 for a simplistic cirrus case, reflected TOA SW flux might change by some 5 W m⁻² even for thin cirrus, hardly a negligible amount. Yet the bias could be even larger, considering that the reviewers have identified several separate potential sources of error. It is worth remembering that relative accuracy is typically easier to achieve than absolute one. This is reflected in the very weak difference found by the Authors between Arctic and mid-latitude cirrus. However, systematic bias is a different matter. The Author’s claim of large reduction of g with respect to previous assumptions must therefore be sufficiently well justified - strong claims require equally strong evidence. This requirement has not been met here.

In the final Author response to all Reviewers, as issued on Nov. 17, 2025, we revised the uncertainty in g to be 0.02, replacing our earlier response to Reviewer 1. Even by taking into account a bias of 0.02, the median g would increase from 0.737 to 0.757, still being significantly lower than values used in optical parameterisations shown in Fig. 8, only matching those of MC6 model.

Reviewer: I note that Reviewer 3 proposed a clear way to test the accuracy of the technique that was used by the Authors which I too strongly recommend - and which would have provided the needed evidence. However, the Authors have chosen not to take advantage of it. Instead, they insist

on relying a computational technique that is known to be inaccurate and/or have been superseded by better ones.

We disagree with this statement. The test proposed by Reviewer 3 was performed, and the results are clearly discussed in Appendix A. The test was performed against 1612 synthetic scattering functions using the well-validated and openly accessible ray tracing code by Andreas Macke. As stated above, we invite future studies to revisit this test as more sophisticated models are developed and made available to the community. We do not know the reasons the IGOM model is not publicly available, but our PHIPS data is.