

# Simulating liquid water distribution at the pore scale in snow: water retention curves and effective transport properties

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## Abstract.

Liquid water flows by gravity and capillarity in snow ~~and modifies drastically~~, drastically modifying its properties. Unlike dry snow, observing wet snow remains a challenge and data from 3D pore-scale imaging are scarce. This limitation hampers our understanding of the water, heat, and vapor transport processes in wet snow, as well as their modeling. Here, we explore a simulation-based approach ~~in which 3D images of dry snow were digitally filled and drained with water through imbibition and drainage simulations driven by capillarity. Time series of wet~~, namely a pore morphology model, to simulate the distribution of liquid water in the pore space of snow images at various water contents were produced. The for various water contents. Liquid water is gradually introduced and then removed by capillarity during wetting (imbibition) and drying (drainage) simulations. This model was applied to a set of 34 3D tomography images of dry snow of varied microstructures. A series of 3D images of wet snow at different stages of drainage and imbibition was obtained. From these images, we examine key properties for the modeling of wet snow processes. First, we describe the water retention curves obtained for imbibition and drainage and for the different microstructures. The classical van Genuchten model is used to reproduce our simulated water retention curves. The obtained model parameters, i.e. the ~~capillary pressure as a function of the shape parameters ( $\alpha_{vg}$  and  $n_{vg}$ ) and the residual water content, are derived for different snow types. New parameters to model the water retention curves of snow are proposed based on the van Genuchten model and compared to existing ones from laboratory experiments~~. From the 3D images, the compared to the ones obtained in laboratory experiments from literature. New parametrizations of these parameters based on snow density, grain size, and the interfacial mean curvature are proposed. Then, we present estimates of hydraulic conductivity, water permeability, effective thermal conductivity, and water vapor diffusivity of wet snow were computed. We analyzed, computed on the simulated wet snow images. We study their evolution in relation to water content, density, and snow type, ~~compared them with existing regressions and presented new ones~~. Our estimates are compared to existing parametrizations of the wet snow properties; new parametrizations are proposed when needed. The proposed relationships are a step forward in improving the physical description. Our simulations are a first step toward a better characterization of the micro-scale distribution of liquid water in snow, and contribute to improving the modeling of the hydraulic and physical properties of wet snow in snowpack scale models.

## 25 1 Introduction

Wet snow is characterized by the presence of liquid water in the ~~snow microstructure. The liquid water, mostly dry snow microstructure itself, composed of air and ice. Liquid water,~~ introduced by rain or melt events, ~~often flows from the surface downwards through,~~ is transported in the snowpack and ~~implies drastic changes causes drastic changes in the microstructures and properties of the snow,~~ which can lead to wet snow avalanches (e.g., Schweizer et al., 2003) ~~and or~~ the release of large amounts of water and flooding (e.g., Singh et al., 1997). ~~Those changes are associated with morphological transformations of the snow microstructure. The rapid morphological transformation of the wet snow microstructure is~~ called wet snow metamorphism (e.g., Wakahama, 1968; Colbeck, 1973; Raymond and Tusima, 1979; Marsh and Woo, 1985) ~~, which are and is~~ characterized by the formation of large, rounded, often clustered grains, referred to as melt forms (MF) (Fierz et al., 2009). ~~The physics of liquid (Fierz et al., 2009). Liquid~~ water transport in snow is ~~complex and a complex phenomenon that~~ involves water flow by gravity and capillarity ~~, which (e.g. Gerdel, 1954; Colbeck, 1976). It can present features such as hysteresis preferential flow (e.g. Hirashima et al., 2014), capillary rise (e.g. Lombardo et al., 2025), capillary barriers (e.g. Quéno et al., 2020), or hysteresis wetting-drying processes, preferential flow or capillary barriers. Water processes (e.g. Adachi et al., 2020). Liquid water transport is coupled with heat transport, which includes heat conduction and convection, as well as latent heat and water vapor transport, driven by heat conduction (e.g. Sturm et al., 1997), heat convection (e.g. Sturm and Johnson, 1991), latent heat and vapor fluxes from phase changes. To predict wet snow, detailed snow cover models rely on a modeling of the liquid (e.g. Calonne et al., 2014b), and vapor diffusion (e.g. Fourteau et al., 2021b). Several models were proposed to simulate wet snow and water transport in snow based on simplified representations, such as the bucket-type approach in the Crocus model (Vionnet et al., 2012). Recently, more sophisticated models of water flow in snow have been presented and describe processes such as water flow by capillarity or preferential flow along water channels (e.g., Daanen and Nieber, 2009; Hirashima et al., 2014; Leroux et al., 2015). These models were little evaluated, and rely on effective wet snow properties, including specific hydraulic properties of snow for water flow, which are often estimated empirically. For unsaturated porous media, the the snowpack, (e.g., Daanen and Nieber, 2009; Hirashima et al., 2014; Leroux et al., 2015), including large-scale operational models as Crocus (Vionnet et al., 2012; Fourteau et al., 2025; Lafaysse et al., 2025) and Snowpack (Lehning et al., 2002; Wever et al., 2014, 2015).~~

The movement of water is classically described by the Richards equation, expressing which expresses the volumetric liquid water content evolution (Richards, 1931): in unsaturated porous media (Richards, 1931):

$$\frac{\partial \theta_w}{\partial t} = \frac{\partial}{\partial t} \frac{\partial}{\partial z} \left( \mathbb{K}_w^u(\theta_w) \left( \frac{\partial h}{\partial t} \frac{\partial h}{\partial z} + 1 \right) \right) - S \quad (1)$$

with  $\theta_w$  (-) the volumetric liquid-water content,  $h$  (m) the liquid pressure head (linearly related to the capillary pressure),  $\mathbb{K}_w^u$  (m  $s^{-1}$ ) the unsaturated hydraulic conductivity, and  $S$  ( $s^{-1}$ ) a source/sink term. To solve this equation, the liquid pressure head and the unsaturated hydraulic conductivity need to be specified. In general porous media applications, the liquid pressure head  $h$  is expressed as a function of the water content  $\theta_w$  through the water retention curve (WRC). The WRC ,describing describes the evolution of capillary pressure in the pores as a function of the water content, which reflects the hydraulic behavior of any the porous medium. This relationship can be provided for-obtained for primary imbibition, which consists in wetting corresponds to

the wetting of a dry porous medium by a liquid until it reaches saturation, ~~or for for primary~~ drainage, which ~~consists in letting the liquid contained in~~ corresponds to the drainage of a fully saturated porous medium ~~drain off~~. WRCs are classically modeled by the van Genuchten (VG) model (van Genuchten, 1980), initially developed and widely used for soil. Such a model includes several hydraulic parameters that depend on the morphology of the material being considered. WRCs have been traditionally measured by imbibition or drainage experiments in the laboratory. For soils, drainage experiments were performed using hanging water columns (drainage by gravity) or by pressure plates (drainage by a pressure-controlled pump). For snow, few WRC measurements have been conducted and are mostly limited to drainage (e.g., Colbeck, 1974; Yamaguchi et al., 2010, 2012; Katsushir  
60  
65 ~~Overall, most authors report large dependencies and for drainage or imbibition of a porous medium at any intermediate state of saturation.~~

WRCs are traditionally derived based on laboratory experiments, in which the liquid pressure head in the porous material under study is measured during a drainage or imbibition experiment, at different water contents. For snow, some WRC measurements are available (e.g., Colbeck, 1974; Coléou et al., 1999; Yamaguchi et al., 2010, 2012; Katsushima et al., 2013; Adachi et al.  
70 ~~They, however, show different limitations. In these studies, the liquid pressure head was typically measured with a coarse vertical resolution on the order of centimeters, except for Adachi et al. (2020), which provide measurements at a spatial resolution of 2 mm based on the magnetic resonance imaging method (MRI), and Lombardo et al. (2025), which provide measurements at 92 μm using neutron radiography. Another limitation concerns the snow samples under study, which, in the majority, were dense melt forms. The measurements of Yamaguchi et al. (2012) are the most extensive, based on 60 snow~~  
75 ~~samples, yet restricted to sieved or natural melt forms and natural rounded grains, with high density values from 360 to 630 kg m<sup>-3</sup>, and grain size from 0.3 to 5.8 mm. The authors showed that the WRCs depend largely on snow density and snow grain size. Recently, magnetic resonance imaging methods were adapted by Adachi et al. (2020) to measure WRCs in snow for both imbibition and drainage. This method enables to capture the liquid water content in 3D at a mm-scale resolution. The authors have revealed a significant hysteresis between both processes, which shows that, as for most porous media, drying and wetting in snow are distinct processes that should be considered separately. Some studies related~~ These results highlight the need to extend our knowledge on the impact of snow microstructure on the WRCs to a broader range of snow types. Finally, most of the WRCs measurements were performed for drainage; only Coléou et al. (1999), Adachi et al. (2020), and Lombardo et al. (2025) provided experimental WRCs for imbibition.

Besides direct measurements, WRCs can be predicted using the widely used van Genuchten (VG) model, initially developed for soils (van Genuchten, 1980). The VG model reads:  
85

$$\theta_w(h) = \theta_w^r + (\theta_w^s - \theta_w^r) \times (1 + (\alpha_{vg} h)^{n_{vg}})^{-m_{vg}} \quad (2)$$

where  $\theta_w^r$  and  $\theta_w^s$  are the residual water content after drainage and the maximum water content reached during imbibition, respectively.  $\alpha_{vg}$ ,  $n_{vg}$  and  $m_{vg}$  are the adjusted parameters of the VG model ~~to experimental WRCs and obtained estimates of the VG parameters for snow. Regressions were presented for these parameters based on snow microstructural properties~~.  
90 ~~that determine the shape of the WRC. These shape parameters depend on the material's morphology and need to be specified. For that, regressions to estimate the shape parameters from microstructural properties were developed by fitting the VG model~~

to WRCs obtained experimentally. For snow, Daanen and Nieber (2009) and Yamaguchi et al. (2010) proposed regressions depending solely on the based on snow grain size, based on developed from a few drainage experiments on similar snow samples composed of rather dense MF dense melt forms. Later, Yamaguchi et al. (2012) presented improved regressions based on additional drainage experiments, which depend on both the both snow density and grain size. Again, this study focuses only on MF samples, limiting the observations using additional drainage experiments, but based on the WRCs of the snow samples made of sieved melt forms only, thus limiting the validation of the regressions to coarse-grained, high-density snow. No To date, no estimates of the VG model parameters were provided shape parameters of the VG model have been presented for imbibition in snow. While the classical experiments of imbibition and drainage enable to capture the overall hydraulic behavior of a snow sample, necessary to resolve the Richard equations, they do not provide the exact distribution of liquid water at the pore scale inside the

The water unsaturated hydraulic conductivity  $\mathbb{K}_w^u$ , which is the second unknown of the Richards equation (Eq. 1), can be expressed as a function of the liquid water content  $\theta_w$  as :

$$\mathbb{K}_w^u(\theta_w) = \mathbb{K}_w^{\text{sat}} \times K_w^r(\theta_w) \quad (3)$$

where  $\mathbb{K}_w^{\text{sat}} = K \rho_w g / \mu_w$  is the saturated hydraulic conductivity of snow microstructure. Accessing 3D wet snow microstructures at varying saturation levels would be critical to estimate the effective properties specific to wet snow, in order to feed macro-scale models. For dry snow, accurate estimates of the effective properties are obtained by computations based on pore scale ( $\text{m s}^{-1}$ ), which depends on the intrinsic permeability of snow  $K$  ( $\text{m}^2$ ), on the water viscosity  $\mu_w$  (Pa s), on the water density  $\rho_w$  ( $\text{kg m}^{-3}$ ), and on the gravitational acceleration  $g$  ( $\text{m s}^{-2}$ ). The intrinsic permeability of snow  $K$  depends solely on the microstructure of the dry snow and can be estimated from empirical parameterizations, such as the one of Shimizu (1970) based on measurements or the one of Calonne et al. (2012) based on numerical computations on 3D images of dry snow obtained by X-tomography (e.g., Kaempfer et al., 2005; Calonne et al., 2011; Zermatten et al., 2011; Calonne et al., 2012, 2014a; Wautier et al., 2015; Parameterizations of effective properties, such as the tomographic images of snow. The term  $K_w^r(\theta_w)$  corresponds to the relative water permeability at a given saturation, and is defined as  $K_w^r(\theta_w) = K_w^u(\theta_w) / K$ ,  $K_w^u(\theta_w)$  being the unsaturated water permeability. The unsaturated water permeability  $K_w^u(\theta_w)$ , or equivalently the relative water permeability  $K_w^r(\theta_w)$ , are classically modeled using the van Genuchten-Mualem (VGM) equation for unsaturated soils, which relates the permeability  $K_w^u$  to the liquid water content  $\theta_w$  (Mualem, 1976; van Genuchten, 1980). The VGM equation is written as:

$$K_w^u(S_e) = K \times S_e^{\tau_{\text{vg}}} \left( 1 - (1 - S_e^{1/m_{\text{vg}}})^{m_{\text{vg}}} \right)^2 \quad (4)$$

$$\text{with } S_e(h) = (1 + (\alpha_{\text{vg}} h)^{n_{\text{vg}}})^{-m_{\text{vg}}} \quad (5)$$

where the shape parameters  $\alpha_{\text{vg}}$ ,  $n_{\text{vg}}$ , and  $m_{\text{vg}}$  are the same as those defined in the VG model (Eq. 2) and where (Eq. 5) is from Mualem (1976). The exponent  $\tau_{\text{vg}}$  describes the effects of the connectivity and tortuosity of the flow paths and is set to  $1/2$  in the following, which is its default value (e.g., Mualem, 1976; Vereecken et al., 2010). The estimation of the unsaturated hydraulic conductivity of snow is thus affected by the same limitations as those described for the estimation of the WRCs of snow.

125 The processes of heat transport and water vapor transport in wet snow are driven by the unsaturated effective thermal conductivity and the of snow  $k^u(\theta_w)$  and the unsaturated effective water vapor diffusivity , have been derived and are used to model heat and mass of snow  $D^u(\theta_w)$ , which have been little studied. Indeed, significant improvements were made on the estimations of these properties for dry snow, thanks to the use of X-ray tomography, which enables access of the 3D distribution of the ice and air structure at the pore scale, and thus, accurate computations (e.g., Kaempfer et al., 2005; Calonne et al., 2011; Zermatten et al., 2014b; Hansen and Foslien, 2015; Calonne et al., 2015; Brondex et al., 2023; Bouvet et al., 2024). However, performing these advances are, however, limited to dry snow. Performing X-ray tomography on wet snow samples is still a challenge, notably due to the lack of contrast between the X-ray absorption of ice and liquid water. The literature studies literature studies generally propose refrozen states of wet snow (e.g., Flin et al., 2011; Avanzi et al., 2017) which do not allow a precise

130 The modeling of heat and vapor transport in dry snow could thus be improved in macro-scale models (e.g., Calonne et al., 2014b; Hansen and Foslien, 2015; Calonne et al., 2015; Brondex et al., 2023; Bouvet et al., 2024). However, performing these advances are, however, limited to dry snow. Performing X-ray tomography on wet snow samples is still a challenge, notably due to the lack of contrast between the X-ray absorption of ice and liquid water. The literature studies literature studies generally propose refrozen states of wet snow (e.g., Flin et al., 2011; Avanzi et al., 2017) which do not allow a precise

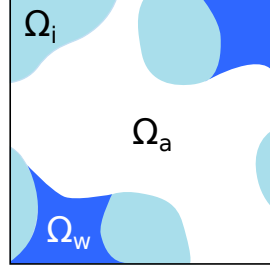
135 imaging of the ice-water interface. Direct 3D imaging of wet snow is presently only achieved by MRI (Adachi et al., 2020) (Adachi et al., 2020; Yamaguchi et al., 2025), with mm-scale images, whereas observing pore-scale pore-scale processes requires a  $\mu\text{m}$ -scale resolution. Because of Due to these limitations, our knowledge of the wet snow properties is not as advanced as that of dry snow. Macro-scale models such as the one derived by Daanen and Nieber (2009), Leroux and Pomeroy (2017) and Moure et al. (2023) rely on effective wet snow properties which are often estimated empirically or directly extrapolated from dry snow as done for the thermal conductivity in Crocus (Lafaysse et al., 2017). In this work, we circumvent this obstacle by exploring a new simulation-based approach in which liquid water is digitally filled into a 3D image of wet snow models rely on simple approaches to estimate the effective properties of wet snow. In many models (Daanen and Nieber, 2009; Leroux and Pomeroy, 2017), the unsaturated thermal conductivity of snow  $k^u$  is derived based on an arithmetic mean, such as  $k^u = k_{dry}^{eff}(1 - \theta_w) + k_w\theta_w$ , with  $k_{dry}^{eff}$  the effective thermal conductivity of dry snow and  $k_w$  the intrinsic thermal conductivity of liquid water. By doing so,

145 the impact of the microstructure and phases connectivity is not fully considered, although their importance was shown in dry snow (Calonne et al., 2011). In the operational Crocus model (Vionnet et al., 2012; Lafaysse et al., 2025), the parameterization of Yen (1981), only valid for dry snow, and then drained out of the porous microstructure. For that purpose, the distribution of liquid water in the microstructure is predicted, following the capillarity laws applied to an ice and air porous medium. As the traditional numerical method based on solving the Navier-Stokes equation in each phase is numerically too costly, we focus here on a is extrapolated to wet snow, which leads to the assumption that ice and water have the same impact on the snow effective thermal conductivity, although ice conducts four times more than water. Finally, the unsaturated effective water vapor diffusivity of snow has not yet been studied, as water vapor transport by diffusion has not yet been implemented to wet snow models. For unsaturated soils, different parameterizations are used depending on the soil nature, such as the ones of Millington and Quirk (1961) and Moldrup et al. (2000) (see Kristensen et al., 2010).

150 Different methods are currently used to compute the WRCs within porous media from 3D images of their microstructure. The first one consists in performing two-phase flow simulations at the pore scale using different numerical methods, such as the lattice-Boltzmann method (Vogel et al., 2005; Ahrenholz et al., 2008), the volume of fluids method (Bhatta et al., 2024), or the phase field models (Prodanović and Bryant, 2006; Jettestuen et al., 2013). Even if these simulations allow for an accurate description of the physical processes in dynamics at the pore scale, they require significant computational resources when

160 applied on complex 3D microstructures. To overcome this drawback, other methods such as the Pore Network Model (PNM) (Vogel et al., 2005; Joekar-Niasar and Hassanizadeh, 2012; Xiong et al., 2016) or the Pore Morphology Method (PMM) (Hilpert and Mille  
can be used. The PNM consists of creating a virtual representation of the porous medium, consisting of pore bodies (nodes) and pore throats (edges) of different sizes connected to each other. It is then possible to simulate the fluid flow and other transport  
165 processes of interest at the meso-scale through this network, with the relevant 1D physics implemented between nodes. The construction of such a model requires the extraction of microstructural parameters from the 3D images, such as pore sizes, throat sizes, coordination number, or shape factor, which is not always straightforward and impacts the accuracy of the modeling (see Xiong et al., 2016, and references herein). In the quasi-static approach (e.g., Hu et al., 2018; Konangi et al., 2021). Of course, wet snow metamorphism effects, characterized by the morphological changes driven by the presence of liquid water such as the rapid rounding and coarsening of grains are not accounted here and the results provided in this study should be considered  
170 as valid only for the initial state of regime, the Pore Morphology Method (PMM) can be used to compute, from a 3D image, the fluid phase distribution through a series of image-processing operations without solving any partial differential equation. The obtained 3D images, corresponding to different values of water content in the porous medium, can then be used to compute the effective properties such as the snow evolution. We thus propose here an exploratory numerical study where a quasi-static model permeability, thermal conductivity, or effective diffusivity (Berg et al., 2016; Becker et al., 2008).

175 In the present paper, the Pore Morphology Method (PMM) is used to compute the WRCs during imbibition and drainage of snow. The ice structure is fixed, i.e. the wet snow metamorphism is not represented. The PMM is applied to 34 experimental 3D tomography images of dry snow presenting diverse snow types and in which water is numerically introduced by capillarity and removed according to an imbibition-drainage pattern. Based on these series of wet snow images, the WRC of snow and its variations with the snow microstructure are first investigated. The VG model is applied to the simulated, presenting a wide  
180 range of snow microstructures, in terms of density, grain size, and shape. A series of 3D images of the distribution of air, ice and liquid water in snow at different stages of drainage or imbibition, so for different liquid water contents, is obtained. The first part of this work is dedicated to the study of the WRCs, which allows to derive its parameters for a large range of microstructures. The obtained parameters are used to propose new regressions are directly obtained from the simulations. The impact of the snow microstructure on the WRC shape is analyzed. The simulated WRCs are compared to experimental WRCs  
185 from the literature, including a comparison of the adjusted parameters of the VG model. New regressions of these parameters are proposed for both imbibition and drainage cases, which are compared to the regressions of Daanen and Nieber (2009), Yamaguchi et al. (2010) and Yamaguchi et al. (2012). Next, numerical computations on the simulated images of wet snow are performed to estimate the effective transport properties of wet snow, as and compared to existing ones. In the second part, the series of 3D snow images at different stages of drainage or imbibition are used to compute the effective properties required  
190 for the modeling of water flow, heat and vapor transport modeling. The investigated in wet snow. The studied properties are the relative permeability (i.e. the unsaturated hydraulic conductivity), the effective thermal conductivity, and the effective water vapor diffusivity. The numerical results are compared with commonly used estimates of commonly used estimates from the literature, such as the VGM model (Mualem, 1976; van Genuchten, 1980) for the unsaturated hydraulic conductivity. New regressions of effective thermal conductivity and vapor diffusivity are presented to account for the level of water saturation.

**1.1 Definition of the micro-scale variables****Figure 1.** Illustration of the micro-scale air, ice and water phases in wet snow.**2 Methods****2.1 Definition of the micro-scale variables**

Within the representative elementary volume (REV) of snow noted  $\Omega$ , we define  $\Omega_i$ ,  $\Omega_w$  and  $\Omega_a$  the volumes occupied by the ice, the water and the air phase, respectively, as illustrated in Figure 1. The **subscripts  $(i)$ ,  $(w)$  and  $(a)$ , are related to quantities defined in  $\Omega_i$ ,  $\Omega_w$  and  $\Omega_a$ , respectively.** The volume fractions  $\theta$  of the different phases (ice, water, air) are written:

$$\theta_i = \frac{\Omega_i}{\Omega}, \quad \theta_w = \frac{\Omega_w}{\Omega}, \quad \theta_a = \frac{\Omega_a}{\Omega}, \quad (6)$$

with  $\theta_i + \theta_w + \theta_a = 1$ . The porosity  $\phi$ , the water saturation  $S_w$  and air saturation  $S_a$  are respectively defined as:

$$\phi = \frac{1 - \Omega_i}{\Omega} = \frac{\Omega_a + \Omega_w}{\Omega}, \quad S_w = \frac{\Omega_w}{\Omega_a + \Omega_w}, \quad S_a = \frac{\Omega_a}{\Omega_a + \Omega_w}, \quad (7)$$

205 which leads to:

$$\phi = 1 - \theta_i, \quad S_w + S_a = 1, \quad \theta_w = \phi S_w, \quad \text{and} \quad \theta_a = \phi S_a = \phi(1 - S_w). \quad (8)$$

**2.2 3D images of snow and microstructural properties**

The set of 3D images of snow used in this study for the simulations corresponds to the one from Calonne et al. (2012) (see the related supplement for details). This set consists of 34 images of different dry snow microstructures obtained by X-ray micro-tomography, covering a **great diversity wide range** of snow properties in terms of density, grain size and snow type. The **imaged snow samples are composed of natural snow collected in the field as well as snow obtained from evolution under controlled environmental conditions in cold laboratory, which replicates natural snow evolution.** The image volumes are cubic, with sides ranging in size from  $\sim 2.5$  to 10 mm and resolution from  $\sim 5$  to 10  $\mu\text{m}$ . Of these 34 images, 5 images representative

**Table 1.** Main characteristics of the 5 selected 3D images.

Name	Snow type	Image size (voxel)	Resolution ( $\mu\text{m}$ )	$\rho$ ( $\text{kg m}^{-3}$ )	$r_{es}$ (mm)
NH2	MF	651	8.6	503	0.53
NH5	MF	1000	9.5	473	0.87
grad3	DH	600	10	369	0.15
0A	RG	700	8.4	315	0.12
fr	PP	1192	4.9	125	0.06

of the diversity of all the images were selected for detailed investigations. They include two images of melt forms at different densities and grain sizes, and one image of depth hoar, rounded grains and precipitation particles. Their main characteristics are shown in Table 1. To characterize these dry snow microstructures, we rely on the snow density  $\rho$  in  $\text{kg m}^{-3}$  (in the following the snow density will always refer to the dry density without liquid water), which is computed from the snow porosity as  $\rho = (1 - \phi)/\rho_i$ , with  $\rho_i$  the ice density taken as  $917 \text{ kg m}^{-3}$ , and the spherical equivalent radius  $r_{es}$  in m, derived from the specific surface area (SSA,  $\text{m}^2 \text{ kg}^{-1}$ ) as  $r_{es} = 3/(\text{SSA} \times \rho_i)$  with  $\rho_i$  the ice density. Both snow density and SSA values were provided by Calonne et al. (2012) (see the related supplement for the detailed table) based on 3D image computations using simple voxel counting and a stereological method (Flin et al., 2011).

### 2.3 Numerical simulations of imbibition and drainage

~~To simulate the distribution of two distinct fluid phases in a porous material, we used the~~

~~The SatuDict module of the Geodict software (Math2Market GmbH) based on a pore morphology method-based algorithm (Thoemen et al., 2008). The model assumes that water flow is in steady state and controlled solely by capillary pressure, and therefore by the pore morphology, using the Young-Laplace equation. Gravity and viscous effects are neglected (Thoemen et al., 2008) based on the Pore Morphology Method (PMM) (e.g., Hilpert and Miller, 2001; Silin and Patzek, 2006; Schulz et al., 2015; Berg et al., 2016) was used to compute water retention curves (WRCs) of snow. This method, valid in a quasi-static regime, is applicable when the gravity and viscous forces are negligible compared to capillary pressure forces, which is a good approximation for small pore sizes and low viscosity fluids as here. Interface dynamics, as generated by phase changes, are not described. The model appears to work well for fluids with contact angles close to 0 (here  $12^\circ$  between ice and liquid water), and for microstructures with rather spherical pores (Hu et al., 2018). Based on the above approximations, we used SatuDict to generate series of wet snow at different water contents from the 34 tomographic images of dry snow. The model provides the 3D distribution of the air and liquid water phases inside the ice structure for different water contents. This can be seen as simulating the process of imbibition or drainage in snow as a sequence of equilibrium states. In more details, the case in snow. Indeed, at the pore scale, capillary forces usually play a much important role than gravity. For air and water, the Bond number, which measures the ratio between gravitational force and surface tension force, is defined as:  $\text{Bo} = (\rho_w - \rho_a)gl^2/\gamma$  where  $\rho_w$  and  $\rho_a$  are the water and the air density, respectively,  $g$  is the gravity,  $\gamma$  the surface tension, and  $l$  is a characteristic length of the snow microstructure~~

at the pore scale, such as the pore size. This dimensionless number varies between  $10^{-3}$  and  $10^{-1}$  for a pore size  $l$  varying  
240 between  $10^{-4}$  and  $10^{-3}$  m as in snow. These estimations show that gravitational forces are negligible at the pore scale in  
comparison to capillary forces. Similarly, it can be shown (Auriault, 1987; Auriault et al., 2009) that within a porous media,  
the viscous stress ( $\sigma_v$ ) at the pore scale is negligible in comparison to the fluid pressure ( $p$ ), which is of the following Jurin  
equation applied for snow is solved:-

$$h = \frac{2\gamma \cos(\psi)}{r\rho_w g}$$

245 where  $h$  is the liquid pressure head in m order of the capillary pressure ( $p_c$ ):  $p \approx p_c \approx (L/l)\sigma_v$  where  $L$  is a macroscopic length,  
i.e. the characteristic size of the snowpack. If we assume that  $l = 10^{-3}$  m and  $L = 0.1$  m, the capillary pressure is around 100  
times larger than the viscous stress.

The PMM uses a sphere with a radius  $r$  as a probe to detect the pore space that is accessible by the non-wetting phase  
(NWP, here the air). This radius is computed from the Young-Laplace equation:  $r = 2\gamma \cos(\psi)/p_c$  where  $p_c$  is the capillary  
250 pressure,  $\gamma = 0.0756 \text{ N m}^{-1}$  is the surface tension at  $0^\circ\text{C}$ ,  $\psi = 12^\circ$  and  $\psi$  is the contact angle between ice and liquid water.  
At  $0^\circ\text{C}$ ,  $\gamma = 0.0756 \text{ N m}^{-1}$  and  $\psi = 12^\circ$  (Knight, 1967). Morphological operations, namely, erosion and/or dilation are used  
in the PMM (Hilpert and Miller, 2001). The algorithm of the PMM can be decomposed into several steps as follows -see e.g.  
Arnold et al. (2023), Fig. 2:

- In drainage condition, the porous medium is initially saturated with the wetting phase (WP, here the water) ( $p_c = 0$ ).  
255 The invading NWP is connected to the inlet, which is the NWP reservoir, and the WP can escape through the outlet,  
the WP reservoir. (i) Then,  $p_c$  is increased incrementally, i.e.  $r$  is the minimum radius of accessible pores,  $g$  is  
the gravitational acceleration in  $\text{m s}^{-2}$  and  $\rho_w$  is the density of water in  $\text{kg m}^{-3}$ . The evolution of the air and liquid  
water in the ice structure can be seen as a series of erosion and dilation operations that depends on the pore distribution  
(Ahrenholz et al., 2008). In this study, we simulated the following sequence: first the imbibition process is simulated  
260 on an initial dry snow image until the saturation is reached, then, the drainage process is simulated on the saturated  
image obtained after imbibition. This configuration can be seen as wetting a snow layer by capillarity from the bottom,  
with liquid rising inside the snow sample, and then draining the snow layer by gravity. This situation can be found  
when liquid water percolated through water channels and pounds over a less permeable layer so that the water flows  
horizontally and can reach a dry snow area where it rises by capillarity (e.g., Coléou et al., 1999; Williams et al., 2010)  
265 -is decreased incrementally. The solid phase is first dilated by a sphere with radius  $r$ . (ii) All the pores connected to the  
NWP reservoir are labeled as NWP. (iii) The NWP is then dilated with the same sphere with radius  $r$ . The remaining  
pores are filled with the WP. The saturation can then be calculated. (iv) All the pores filled by the WP disconnected from  
the WP reservoir are considered as WP residual, and are no longer considered in the next steps. All these steps (i to iv)  
are repeated by increasing the value of the pressure, i.e. by decreasing the value of  $r$ . In the present case, the radius  $r$   
270 was decreased gradually with a step of 2 pixel size. For both imbibition and drainage, the boundary conditions consist  
of an air reservoir located at the top of the snow sample and a water reservoir located at the bottom. Periodic conditions  
are applied on the sides of the samples. For imbibition, the simulation starts with the 3D image of dry snow. A minimum

pore radius is defined according to Eq. (??), which then increases step by step. At each time step, a pore is filled with liquid water if (

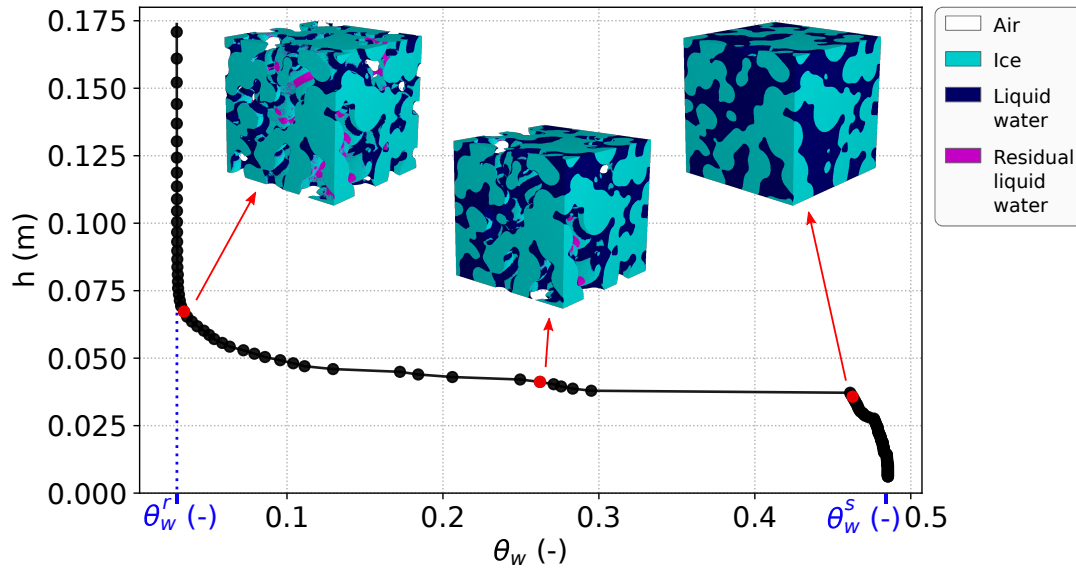
275 – In imbibition condition, the porous medium is initially saturated with the NWP. The invading WP is connected to the inlet, which is the WP reservoir, and the NWP can escape through the outlet, the NWP reservoir. (i) ~~the pore radius is below the minimum pore radius, and~~ (Then,  $p_c$  is decreased incrementally, i.e.  $r$  is increased incrementally. The solid phase is first dilated by a sphere with radius  $r$ . (ii) ~~the liquid water phase is~~ The NWP is then dilated with the same sphere with radius  $r$ . (iii) All the pores connected to the ~~liquid water reservoir.~~ For drainage, WP are now labeled as WP. The remaining pores are NWP. The saturation can then be calculated. (iv) All the pores filled by the NWP disconnected from the NWP reservoir are considered as NWP residual, and are no longer considered in the next steps. As for the drainage condition, all these steps (i to iv) are repeated for the next value of the pressure, i.e. the next value of  $r$ . In the present case, the radius  $r$  was increased gradually with a step of 2 pixel size.

In imbibition condition, if the step (iv) is ignored, ~~the simulation starts with the~~ PMM also allows to compute the Mercury Injection Capillary Pressure (MICP) curves, which are a commonly-used technique for measurements of porosity or pore throat size distribution, for instance (Hilpert and Miller, 2001; Berg et al., 2016). As underlined in Hilpert and Miller (2001), ~~the accuracy of the PMM may depend on the resolution and size of the 3D image of water saturated snow obtained at the end of the imbibition simulation.~~ A maximum pore radius is defined according to Eq. (??), which then decreases step by step. At each time step, a pore is filled with air if (i) the pore radius is larger than the maximum pore radius, (ii) the entering air phase is connected to the air reservoir, and (iii) ~~the initial liquid water phase of the pore is~~ images and of the definition of the structural element. Finally, it is worth mentioning that the boundary conditions applied on the four sides of ~~the 3D images that are not~~ connected to the ~~liquid water reservoir.~~ Those conditions enable residual water or residual air to remain trapped in the porous media after the imbibition and drainage simulations, respectively. The output of the algorithm is a sequence of quasi-stationary three-phase distributions, which are used in SatuDict. ~~NWP or WP reservoirs may also play~~ a role on the WRC simulations, even if this point has not been discussed in the literature, to the best of our knowledge. In Vogel et al. (2005), impervious boundary conditions are applied, whereas other conditions, such as symmetry, displaced fluid outlet, or invading fluid inlet, may also be applied (Berg et al., 2016). These boundary conditions may lead to different values of the NWP (air) residuals during the imbibition process, since these residuals can be trapped or not at the boundaries. Depending on the boundary conditions, the maximum water saturation ( $\theta_w^s$ ) may range from 45% to 90% of the porosity. Despite such large differences, the values of  $\alpha_{vg}$  and  $n_{vg}$  in the van Genuchten (VG) model (van Genuchten, 1980) remains almost constant (Likos et al., 2014; Farooq et al., 2024). The impact of the boundary conditions is less pronounced in drainage conditions, since the water residuals are mainly located at the junction between grains.

In the present study, the PMM implemented in the SatuDict software was used to compute the ~~saturation-dependent water retention curve (WRC).~~ Finally, ~~we ensured that the simulations were performed on~~ WRCs of the 34 snow samples. We computed (i) a primary imbibition curve assuming that there is no air (NWP) residuals as in MICP experiments, thus  $\theta_w^s = \phi$  in Eq. 9, and then (ii) a primary drainage curve until reaching the water (WP) residuals ( $\theta_w^r$  in Eq. 9). In both cases, symmetric

boundary conditions are applied on the four sides of the volumes. The series of 3D images of sufficient size, of at least the size of the REV using increasing volumes (see Fig. 4. a) snow images at different stages of drainage were then used to compute the relative permeability (i.e. the unsaturated hydraulic conductivity), the effective thermal conductivity, and the effective water vapor diffusivity.

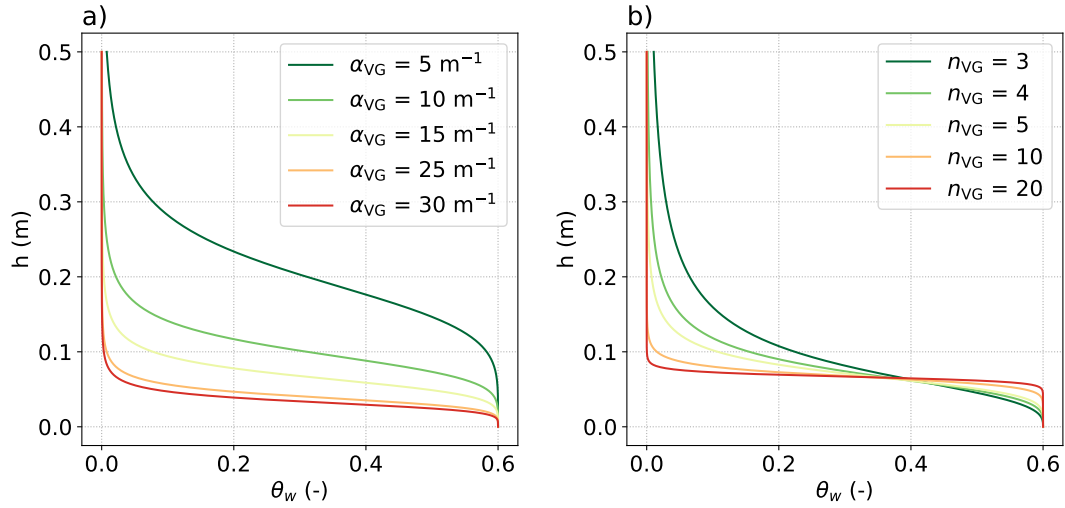
## 2.4 Water retention curve analysis



**Figure 2.** Example of a water retention curve estimated from a drainage simulation on a the 3D tomographic snow sample of melt forms. The corresponding drying path is represented by the arrows NH5 (MF). The simulated 3D water distribution in the pores is shown at 3 different stages.

WRCs from the VG model: influence of a) the parameter  $\alpha_{VG}$  (with a fixed  $n_{VG} = 5$ ) and of b) the parameter  $n_{VG}$  (with a fixed  $\alpha_{VG} = 15$ ).

Results from the SatuDict module software are used to determine the WRC, by expressing the liquid pressure head  $h$  as a function of the liquid water volumic fraction  $\theta_w$ , for both imbibition and drainage. An example of the WRC obtained from a drainage simulation on a small MF sample is shown in Fig. 2. The plot should be read from right to left, as water is gradually drained out of the porous microstructure. The liquid pressure head increases with decreasing water content, characterized by a sharp rise at the very beginning and very end of drainage and a near constant value for the intermediate water contents. The 3D images show the distribution of the air (transparent and yellow phases) and water (dark blue) in the pore space of the snow at different points on the WRC. WRCs also provide the residual water content  $\theta_w^r$ , being the remaining water content after drainage, and the saturated water content  $\theta_w^s$ , being the maximum amount of water the snow volume can store. Here,  $\theta_w^s = \phi$  since no air residuals are simulated. Both parameters are illustrated in the figure as the end-points endpoints of the WRC.

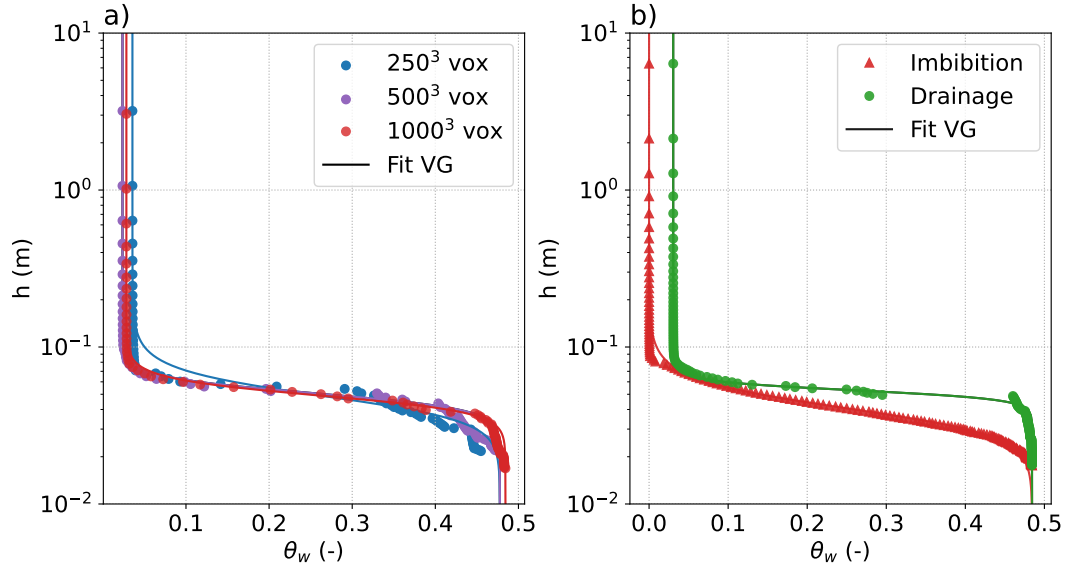


**Figure 3.** WRCs from the VG model: influence of a) the parameter  $\alpha_{VG}$  (with a fixed  $n_{VG} = 5$ ) and of b) the parameter  $n_{VG}$  (with a fixed  $\alpha_{VG} = 15 \text{ m}^{-1}$ ).

To further analyze the WRCs estimated from the different snow images, the curves are fitted to the van Genuchten (VG) model (van Genuchten, 1980), which reads as follows:

$$325 \quad \theta_w(h) = \theta_w^r + (\theta_w^s - \theta_w^r) \times \left( 1 + (\alpha_{VG} h)^{n_{VG}} \right)^{-\frac{m_{VG}}{n_{VG}}} \quad (9)$$

where  $\alpha_{VG}$ ,  $n_{VG}$  and  $m_{VG}$  ( $\text{m}^{-1}$ ),  $n_{VG}$  (-) and  $m_{VG}$  (-) are the unknown parameters that determine the shape of the WRC. These parameters depend on the snow microstructure and can be estimated by fitting the VG model to WRCs obtained experimentally (Daanen and Nieber, 2009; Yamaguchi et al., 2010, 2012). The parameter  $m_{VG}$  is usually approximated by  $m_{VG} = 1 - 1/n_{VG}$  (e.g., Yamaguchi et al., 2010). The influence of the parameters  $\alpha_{VG}$  and  $n_{VG}$  on the WRC is illustrated in Fig. 3.  $n_{VG}$  defines the steepness of the curve inflections and  $\alpha_{VG}$  defines the inverse of  $\alpha_{VG}$  is related to the values of  $h$  at which inflections occur. Roughly, the parameter  $\alpha_{VG}$  can be related to the average pore size and  $n_{VG}$  to the  $n_{VG}$  to the width of the pore size distribution (e.g., Yamaguchi et al., 2010) (e.g., Yamaguchi et al., 2010; van Lier and Pinheiro, 2018). In what follows, the WRCs of each imbibition and drainage simulation are considered and used to fit the VG model. For fitting,  $\theta_w^s$  is set equal to the snow sample porosity and we took  $\theta_w^s = \phi$  for both imbibition and drainage.  $\theta_w^r = 0$  for imbibition, and  $\theta_w^r$  is set to 0 for imbibition and to equals the minimum value of the  $\theta_w$  obtained from the drainage simulations for drainage. These choices are justified by the fact that our simulations always start with imbibition, i.e. start with a primary imbibition, without any liquid water in the pores, and end with drainage, at the end of which the volume of the air residuals is negligible (around 0.03% in average) as compared to the overall porous volume (see a complementary discussion on this topic in subsection 3.1.3) that we do not simulate entrapped air in imbibition but residual water in drainage. Finally, as mentioned above, simulations of-



**Figure 4.** a) Influence of the volume size taken for drainage simulations on the WRC, for the sample NH5 of melt forms. Simulation data are represented by symbols and the fitted VG models by solid lines. b) Example of hysteresis of the WRC between imbibition and drainage, for the snow sample NH5. The corresponding path is represented by the arrows.

Finally, we evaluated the REV of the WRC on our 3D images, by performing imbibition and drainage were performed on REV's so that we obtain representative distributions of the fluids in the pores and representative WRCs. Figure 4. a illustrates the impact of the image size on the simulated WRC. When volumes smaller than the REV are taken for simulation, simulations from several sub-volumes of increasing sizes within the same sample, as in Hilpert and Miller (2001). The size of the REV was assumed to be reached once values did not vary significantly when the size of the sub-volumes of computation increased. Fig. 4.a shows an example for the sample NH5 (MF), which is the most critical sample because of its large grains. We see that the REV needed to obtain representative distributions of the fluids in the pores and representative WRCs is rather large compared to typical REV of thermal conductivity, density or SSA (e.g., Calonne et al., 2011; Flin et al., 2011). The maximum sizes available for all the obtained WRCs are degraded and differ from the WRC obtained for a REV. Such tests were performed to assess the REV for WRCs of all our snow samples and ensure 3D images were then used, giving satisfactory results.

## 2.5 Computation of the effective transport properties of wet snow

Next, we study the effective transport properties of wet snow based on the 3D image series obtained from the drainage simulations. We focus on the properties involved in the processes of heat, water vapor and liquid water transport, which are key for wet snow modeling. For the process of liquid water flow, the driving effective property is namely the water unsaturated hydraulic conductivity  $\mathbb{K}_w^u$ , expressed in Richards equation (Eq. 1).  $\mathbb{K}_w^u$  can be expressed as a function of  $\theta_w$  by  $\mathbb{K}_w^u(\theta_w) = \mathbb{K}_w^{\text{sat}} \times K_w^r(\theta_w)$ .  $\mathbb{K}_w^{\text{sat}} = K \rho_w g / \mu_w$  is the saturated hydraulic conductivity of snow, which depends on the

intrinsic permeability of snow  $K$ ,  $\mu_w$  (Pa s) being the water viscosity.  $K$  depends solely on the microstructure of the dry snow and can be estimated from empirical parameterizations, such as the one of Shimizu (1970) based on measurements or the one of Calonne et al. (2012) based on numerical computations on 3D tomographic images of snow. The term  $K_w^r(\theta_w)$  corresponds (linked to the relative water permeability at a given saturation, and is defined as  $K_w^r(\theta_w) = K_w^u(\theta_w) / K_w^r(\theta_w)$ ),  $K_w^u(\theta_w)$  being the unsaturated water permeability. The unsaturated water permeability  $K_w^u(\theta_w)$ , or equivalently the relative water permeability  $K_w^r(\theta_w)$ , are classically modeled using the van Genuchten-Mualem (VGM) equation for unsaturated soils, which relates the permeability  $K_w^u$  to the liquid water content  $\theta_w$  (Mualem, 1976; van Genuchten, 1980). The processes of heat transport and water vapor transport are driven by the unsaturated effective thermal conductivity of snow  $k^u(\theta_w)$  and the unsaturated effective water vapor diffusivity of snow  $D^u(\theta_w)$ . Both properties have been widely estimated and parameterized for the case of dry snow (e.g., Calonne et al., 2011, 2014b; Fourteau et al., 2021a, b) but very little for the case of wet snow. To access  $K_w^u(\theta_w)$ ,  $k^u(\theta_w)$  and  $D^u(\theta_w)$ , the Geodict software was used to compute the 3D tensors of these three transport properties of wet snow: the intrinsic water permeability  $K_w^u$  in  $m^2$ , the effective thermal conductivity  $k^u$  in  $W m^{-1} K^{-1}$  and the effective water vapor diffusivity  $D^u$  in  $m^2 s^{-1}$ . Computations were performed on the series of 3D images of wet snow obtained from the imbibition and drainage simulations, so for different water contents, snow densities and microstructures. For each property, a specific boundary value problem, resulting from a homogenization technique, is solved on the REV applying periodic boundary conditions on the external boundaries of each volume. The equations to be solved are provided in the Supplement and correspond to Eq. (S.1) - (S.4) for the intrinsic water permeability  $K_w^u$ , Eq. (S.5) - (S.15) for the effective thermal conductivity  $k^u$ , and Eq. (S.16) - (S.20) for the effective vapor diffusivity  $D^u$ . Computations of the thermal conductivity were carried out using the thermal properties of ice, air and liquid water at  $0^\circ C$  ( $k_i = 2.14 W m^{-1} K^{-1}$ ,  $k_a = 0.024 W m^{-1} K^{-1}$ ,  $k_w = 0.556 W m^{-1} K^{-1}$ ). As the non-diagonal terms of the tensors  $K_w^u$ ,  $k^u$  and  $D^u$  are negligible, we consider only the diagonal terms, that are seen as the eigenvalues of the tensors (see e.g., Calonne et al., 2011). In the following,  $K_w^u$ ,  $k^u$  and  $D^u$  refer to the average of the diagonal terms of  $K_w^u$ ,  $k^u$  and  $D^u$ .

### 3 Results and discussion

#### 3.1 Water retention curves

##### 3.1.1 Hysteresis of the WRCs

A first result is that the WRCs of the snow samples present generally significant hysteresis, i.e. as they differ between imbibition and drainage. A Following Figure 4.b, a steeper WRC is found for imbibition compared to drainage. This is illustrated in Fig. 4.b for the, here for the large melt forms sample NH5 composed of large MF, showing significantly different WRCs between both processes. This result indicates that the imbibition and drainage are irreversible processes and should be considered separately, as already pointed out by previous works (e.g., Adachi et al., 2020). Both processes can be linked with the following path: (i) wetting of a dry snow sample until it reaches the saturated water content  $\theta_w^s$ , and (ii) drainage of the sample until it reaches the residual water content  $\theta_w^r$  (following the arrows in Fig. 4. b).

### 3.1.2 WRCs of different snow microstructures and their related VG fits

390 Figure 5 presents vertical cross-sections of the drainage and imbibition simulations at three stages of water saturation for the five selected snow samples described in Table 1. The pore-scale distribution of liquid water in the microstructures can be observed. For imbibition, the small pores are first filled, and the larger pores are filled last. For drainage, it is the other way around, with water escaping the large pore first. Residual liquid water at the end of the drainage simulation is shown in pink. Air bubbles entrapped in the ice skeleton are shown in yellow (for instance, in the bottom right of the 0A (RG) sample). This  
395 figure also highlights the fact that, for a given saturation, the liquid water distribution is different depending on the snow types, as well as between the imbibition and drainage processes.

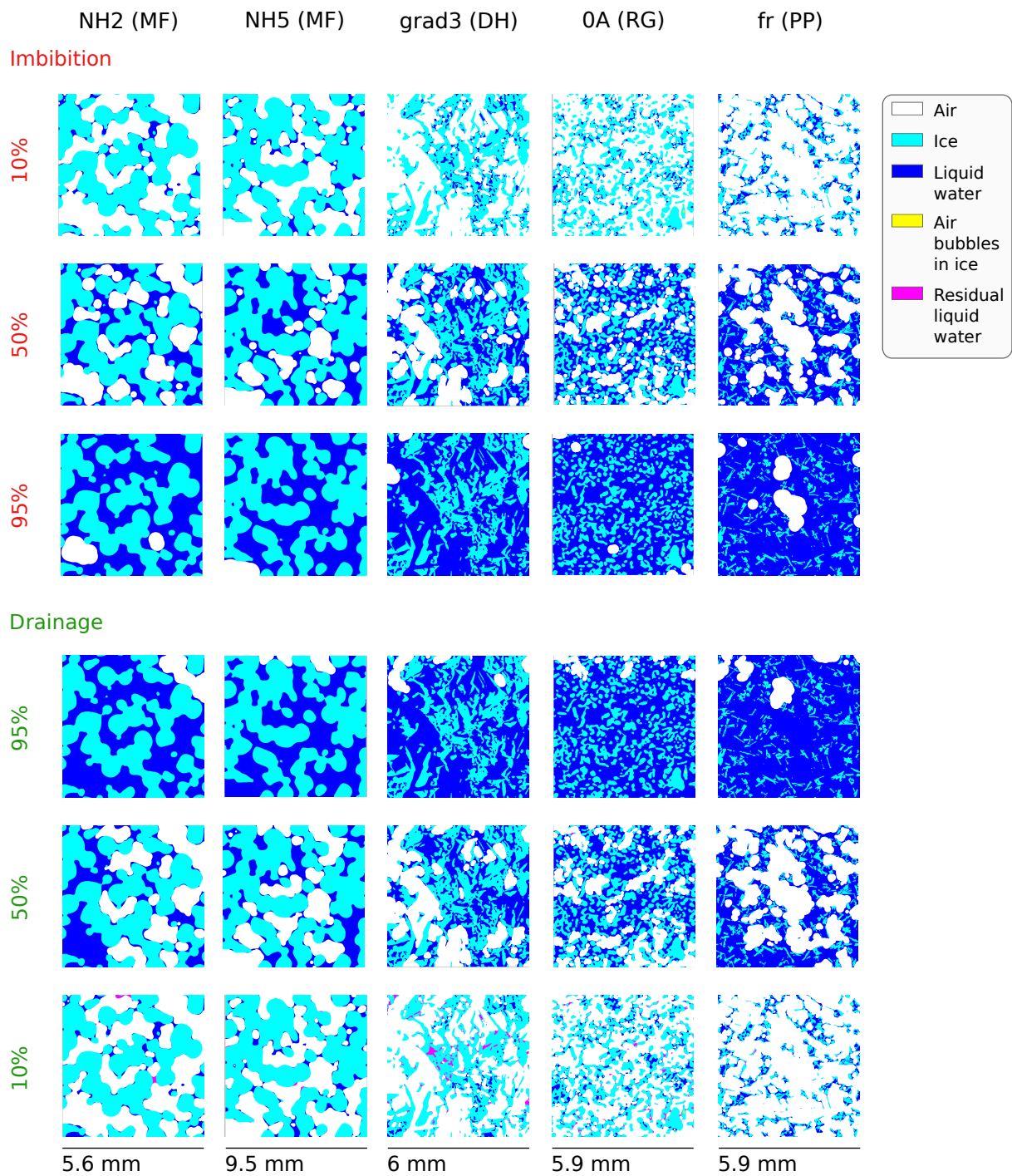
### 3.1.3 ~~WRCs of different snow microstructures and their related VG fits~~

Figures 6.a and 6.b present the WRCs of the imbibition and drainage simulations applied to the 5 selected snow samples presented in Table 1. The influence of the snow geometrical properties can be observed. Snow samples presenting small grains  
400 (0.06 - 0.15 mm), such as the samples fr (PP), 0A (RG) and grad3 (DH), show higher pore pressures at a given water content than the samples NH2 (MF) and NH5 (MF) presenting coarse grains (0.53 - 0.87 mm). Snow density has also a direct influence on the WRCs, as it limits the maximum water content that can be reached. ~~In a more subtle way, it also seems to influence the shape of the WRCs, which~~ More subtly, the WRCs tend to show ~~smoother inflections for lower densities, as seen when comparing the WRC of the lightest sample fr and the denser sample NH2, for example.~~ sharper transitions for the most evolved  
405 snow microstructures.

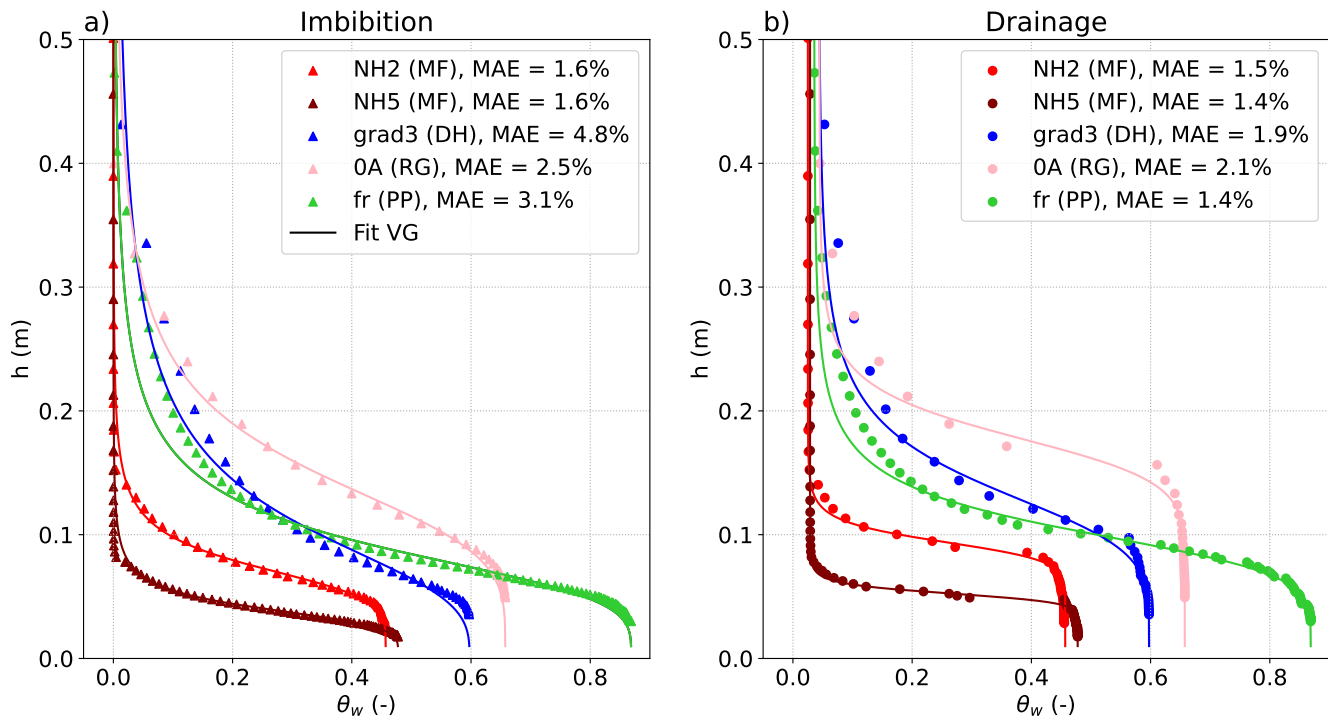
For each microstructure, we also present the related VG fit (Eq. 9) that best ~~reproduce~~ reproduces the simulated WRC. In other words, the VG parameters  ~~$\alpha_{VG}$ ,  $n_{VG}$ ,  $\alpha_{vg}$ ,  $n_{vg}$~~  and  $\theta_w^r$  were optimized to best fit the simulated WRC of each snow sample for imbibition and for drainage. ~~As a consequence, the~~ The fitting is rated in terms of MAE (mean absolute error) on  $\theta_w$  and expressed as a percentage. The fitted WRCs show overall good agreement with the simulated WRCs, ~~with an  $R^2$  greater than~~  
410 ~~0.98 for each sample~~, as illustrated for the five selected snow samples in Fig. 6a. and b. ~~Overall, the~~ The fits show slightly better results for ~~imbibition than for drainage, due to the sharper inflections of WRCs for the latter one.~~ drainage than for imbibition, and for MF samples compared to the other snow types. In addition to that, we chose a simple parameterization of the VG formulation, which could be refined using more parameters, such as  $m_{vg}$ , which could enable fitting both the left and right inflection points of the WRCs, especially for drainage.

### 415 3.1.3 Analysis of the VG parameters

Figure 7 presents the VG parameters  ~~$\alpha_{VG}$ ,  $n_{VG}$ ,  $\alpha_{vg}$ ,  $n_{vg}$~~  and  $\theta_w^r$  obtained by fitting the VG model to our simulated WRCs for all our dataset. The parameters, obtained for imbibition and drainage, are expressed as a function of the term  $\rho/d$  or  $\rho$ , following Yamaguchi et al. (2012), and compared to the regressions suggested by Yamaguchi et al. (2012) (black solid curves in Fig. 7). In addition, the measurement data of  ~~$\alpha_{VG}$  and  $n_{VG}$  from Yamaguchi et al. (2012) and Adachi et al. (2020)~~  $\alpha_{vg}$  and  $n_{vg}$  from  
420 Yamaguchi et al. (2012); Katsushima et al. (2013); Adachi et al. (2020) and Lombardo et al. (2025) are also shown ~~with black~~



**Figure 5.** Vertical cross-sections of the 5 selected snow samples with drainage and imbibition processes at 3 stages of effective saturation. The cross-sections are taken in the center of the samples. The reservoir of the wetting phase is located on the bottom boundary and the reservoir of the non-wetting phase is located on the top boundary.



**Figure 6.** Numerical imbibition and drainage WRCs for different types of snow samples with the corresponding VG fits. The curve colors represent the different snow types, the MAE on  $\theta_w$  of the fits are expressed in percent.

and gray markers. They correspond to values derived from WRCs obtained from laboratory experiments of imbibition and drainage. In As specified in subsection 2.2, in all our computations,  $d$  was estimated based on the SSA, following the formula  $d = 2 \times r_{es} = 6 / (\text{SSA} \times \rho_i)$  with  $\rho_i = 917 \text{ kg m}^{-3}$  the ice density.

#### ~~$\alpha_{VG}$~~ $\alpha_{vg}$ parameter:

425 Overall, the parameter  ~~$\alpha_{VG}$~~   $\alpha_{vg}$  decreases with increasing  $\rho/d$ , so when density increases and/or grain size decreases. ~~Values obtained from the simulations of imbibition (triangle symbols)~~ This trend is in overall consistent with the measurements of Yamaguchi et al. (2012), Adachi et al. (2020), Katsushima et al. (2013), and Lombardo et al. (2025). Moreover,  $\alpha_{vg}$  values from the imbibition simulations are systematically larger than the ones from the ~~simulations of drainage (circle symbols), drainage simulations.~~ This is in agreement with the observation of Adachi et al. (2020). The  ~~$\alpha_{VG}$~~  hysteresis reported by Adachi et al. (2020) and with the high  $\alpha_{vg}$  values obtained from imbibition experiments by Lombardo et al. (2025)

430 The  $\alpha_{vg}$  values from our simulations and the regression of Yamaguchi et al. (2012) are overall in good agreement for drainage, with a MAE of 25%, a little less so for imbibition, with a MAE of 38%.

Following the formulation of Yamaguchi et al. (2012), we proposed two new regressions of  $\alpha_{VG}$  that  $\alpha_{vg}$ , which allow reproducing the different behaviors between imbibition and drainage, as shown in the figure. Their expressions are provided in Table 2. They are shown in Figure 7.a and .b, and have a MAEs of 17% when compared to our data. The proposed regressions were derived from our snow samples, snow samples with  $\rho/d$  values comprised between  $0.25 \times 10^6$  and 1.3 and consequently do not perfectly capture the steep evolution of  $\alpha_{VG}$  for  $\times 10^6 \text{ kg m}^{-4}$ , and do not cover the lowest values of  $\rho/d$  ( $< 0.25$ ) reported by the measurements of Yamaguchi et al. (2012) and Adachi et al. (2020).

#### **$n_{VG}$ parameter:**

For the parameter  $n_{VG}$ , the  $\times 10^6 \text{ kg m}^{-4}$ , for which a steep evolution of  $\alpha_{vg}$  was reported (Yamaguchi et al., 2012; Adachi et al., 2020). Still, we derived our regressions with an exponential form to be consistent with these observations and following Yamaguchi et al. (2012).

We use the  $\alpha_{vg}$  parameter to quantify the degree of hysteresis of the WRCs, based on the ratio  $\alpha_{vg}$  from imbibition over  $\alpha_{vg}$  from drainage. For our set of images, this ratio ranges from 1.19 to 1.42, with an average value of 1.28. It is consistent with the ratios measured by Adachi et al. (2020), for which values of 1.46, 1.52, and 1.38 are found for the S, M, and L samples, respectively. No correlation was found between this ratio and grain type, grain size, or density. Our ratios and the ones of Adachi et al. (2020) tend to confirm hysteresis ratios around 1.5 for snow, which is lower than the classical value of 2 used for soils (Leroux and Pomeroy, 2017).

#### **$n_{vg}$ parameter**

The results obtained from the simulations are more surprising. Values from imbibition remain around the value  $n_{VG} = 4.7$ , showing little variations regarding the parameter  $n_{vg}$ . For imbibition, the  $n_{vg}$  values show little variation for the whole range of  $\rho/d$ . They do not agree with the regression of Yamaguchi et al. (2012), which predicts a gradual increase of  $n_{VG}$  and are around 4.7 (Fig. 7.d). These results are rather consistent with the  $n_{vg}$  values of Lombardo et al. (2025) but not with Adachi et al. (2020). The latter reports a correlation of the  $n_{vg}$  with  $\rho/d$ . We thus suggest using a constant value of 4.7 to estimate  $n_{VG}$  for imbibition. For the drainage simulations, values are widely spread that follows the regression of Yamaguchi et al. (2012) based on drainage experiments, not observed in our work and the one of Lombardo et al. (2025). For drainage, our  $n_{vg}$  values are spread and show little correlation with  $\rho/d$ , and do not seem to. Looking at the experimental data, a large spread is also observed in Katsushima et al. (2013), Lombardo et al. (2025), and for the rounded grains samples of Yamaguchi et al. (2012). Our estimated  $n_{vg}$  values overall do not follow the regression of Yamaguchi et al. (2012). The observed scatter could come, in part, from the performance of the fit of the VG model. As seen in Fig. 6, the fitted VG model follows less closely the simulated data from drainage compared to imbibition, especially near the second inflection point. Quality fits at that location are more difficult because the inflection is actually not perfectly well described in the drainage simulations, with a lack of simulated data, compared to imbibition. This roughness in the data for drainage may affect the fit quality, and thus the derived VG parameters. It is however unlikely to fully explain the observed scatter. Another explanation for this scatter could be related to the relationship between  $n_{VG}$  and, although the  $n_{vg}$  values of the melt forms samples are closer to this regression compared

to the other snow types. Let us recall that Yamaguchi et al. (2012) presented a regression based on drainage experiments on sieved melt forms only (black filled dots in Fig. 7.d), which was in good agreement with  $n_{vg}$  values estimated on other samples of natural melt forms (black circles), but in poor agreement for samples of natural rounded grains (black stars), for which  $n_{vg}$  appear to be independent of  $\rho/d$ . Indeed, while Yamaguchi et al. (2012) found a similar correlation between  $n_{VG}$  and  $\rho/d$  for both of the MF sample categories (the samples used for the regression (black disks) and the independent ones (circles)), they did not succeed for the RG samples (stars). Yamaguchi et al. (2012) attributed these different behaviors between snow types samples to differences in pore-size uniformity, which seems coherent with the definition of  $n_{VG}$ . To test this hypothesis  $n_{vg}$ .

To test the hypothesis that  $n_{vg}$  could depend on the pore-size uniformity, we characterized pore size variations by using the distributions of mean curvature of the snow ~~microstruture~~ microstructure, which is a standard parameter used for snow (e.g. see Lesaffre et al., 1998; Brzoska et al., 1999). The mean curvature was calculated at each point on the surface of the 3D images and represented as a statistical distribution (see Flin et al. (2004, 2005), Calonne et al. (2014a), Bouvet et al. (2022) and the supplement for additional information). Figure 8 displays  ~~$n_{VG}$~~  shows  $n_{vg}$  as a function of the standard deviation interquartile range of the mean curvature  $\sigma_{MC}$  for ~~IQR<sub>MC</sub>~~ of each 3D snow image for both imbibition and drainage, thus describing the relationship between  $n_{vg}$  and the mean curvature uniformity for both processes. A trend can be observed, in the form of an inverse function between  ~~$n_{VG}$~~  and  $\sigma_{MC}$ . Lower values of  $\sigma_{MC}$  so that low IQR<sub>MC</sub> values tend to be correlated to larger values of  ~~$n_{VG}$~~ , which large  $n_{vg}$  values, following the form of an inverse function. The trend is overall similar for imbibition and drainage, with lower values for imbibition. The observed trend is consistent with the fact that, during drainage, water leaves the pores more or less all at once for snow with rather uniform pore shapesizes, such as melt forms (in red), as showed by very sharp WRCs and modeled by large  $n_{vg}$  values (see Fig. 3.b). The lower limit of IQR<sub>MC</sub> corresponds to a material with uniform pore sizes and higher values of  $n_{vg}$  (step function). On the other extend, for snow types showing large pore shape-variabilities size variability, such as fresh snow (in light green),  ~~$n_{VG}$  is low~~  $n_{vg}$  values are smaller and the resulting WRC shows a smoother transition as a function of the water content, i.e. the drainage is more gradual. The same considerations apply to imbibition, while the impact is less prominent, due to the rather large snow porosity (at least greater than 0.4 for each sample of our dataset). The imbibition is thus less dependent on the local microstructure than the drainage process. In conclusion, to estimate  ~~$n_{VG}$  for drainage~~  $n_{vg}$ , our results do not support the use of the  $\rho/d$  ratio but rather of more refined parameters, such as the mean curvature distribution (see Fig. 8 and Table 2 for the detailed regression proposed). This can be seen as a limitation for larger-scale modeling as this parameter can currently only be derived from 3D images.

### $\theta_w^r$ parameter:

The last parameter required for our use of the VG model is the residual water content  $\theta_w^r$ . As already mentioned, this parameter was set to 0 for imbibition as simulations were performed on fully dry snow images, and was only determined for drainage as the minimum value of water content reached during the drainage simulations. Two distinct groups are observed (Fig 7e,e). Samples with density below  $450 \text{ kg m}^{-3}$  show values centered around 0.046, which slightly increase with density from about 0.036 to 0.061. Samples of melt forms with density above  $450 \text{ kg m}^{-3}$  show smaller values around 0.029, including the NH2 and NH5 samples. This division is probably due to the fact that the denser snow samples, composed of MF grown under

500 conditions of liquid water saturation, show large pores which can hold little water by capillarity. All the values of  $\theta_w^r$  from our simulations are larger than the value of 0.02 proposed in Yamaguchi et al. (2012). Following our results, we propose to approximate  $\theta_w^r$  by two constant values depending on the snow type: 0.029 for melt forms and 0.046 for the other snow types (Fig. 7 and Table 2).

### $\theta_w^s$ parameter:

505 To complete the picture, it seems worth discussing the saturated water content  $\theta_w^s$ . This parameter is here approximated by the snow porosity  $\phi$ , as opposed to observations from the snow ~~drainage and imbibition experiments (Yamaguchi et al., 2012; Adachi et al., 2020)~~ which found  $\theta_w^s \approx 0.9\phi$  imbibition and drainage experiments (Yamaguchi et al., 2012; Katsushima et al., 2013; Adachi et al., 2020) which report  $\theta_w^s$  ranging from  $0.6\phi$  to  $0.9\phi$  with the experimental challenge of filling complex geometries with melt and freezing processes occurring during the imbibition. In porous media such as soils and sands, imbibition and drainage experiments show similar results, with  $\theta_w^s$  up to 30% of the porosity. Those also show a large range between  $0.3\phi$  to around  $\phi$ . Low values of  $\theta_w^s$  could either be are either seen as underestimated due to the experimental limits linked to the challenge of filling complex geometries, or could reflect as reflecting the real physical processes at stake (e.g., Clayton, 1999; Cho et al., 2022). Therefore, two approaches are commonly used in the literature, either the approximation  $\theta_w^s = \phi$  is taken or  $\theta_w^s$  is adjusted to experimental data. The experiments of Likos et al. (2014) and Farooq et al. (2024) showed that having  $\theta_w^s$  smaller than  $\phi$  generally implies greater  $\alpha_{VG}$   $\alpha_{vg}$  values, but has no significant impact on the  $n_{VG}$   $n_{vg}$  values. For snow purposes, the actual values of the saturated water content still remain unclear and should be further investigated to refine the VG parameters.

### Conclusion on the VG parameters:

The regressions of Yamaguchi et al. (2012) are in good agreement with our simulations for  $\alpha_{VG}$   $\alpha_{vg}$ , especially for drainage, but not for  $n_{VG}$   $n_{vg}$  for both imbibition and drainage, and not for  $\theta_w^r$ . We recall that the regressions of Yamaguchi et al. (2012) are based on drainage experiments only and realized on a limited number of snow types, mainly composed of dense MF, which may explain some of the observed discrepancies. For  $\alpha_{VG}$   $\alpha_{vg}$ , we proposed new regressions based on  $\rho/d$  for both imbibition and drainage, for a wide range of  $\rho/d$  values. For  $n_{VG}$   $n_{vg}$ , a constant value was suggested for imbibition, but no estimates for drainage could be proposed using  $\rho/d$ . A parameter that captures the pore size distribution of snow such as the standard deviation interquartile range of the mean curvature seems to be required. For  $\theta_w^r$ , two mean values were provided depending on the snow type for drainage.

~~Finally, we should also recall that comparing results from experiments and from our numerical simulations is not straightforward. As already mentioned, in the experiments, the snow microstructure can evolve rapidly when in contact with liquid water, whereas in the simulations, the ice skeleton is fixed and defined by the provided tomography image, always remaining in its initial stage. This could also explain some differences in the WRCs.~~

530 **3.1.4 Evaluation Application of the proposed different VG model models on two distinct representative snow samples**

	$\alpha_{VG}-\alpha_{vg} \text{ (m}^{-1}\text{)}$	$n_{VG}-n_{vg} \text{ (-)}$	
Daanen and Nieber (2009)	$30 \times d + 12$	$0.8 \times d + 3$	
Yamaguchi et al. (2010)	$7.3 \times d + 1.9$	$-3.3 \times d + 14.4$	
Yamaguchi et al. (2012)	$4.4 \times 10^6 \times (\rho/d)^{-0.98}$	$1 + 2.7 \times 10^{-3} \times (\rho/d)^{0.61}$	
This work, imbibition	$8.9 \times 10^4 \times (\rho/d)^{-0.66} 4.4 \times 10^6 \times (\rho/d)^{-0.95 \pm 0.003}$	$4.7(3.8 \pm 0.3) + (3.6 \pm 1.7)/IQR_{MC}$	
This work, drainage	$4.0 \times 10^4 \times (\rho/d)^{-0.61} 4.4 \times 10^6 \times (\rho/d)^{-0.97 \pm 0.003}$	$7 + 21/\sigma_{MC} (6.4 \pm 1.6) + (25.6 \pm 10.3)/IQR_{MC}$	$0.046$

**Table 2.** Regressions of the shape parameters of the VG model parameters  $\alpha_{VG}-\alpha_{vg}$ ,  $n_{VG}-n_{vg}$ , and  $\theta_w^r$  proposed by Daanen and Nieber (2009), Yamaguchi et al. (2010), Yamaguchi et al. (2012) and from this work for both imbibition and drainage. For the derived regressions, standard deviations are given for each coefficient.

Here, we compare our results with the different VG models suggested for snow in the literature (see Table 2). All these studies present estimates of the VG parameters  $\alpha_{VG}$ ,  $n_{VG}$ , and  $\theta_w^r$  based on microstructural properties, which allows using the VG model for any snow sample providing the knowledge of such properties. In other terms, the regressions of Table 2 are of particular interest for the practical determination of snow WRCs.

535 Here, we applied different VG models to predict the WRCs for drainage and imbibition for two imaginary snow samples. The properties of those samples have been chosen to be representative of melt forms (sample 1:  $d = 1.5 \text{ mm}$ ,  $\rho = 450 \text{ kg m}^{-3}$ ,  $IQR_{MC} = 5 \text{ mm}^{-1}$ ) and precipitation particles (sample 2:  $d = 0.1 \text{ mm}$ ,  $\rho = 130 \text{ kg m}^{-3}$ ,  $IQR_{MC} = 15 \text{ mm}^{-1}$ ). We present predictions based on the regressions of the shape parameters  $\alpha_{vg}$ ,  $n_{vg}$ , and  $\theta_w^r$  from Yamaguchi et al. (2010), Yamaguchi et al. (2012), Daanen and Nieber (2009), and from this study for both imbibition and drainage. These regressions of the shape parameters are provided in Table 2. Daanen and Nieber (2009) and Yamaguchi et al. (2010) present a regression of  $\alpha_{VG}$  and  $n_{VG}$   $\alpha_{vg}$  and  $n_{vg}$  based on grain size, while Yamaguchi et al. (2012) include both grain size and snow density, using the variable  $\rho/d$ , with  $d$  the mean grain diameter. In all these studies, which are based on drainage measurements, a constant value of  $\theta_w^r \neq 0$  is proposed. In the present study, which is based on both imbibition and drainage simulations, we propose specific parameterizations depending on imbibition or drainage. Overall, the resulting WRCs provide accurate results on all our dataset. To illustrate this effect in more detail, we focused on four images whose VG parameters are the farthest from the provided regressions, namely E2B, for parameter  $\alpha_{VG}$  in imbibition; P11 for  $\theta_w^r$  in imbibition; grad3 for  $n_{VG}$  in imbibition and I23 for  $n_{VG}$  in drainage (see Fig. 7, Fig. 8 and related supplement tables) and plotted the simulated WRCs as well as their different VG models (Fig. ??). While the four images correspond to the worst cases for the regressions proposed in this work, our VG model still behaves very well as compared to We recall that the latter three regressions were developed based on drainage measurements only.

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550 Figure 9 presents the WRCs for the two representative samples for the different VG models. These curves are expressed as a function of the effective saturation (e.g., Mualem, 1976; d'Amboise et al., 2017), defined as:

$$S_e = \frac{\theta_w - \theta_w^r}{\theta_w^s - \theta_w^r}. \quad (10)$$

Our VG models are closer to the ~~the other models of the literature~~ VG model of Yamaguchi et al. (2012) for both samples, the one of Yamaguchi et al. (2010) and Daanen and Nieber (2009) being consistently above and below our estimates, respectively. Besides, the difference of WRCs estimated for imbibition and drainage on the same sample is lower than the difference between models. A larger spread of the curves can be observed for sample 2, which tends to show the highest sensitivity of the VG shape parameters for low-density and small-grain snow.

### 3.2 Effective wet snow transport properties

Unsaturated effective properties were computed on the simulated 3D images of wet snow obtained for different stages of the drainage simulations. We present here the results for the effective water permeability  $K_w^u$ , the effective thermal conductivity  $k^u$ , and the effective water vapor diffusivity  $D^u$  (see Sec. 2.5). ~~These properties are classically expressed as a function of the effective saturation (e.g., Mualem, 1976; d'Amboise et al., 2017), defined as:~~

$$S_e = \frac{\theta_w - \theta_w^r}{\theta_w^s - \theta_w^r}.$$

#### 3.2.1 Water permeability and hydraulic conductivity

565 First, we study the effective water permeability  $K_w^u$ . To compare all the samples together, we use the relative water permeability  $K_w^r(\theta_w) = K_w^u(\theta_w)/K$ , with  $K$  the intrinsic permeability of the saturated media. In all our computations, the values of  $K$  used are the numerical estimations from Calonne et al. (2012). The evolution of the relative permeability with the effective saturation is shown in Fig. 10. ~~and indicates. The relationship describes an exponential increase. The evolution of all the snow samples seems, which tends, for all samples, to merge into a single curve, which indicates. This shows~~ that the water permeability is ~~primarily at first order~~ driven by the water content and the snow density ~~and is little, or to a way lesser extent, influenced by the morphology of the snow microstructure, and that other dependencies with other microstructural parameters are, if any, of lesser strength.~~ We compare our results with the van Genuchten-Mualem (VGM) model of relative water permeability Mualem (1976), which writes as:

$$K_w^u(S_e) = K \times S_e^{1/2} \left( 1 - (1 - S_e^{1/m_{VG}})^{m_{VG} m_{VG}} \right)^2 \quad (11)$$

$$575 \text{ with } S_e(h) = \left( 1 + (\alpha_{VG} h)^{n_{VG}} \right)^{-m_{VG} - m_{VG}} \quad (12)$$

where the ~~VG parameters are the ones defined in Eq. 9 from~~ shape parameters  $n_{VG}$ ,  $\alpha_{VG}$ ,  $m_{VG} = 1 - 1/n_{VG}$  and  $\tau_{VG} = 1/2$  are ~~the parameters of~~ the VG model (van Genuchten, 1980) ~~and where Eq. (5) is from Mualem (1976). The VG parameters used here are the ones from the fits of the drainage WRCs (Fig. 6.b) defined in Eq. 2. Here, we used the shape parameters fitted to the~~

WRC of each image. For the 5 reference snow samples, good agreements are found between the VGM model and our numerical  
 580 computations (Fig. 10.b). ~~Unsaturated hydraulic conductivity  $\mathbb{K}_w^u$  as a function of the effective saturation for the whole set of  
 snow samples (a) and for the 5 selected ones (b). The dry density of the snow samples is represented by the colorbar. The VGM  
 model, using the values of intrinsic permeability from Calonne et al. (2012) and the fitted VG parameters on each snow sample  
 is shown with the solid lines. The VGM model, using the values of intrinsic permeability from Calonne et al. (2012) and the  
 regression from Yamaguchi et al. (2012) is shown with the dashed lines.~~

585 From the relative water permeability, the unsaturated hydraulic conductivity  $\mathbb{K}_w^u$  can be obtained as (see Sec. 2.5):

$$\mathbb{K}_w^u(\theta_w) = \mathbb{K}_w^{\text{sat}} \times K_w^r(\theta_w) \quad (13)$$

$$\text{with } \mathbb{K}_w^{\text{sat}} = K \rho_w g / \mu_w \quad (14)$$

$K$  depends on the snow microstructure and is estimated using the parameterization of Calonne et al. (2012) based on the density  
 and the spherical equivalent radius of snow. Figure 11 presents the unsaturated hydraulic conductivity  $\mathbb{K}_w^u$  as a function of the  
 590 effective saturation, showing an a non-linear increase with increasing saturation. As  $\mathbb{K}_w^u$  contains a  $K$  factor as compared  
 to the relative permeability  $K_w^r$ , an effect of the microstructure can now here be observed: for a given water saturation,  
 $\mathbb{K}_w^u$  varies with density, so that a lighter snow shows a larger  $\mathbb{K}_w^u$  values, and inversely. ~~Again, estimates of  $\mathbb{K}_w^u$  using This  
 time, the VGM model combined is used with the shape parameter estimated from the regressions proposed in this study  
 for drainage (given in Table 2). Combined with the parameterization of Calonne et al. (2012) and the fitted VG parameters  
 on each snow sample show overall good agreements~~, they provide estimates of the unsaturated hydraulic conductivity that  
 are overall in good agreement with the computed data. ~~For a sake of comparison,  $\mathbb{K}_w^u$  using The predictions of the unsaturated  
 hydraulic conductivity from the VGM model using the shape parameters from Yamaguchi et al. (2012) (Table 2) are also shown  
 (dashed lines). Both VGM models are overall fairly close. For the melt forms samples, a slight improvement is found using  
 our regressions, with MAE values around 10% for Yamaguchi et al. (2012) and around 9% for our model. These similarities  
 can be related to the fact that, on one hand, the regression of Yamaguchi et al. (2012) provides slightly better estimates of  
 $\alpha_{\text{VG}}$  for this snow type, compared to our regression (Fig. 7.b); on the other hand,  $n_{\text{VG}}$  values are better estimates from our  
 regression (Fig. 7.d and Fig. 8). For the other snow types, the VGM model using our shape parameter estimates provides better  
 predictions of unsaturated hydraulic conductivity, showing MAE values around 15% compared to the VGM model with  $\alpha_{\text{VG}}$ ,  
 $n_{\text{VG}}$  and  $\theta_r$  from Yamaguchi et al. (2012) is shown with the dashed lines. Overall, the difference between the three using the  
 605 estimates of Yamaguchi et al. (2012), showing MAEs around 22%. Indeed, for all the snow types excluding melt forms, both  
 shape parameters  $\alpha_{\text{VG}}$  and  $n_{\text{VG}}$  are overall better estimated using our regression (as optimized to best match our numerical  
 simulations). This highlights the advantage of considering a large diversity of snow to develop the regressions of the shape  
 parameter, so that the regressions can be applied more widely. However, we point out that the VGM model based on the shape  
 parameter estimates of Yamaguchi et al. (2012), which only required the knowledge of density and grain size, still allows for  
 610 fair estimates of the ~~hydraulic conductivity is small, especially for high saturations, unsaturated hydraulic conductivity, with  
 MAEs ranging from 10 to 20%, when compared to the simulations on our five samples.~~~~

### 3.2.2 Thermal conductivity

The unsaturated effective thermal conductivity of wet snow  $k^u$ , which accounts for heat conduction in the ice, air and liquid water, is presented for different saturation levels in Fig. 12. As expected, thermal conductivity increases with increasing saturation, as liquid water conducts better than air. Values of thermal conductivity of fully saturated snow are increased of about 0.5 to 0.6 W m<sup>-1</sup> K<sup>-1</sup> compared to the ones of dry snow, which means multiplying by 6 the thermal conductivity of a fresh snow sample or by 2 that of a melt form sample. The major impact of snow density is also shown, as already reported for dry snow (e.g., Sturm et al., 1997; Calonne et al., 2011). Density also influences the steepness of the linear conductivity-saturation relationship, such as dense snow shows less steep slopes than light snow. Indeed, the pore space available for a conductivity gain due to an increase of water content is smaller for denser snow. To represent the evolution of thermal conductivity with both density and liquid water content, we propose the following regression based on our data:

$$k^u(\rho, \theta_w) = k_{\text{Calonne}}^{\text{dry}}(\rho) + \theta_w (1.68 \times 10^{-3} \rho + k_w - k_a) \quad (15)$$

$$\text{with } k_{\text{Calonne}}^{\text{dry}}(\rho) = k_a + \rho (2.5 \times 10^{-6} \rho - 1.23 \times 10^{-4}) \quad (16)$$

where  $k_{\text{Calonne}}^{\text{dry}}(\rho)$  is the parameterization of thermal conductivity for dry snow of Calonne et al. (2011) and  $\rho$  is the dry snow density. The choice of the regression form was motivated by a concern for simplicity and to respect the two extreme cases: a volume fully made of liquid water ( $\rho = 0$  kg m<sup>-3</sup> and  $\theta_w = 1$ ) and a volume fully made of air ( $\rho = 0$  kg m<sup>-3</sup> and  $\theta_w = 0$ ). The case of a volume fully made of ice is not considered, as the regression of Calonne et al. (2011) is only valid for densities corresponding to snow ( $\rho \leq 550$  kg m<sup>-3</sup>). As illustrated in Fig. 12.a and Fig. 12.b, the proposed regression reproduces well the computed data, ~~yet slightly underestimating the values for the densest samples.~~ In Fig. 12, the self-consistent (SC) estimate of thermal conductivity for a 3-phases composite aggregate of spherical inclusions (Torquato, 2005) is included for comparison. This analytical model is derived by solving the polynomial expression

$$\sum_{\alpha=\{i,a,w\}} \theta_{\alpha} \frac{k_{\alpha} - k^u}{k_{\alpha} + 2k^u} = 0 \quad (17)$$

where  $\theta_{\alpha}$  and  $k_{\alpha}$  are the volume fraction and thermal conductivity of the phase  $\alpha$  respectively. The SC model prediction is consistent with the numerical values computed on the 3D images.

For several models of the literature, the thermal conductivity of snow as a function of the water content is needed, including snowpack models, such as the Crocus model (Vionnet et al., 2012) (Vionnet et al., 2012; Lafaysse et al., 2025), or detailed models of wet snow processes, such as the one of Leroux and Pomeroy (2017). They often rely on simple approximations to include the effect of water on the effective snow thermal conductivity. In the current version of Crocus, the thermal conductivity estimate from Yen (1981)  $k_{\text{Yen1981}}$ , developed for dry snow, is applied to wet snow by simply accounting for the volume fraction of both ice and liquid water, as:

$$k_{\text{Crocus}}^u(\rho, \theta_w) = k_{\text{Yen1981}}^{\text{dry}}(\rho + \theta_w \times \rho_w) = k_i \times \left( \frac{\rho + \theta_w \times \rho_w}{\rho_w} \right)^{1.88} \quad (18)$$

The model of Leroux and Pomeroy (2017) relies on a weighted average of the thermal conductivity of water and of dry snow, such as:

$$k_{\text{Leroux2017}}^u(\rho, \theta_w) = k_{\text{Calonne}}^{\text{dry}}(\rho) \times (1 - \theta_w) + k_w \times \theta_w. \quad (19)$$

645 To evaluate these two approaches, Fig. 13 presents a comparison with our data from computations and our proposed regression, for 3 snow samples. Major differences between the estimates are observed for each sample. The approximation used in Crocus largely overestimates the thermal conductivity of wet snow, up to a factor 3 for the case of light snow at full saturation. The formulation of Leroux and Pomeroy (2017) leads overall to large underestimations compared to our data. This comparison shows that the water distribution in the pore space plays an important role on the thermal conductivity of wet snow and that  
650 considering the bulk water content only is not sufficient. This motivates further studies to improve the modeling of wet snow conductivity and test the regression proposed here.

### 3.2.3 Water vapor diffusivity

Extending the description of water vapor diffusion of Calonne et al. (2014b) and Bouvet et al. (2024) to the wet snow problem using a similar method as for the heat transport and the liquid water flow of Moure et al. (2023), would involve the effective  
655 unsaturated water vapor diffusion  $D^u$ . As done for the water permeability, the unsaturated diffusivity  $D^u$  is normalized, here by the value of the dry effective vapor diffusivity  $D$  using the numerical estimate from Calonne et al. (2014b). ~~Values of the closed pore water content  $\theta_w^{\text{CP}}$  as a function of the snow porosity (circles). The colors represent the snow types. The saturation line  $\theta_w = \phi$  is represented in gray and the proposed fit is shown in blue.~~

Figure 14 shows  $D^u/D$  as a function of  ~~$\theta_w/\phi$~~  the water saturation  $S_w \equiv \theta_w/\phi$ , with  $\phi$  the porosity of dry snow. ~~The term  $\theta_w/\phi$  provides the proportion of the pore space occupied by liquid water.~~ Very similar evolutions of the relative unsaturated water vapor diffusivity with the term  $\theta_w/\phi$  are shown by all the samples, which highlights the strong correlation of the unsaturated diffusivity with both porosity and liquid water content. Diffusivity decreases exponentially as the proportion of liquid water increases, and for a given proportion of liquid water, diffusivity is higher for ~~low density~~ low-density snow, and inversely. Looking now at the maximum value of  $\theta_w/\phi$  reached for each sample, we observe that this value varies, as suggested  
665 by Fig. 14.b. The domain of definition of  $D^u$  as a function of  $\theta_w/\phi$  is determined by the value of water content at which the pore space is obstructed by liquid water, and so when  $D^u$  can no longer be estimated. This specific value of water content is referred to here as the closed pore water content  $\theta_w^{\text{CP}}$ . It is similar to the close-off density at which the air can no longer diffuse in the pore space, as used in Calonne et al. (2022) for dry snow and firn samples. Values of  $\theta_w^{\text{CP}}$  for all our snow samples are presented as a function of the dry porosity in Fig. 15. The closed pore water content  $\theta_w^{\text{CP}}$  is significantly smaller than the  
670 dry porosity, in-on average 17% smaller. ~~For comparison, the saturated water content  $\theta_w^s$  is in average only 0.03% smaller than the porosity (see section 2.4).~~ This means that vapor diffusion in wet snow is no longer possible long before saturation is reached.  $\theta_w^{\text{CP}}$  increases with increasing porosity (decreasing density), but does not seem specifically impacted by the type of

snow considered. A fit is estimated here as  $\theta_w^{\text{CP}} = \phi - 0.17$  (in blue):

$$\theta_w^{\text{CP}} = \phi - 0.17 \quad (20)$$

675 shown in black in Fig. 15).

Based on our data, a simple regression is proposed to estimate  $D^u$  from the liquid water content and the dry snow porosity:

$$D^u(\phi, \theta_w) = \begin{cases} D_{\text{SC}} \times \left(1 - \frac{\theta_w}{\phi}\right)^3 = D_{\text{SC}} \times \left(1 - \frac{\theta_w}{1 - \rho/\rho_i}\right)^3 & \theta_w \leq \theta_w^{\text{CP}} \\ 0 & \theta_w > \theta_w^{\text{CP}} \end{cases} \quad (21)$$

$$D_{\text{SC}} = D_v(3\phi - 1)/2 \quad (22)$$

$$\theta_w^{\text{CP}} = \phi - 0.17 \quad (23)$$

680 with  $D_{\text{SC}}$  the self consistent estimate of effective vapor diffusivity as used in Calonne et al. (2014b). ~~Estimates with this proposed regression are~~ This relationship follows the general form of estimates of  $D^u/D_{\text{dry}} = (1 - \theta_w/\phi)^a$  classically used for soils (Kristensen et al., 2010) with  $a = 10/3$  in Millington and Quirk (1961) and  $a = 5/2$  in Moldrup et al. (2000). The regression is shown in Fig. 14 by the black line. ~~This regression is a first suggestion that could be improved by refining the effect of the density, as it predicts too high values for the denser snow samples compared to our simulations and too low values for the lighter snow samples (left panel of Fig. 14). The values of  $\theta_w^{\text{CP}}$  could also be refined by calculating  $D_u^{\text{eff}}$  for more values of saturation,~~ along with the soil models of Millington and Quirk (1961) and Moldrup et al. (2000) that are displayed with gray dotted and dashed lines. The latter two models fairly represent the behavior of  $D^u$  and bound the regression for snow with higher and lower values. Both are used for predicting  $D^u$  in structureless natural soils.

685

### 3.2.4 Sum up on the effective properties

690 The analysis of the effective properties based on the images obtained from the drainage simulations showed interesting results. As already mentioned, it should be kept in mind that many of these images are likely far from natural snow microstructure as the ice structure remains unchanged in the presence of liquid water in the simulations. Yet, they offer new insights on how the liquid water distribution in snow could influence the effective transport properties, for a variety of microstructures. It also enables a comparison between the current parameterizations of the transport properties, and suggestions of new estimates.

695 Especially, we showed that the classic VGM parameterization for the relative water permeability and the unsaturated hydraulic conductivity reproduces well our simulated data and seems thus a good choice for both properties. For the effective unsaturated vapor diffusivity and the effective unsaturated thermal conductivity, we proposed new regressions, which, for the conductivity, differs strongly from some of the current parameterizations used in snow modeling. Finally, as all the above effective properties were ~~only~~ computed on images from drainage simulations only, it could be interesting to compare those estimates to the

700 imbibition case.

## 4 ~~Conclusion~~ Main limitations

~~In this study, a quasi-static model~~The proposed study present different limitations:

- In the present work, the Pore Morphology Method (PMM) has been used. This method is valid in quasi static regime, and when capillary forces dominate in comparison to gravity and viscous forces. Dynamic effects than can occur in practice can not be captured as in two phase flow simulations (Vogel et al., 2005; Ahrenholz et al., 2008; Bhatta et al., 2024; Prodanović and I  
705 or using a Pore Network Model (PNM) (Vogel et al., 2005; Joekar-Niasar and Hassanizadeh, 2012; Xiong et al., 2016) . Despite such limitations, the PMM provides good estimations of the WRCs for porous media whose wetting phase shows generally spherical menisci, which is the case for snow (Hilpert and Miller, 2001; Vogel et al., 2005).
- As it has been already underlined, the boundary conditions applied on the four sides of the 3D images not linked  
710 to the WP or NWP reservoir may play an important role for the residuals after drainage or imbibition. While little influence was reported for the case of drainage, the amount of residual air (or entrapped air) during imbibition may vary significantly depending on the chosen boundary conditions (see Sec. 2.3). This point, which concerns all the methods (PMM, PNM, two-phases flow simulations) to describe two-phase flows, has been little discussed in the literature, to the best of our knowledge, except in Galindo-Torres et al. (2016) and Zhang et al. (2025) in the case of lattice Boltzmann  
715 simulations. For our snow samples, preliminary tests showed that the maximum water saturation ( $\theta_w^s$ ) ranges from 45% to 90% of the porosity, depending on the applied boundary conditions (symmetry, wall or displaced fluid outlet). More generally, knowledge on the residual air in snow seems limited. Estimates based on measurements remain an experimental challenge and show large differences (Yamaguchi et al., 2012; Katsushima et al., 2013; Adachi et al., 2020) . Further work would be required to validate the proposed approach through refined comparisons with experimental data,  
720 for example, with imbibition experiments that combine measurements of the microstructure by X-ray tomography and measurements of liquid water content by neutron radiography (see e.g., Tengattini et al., 2020). At this stage, given the uncertainty, the imbibition curve was simulated assuming that there is no air residuals, as in the Mercury Injection Capillary Pressure experiments (e.g. Hilpert and Miller, 2001; Berg et al., 2016), thus  $\theta_w^s = \phi$ . We evaluated the impact of this simplification on the shape parameters of the VG model. Even with  $\theta_w^s$  as low as  $0.6\phi$ , the shape parameters  
725 remain almost constant, with an effect only visible for  $\alpha_{vg}$ , with differences around 10% of the value compared to the case  $\theta_w^s = \phi$ . This was also reported in the experiments of Likos et al. (2014) and Farooq et al. (2024), which showed that having  $\theta_w^s$  smaller than  $\phi$  generally implies greater  $\alpha_{vg}$  values, but has no significant impact on the  $n_{vg}$  values.
- Since the PMM is applied to 3D images, uncertainties can arise from the size and resolution of the images under  
730 consideration. The effects of both parameters on the results are available in Hilpert and Miller (2001) and Vogel et al. (2005) , and are assumed to be transferable to snow. In the present study, we checked that our snow images correspond to representative elementary volumes. Uncertainties remain regarding the side length of the melt forms images, for which the maximum available sizes were taken, but they still present a limited number of heterogeneities.
- Our simulated WRCs were compared to WRCs measured during experiments of drainage and or imbibition. Such a comparison is not straightforward, as, in the experiments, the snow microstructure can evolve rapidly when in contact

735 with liquid water, whereas, in the simulations, the ice skeleton is fixed and defined by the provided tomography image,  
always remaining in its initial stage. The comparison simulation-experiment was mainly done through the comparison  
of the shape parameter of the VG model derived from the WRCs. Experimental estimates remain, however, limited, often  
focusing either on imbibition or on drainage, or studying only a small range of snow types (Yamaguchi et al., 2010, 2012; Katsushima  
740 . While estimates of  $\alpha_{vg}$  are rather consistent between all the measurements and our simulations for both imbibition and  
drainage (Fig. 7.a and b.), estimates of  $n_{vg}$  differ significantly (Fig. 7.c and d.). Hence, it is difficult to conclude on the  
evaluation of our simulations. Again, dedicated studies would be required to provide further experimental data.

– Finally, the uncertainties of the WRCs simulations are not necessarily transferred to the estimates of the effective  
transport properties of wet snow. The simulations provide the 3D skeleton of the air, ice, and liquid water, for which  
the distribution of each phase in space can contain errors, as discussed above. However, only the unsaturated hydraulic  
745 conductivity depends at the first order on both the volumetric fraction and the 3D distribution of the phases. Its predictions  
with the VGM model inherit from the uncertainties on the estimates of the VG parameters used to model the WRCs. In  
contrast, the unsaturated thermal conductivity and water vapor diffusivity of snow depend, at first order, mainly on the  
volumetric fraction of the phases, and the contribution of the phase distribution is secondary.

## 5 Conclusion

750 In this study, a Pore Morphology Model (PMM) was used to simulate the distribution of liquid water in the pore space of  
snow for various water contents. Liquid water ~~is~~ was gradually introduced and then removed by capillarity during wetting  
(imbibition) and drying (drainage) simulations. ~~Such simulations were applied on~~ This model was applied to a set of 34 3D  
tomography images of dry snow presenting various microstructures. ~~Unlike natural snow~~ For each dry snow image, a series  
755 of 3D images of wet snow at different stages of imbibition or drainage was produced. Unlike what happens in nature, the ice  
matrix is fixed and does not evolve with the liquid water in the simulations (no wet snow metamorphism). The simulations  
were performed with a saturation water content set to the porosity value (no air residuals). This work constitutes an exploratory  
numerical work to study (i) ~~to study~~ the water retention curves (WRCs) of snow and (ii) the effective transport properties of  
wet snow ~~and~~, notably how they are influenced by the water distribution at the pore scale. Both points are critical to better  
understand and model water flow, heat, and vapor transport in wet snow (e.g., Leroux and Pomeroy, 2017; Moure et al., 2023)

760 .

The WRCs of snow were derived from the simulated wet snow images, for both imbibition and drainage. ~~From each initial~~  
image of dry snow, two series of 3D images were produced and correspond to different stages of the simulations of imbibition  
or drainage, which were used to derive the corresponding WRCs. ~~We confirmed~~ We confirm the hysteresis of both processes  
in snow and highlighted and highlight the dependency of the WRC on the microstructural features of snow, such as the pore  
765 size and the density snow density and interfacial mean curvature distribution. We then ~~compared our results to reproduced~~  
our WRCs with the model of van Genuchten (1980) ~~, widely used to model the WRCs of porous materials. For that, the~~  
model was fitted to our simulated WRCs by adjusting and derived new values of the model parameters  $\alpha_{VG}$ ,  $n_{VG}$  and  $\theta_w^*$ ,

770 ~~which are the shape parameters  $\alpha_{vg}$  and  $n_{vg}$  and the residual water content  $\theta_w^r$  (the saturated water content  $\theta_w^s$  being set to the porosity value in our study). Comparing with previous estimates of those parameters, we found that the latest regressions from Yamaguchi et al. (2012) provide the closest values to our fitted parameters, with values from previous experimental studies (Yamaguchi et al., 2012; Katsushima et al., 2013; Adachi et al., 2020; Lombardo et al., 2025), we point out an overall fair agreement for  $\alpha_{VG}$  for both imbibition and drainage, but poor agreement for  $n_{VG}$   $\alpha_{vg}$  but significant differences for  $n_{vg}$  and  $\theta_w^r$ . New regressions and estimates of the parameters of the model of van Genuchten (1980) were proposed, even between the experimental literature values themselves. Possible causes of these discrepancies are discussed. We propose new parameterizations of  $\alpha_{vg}$ ,  $n_{vg}$ , and  $\theta_w^r$  for both imbibition and drainage processes, fitting our numerical results optimized for our simulated WRCs. They should contribute to a more generalized use of this model to predict the WRCs of snow, as needed for water flow modeling. Dedicated investigations should however continue to refine those regressions, however, be performed to further evaluate those parameterizations, especially concerning the exact values of the saturated water content  $\theta_w^s$ , which remains an open question. The series-~~

775 ~~Effective transport properties of wet snow images simulating drainage were then used to estimate the effective transport properties of wet snow were then estimated based on the series of images from the drainage simulations. They include the unsaturated hydraulic conductivity, water permeability, thermal conductivity, and water vapor diffusivity. These properties were computed using the Geodict software, solving boundary problems resulting from the homogenization process applied to the heat, vapor, and water transport equations in wet snow (e.g., Leroux and Pomeroy, 2017; Moure et al., 2023). Results allowed to-~~ We describe the relationships of ~~these effective properties with the water content as well as the~~ the transport properties depending on water content and dry snow density. ~~The computed properties were compared to commonly-used parameterizations. Especially, we showed~~ We show that the relative water permeability and the unsaturated hydraulic conductivity can be well reproduced by the VGM model (Mualem, 1976; van Genuchten, 1980), combined with the estimates of intrinsic permeability for dry snow of Calonne et al. (2012). For the effective thermal conductivity ~~of wet snow, large discrepancies were reported when comparing our computations to-~~, we report large discrepancies between our numerical results and some current estimates used in snow models, such as the Crocus model ~~(Vionnet et al., 2012)~~ (Vionnet et al., 2012; Lafaysse et al., 2025) or the model of Leroux and Pomeroy (2017). A new regression depending parametrization based on both snow density and water content was suggested. Finally, a regression was also proposed to predict the water vapor diffusivity of wet snow, based on the dry snow diffusivity estimated with the self-consistent model, the water content, and the snow density. ~~All the above results-~~

780 ~~Results of this study~~ are a first step toward a better understanding of the influence of the micro-scale characterization of the distribution of liquid water ~~on the hydraulic and in the pore space of snow, as well as a better modeling of the~~ physical properties of wet snow. Future studies ~~should will~~ take into account additional processes, ~~neglected in this study-~~ such as the transformation of the microstructure by phase changes and the movement of water by gravity.

. The equations of the boundary problems that were solved to compute the effective wet snow properties are available in the Supplement.  
800 The Supplement also includes detailed presentations of the 34 images used in this study, with property tables, downward and upward views, and mean curvature histograms. Finally, the computed values of imbibition and drainage simulations on our snow samples, the resulting VG parameters, and the numerical estimations of conductivity, water vapor diffusivity, and water permeability are also in the Supplement.

. FF, CG and NC proposed the study. FF and NC acquired and prepared the image dataset. NA, LB and CG conducted the imbibition and drainage numerical simulations. The analyses and interpretations were carried out by LB, NA, CG, NC, and FF. LB and NA prepared the  
805 manuscript with contributions from all co-authors.

. The contact author has declared that none of the authors have any competing interests.

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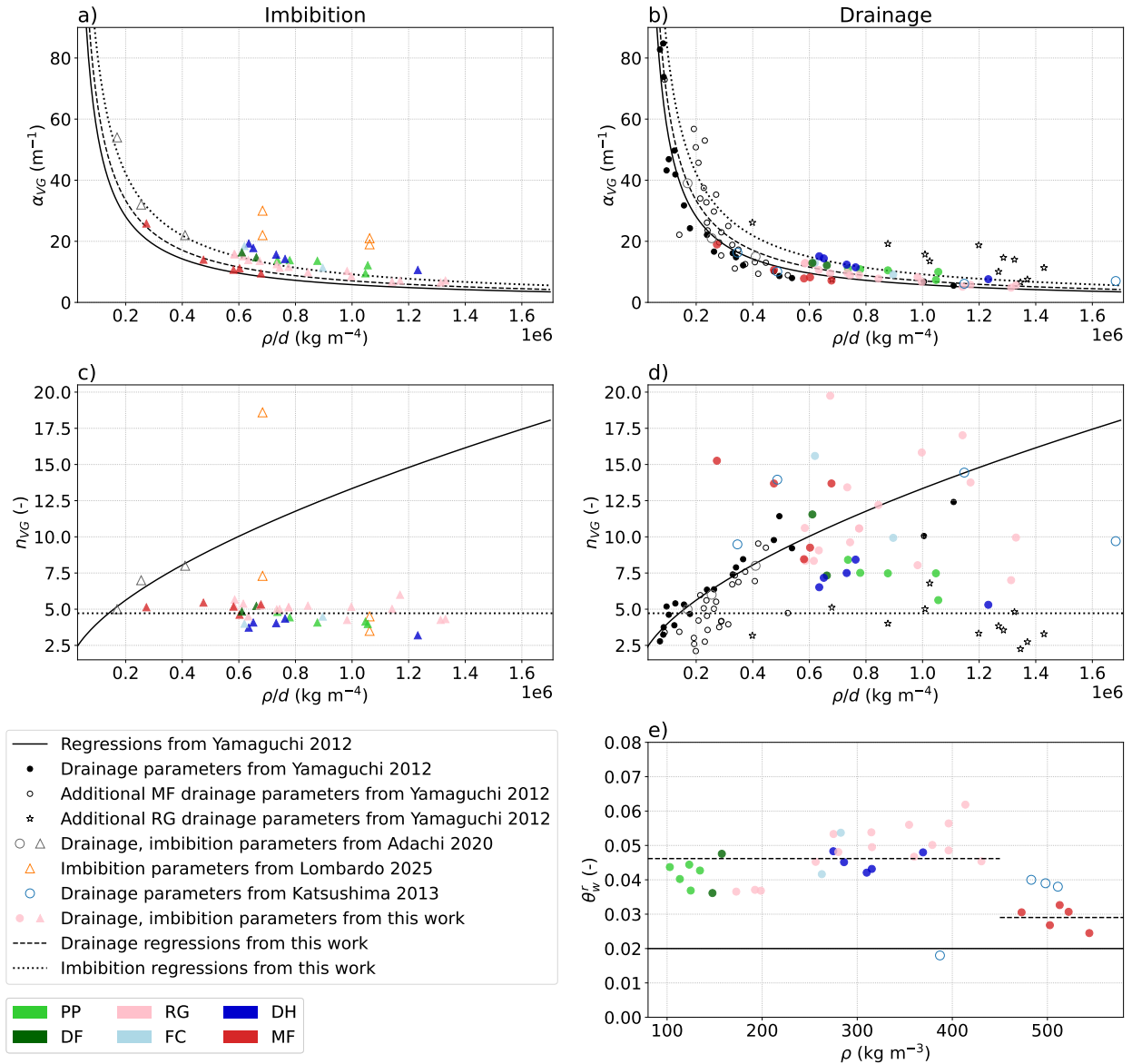
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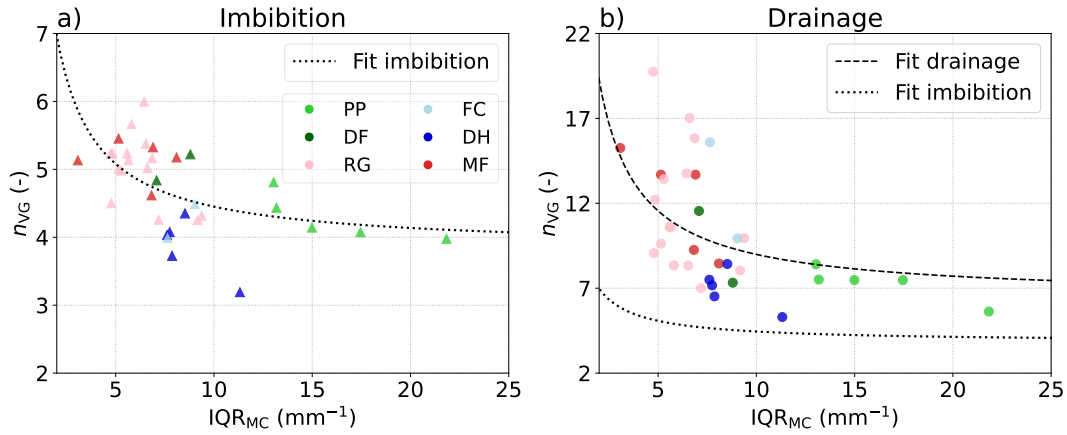
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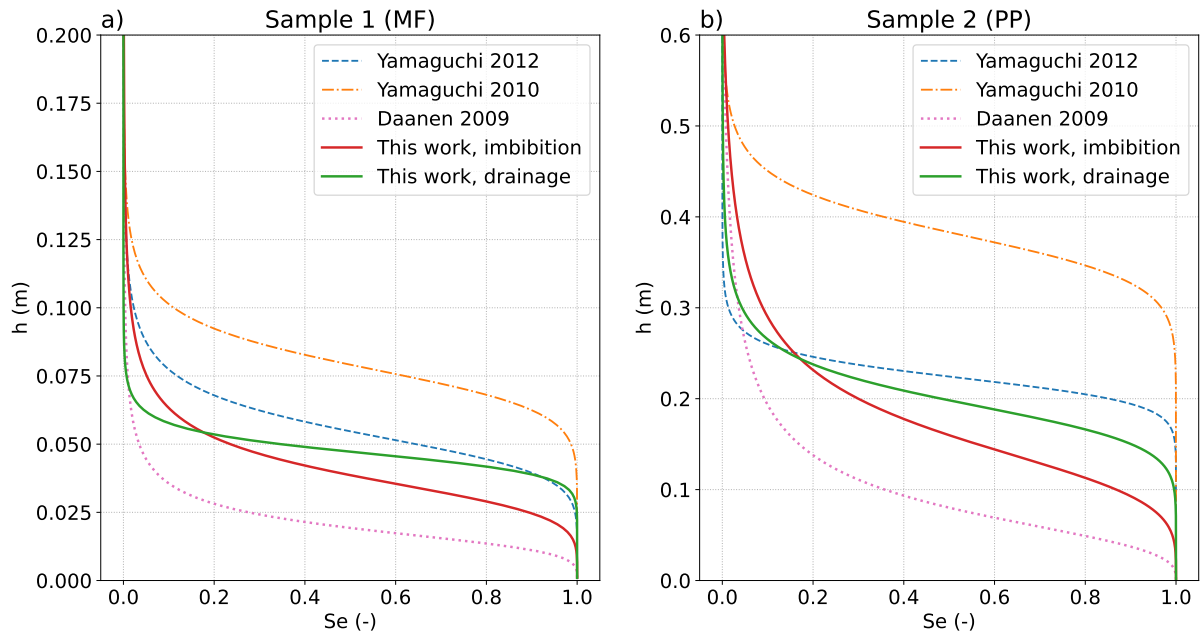
a)  $\alpha_{VG}$ , b)  $n_{VG}$  and c)  $\theta_w^r$  parameters of the VG model as a function of  $\rho/d$  or  $\rho$ . The regressions of Yamaguchi et al. (2012) are shown by black lines, the parameter values from drainage measurements used to deduct those regressions are shown by black disks ("S-samples", composed of refrozen melt forms – see Yamaguchi et al. (2012)). The additional drainage measurements from Yamaguchi et al. (2012) are shown by circles (MF samples) and stars (RG samples). Imbibition and drainage measurements from Adachi et al. (2020) are shown by empty gray markers. From this work, parameters from imbibition and drainage simulations are shown by colored disks and triangles, respectively, the colors showing the snow types. The proposed regressions based on our simulated data are shown by dashed and dotted lines (see Table 2).



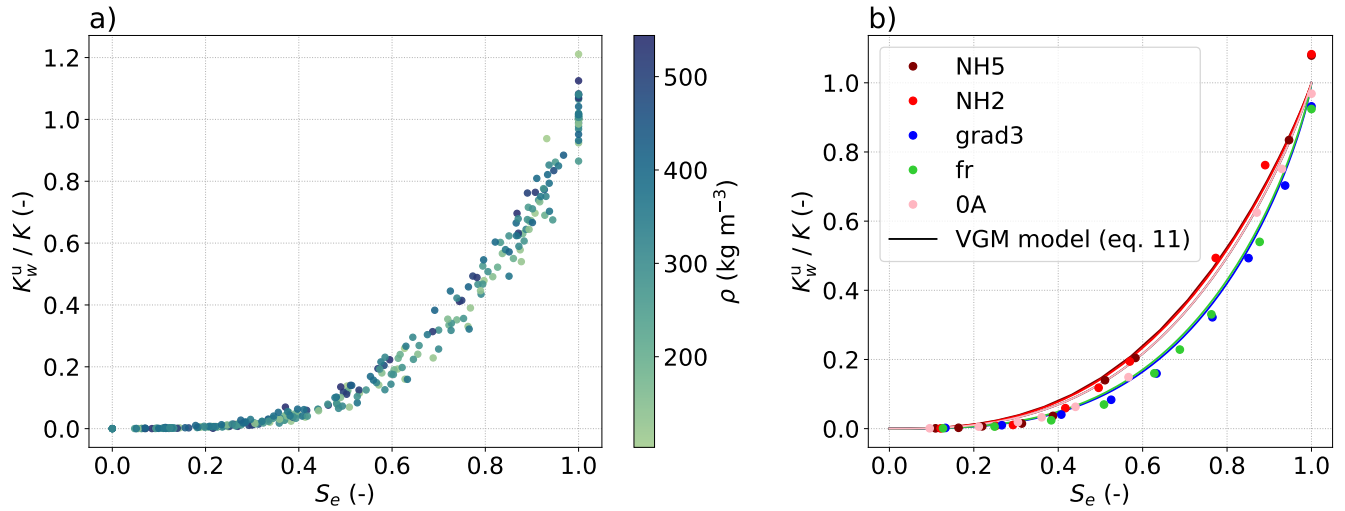
**Figure 7.** a)  $\alpha_{VG}$ , b)  $n_{VG}$  and c)  $\theta_w^r$  parameters of the VG model as a function of  $\rho/d$  or  $\rho$  for imbibition and drainage. The regressions of Yamaguchi et al. (2012) are shown by black lines, the values used to deduct those regressions are shown by black disks ("S-samples", composed of refrozen MF - see Yamaguchi et al. (2012)). The additional drainage measurements from Yamaguchi et al. (2012) are shown by circles (MF samples) and stars (RG samples). Measurements from Adachi et al. (2020) are shown by empty gray markers. Measurements from Lombardo et al. (2025) and Katsushima et al. (2013) are shown by empty orange triangles and blue circles. From this work, parameters from imbibition and drainage simulations are shown by colored disks and triangles, respectively, the colors showing the snow types. The



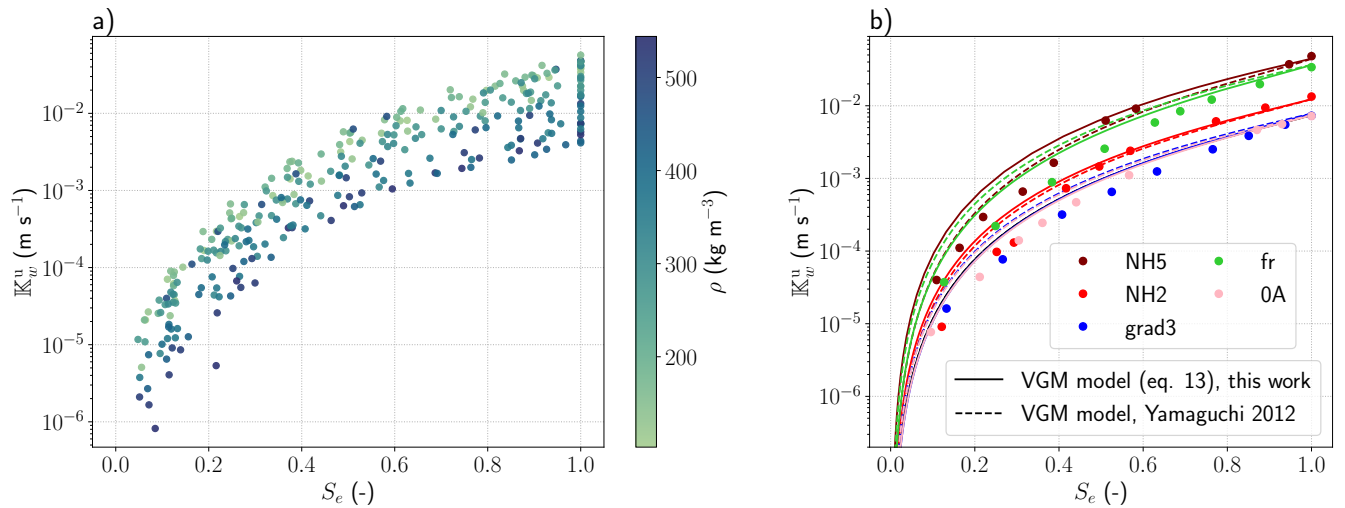
**Figure 8.** The  $n_{VG}$  parameter for drainage as a function of the standard deviation  $\sigma_{MC}$  interquartile range  $IQR_{MC}$  obtained for the mean curvature distribution computed on each 3D dry snow image. Regressions for imbibition and drainage are shown with dotted and dashed lines (see Table 2).



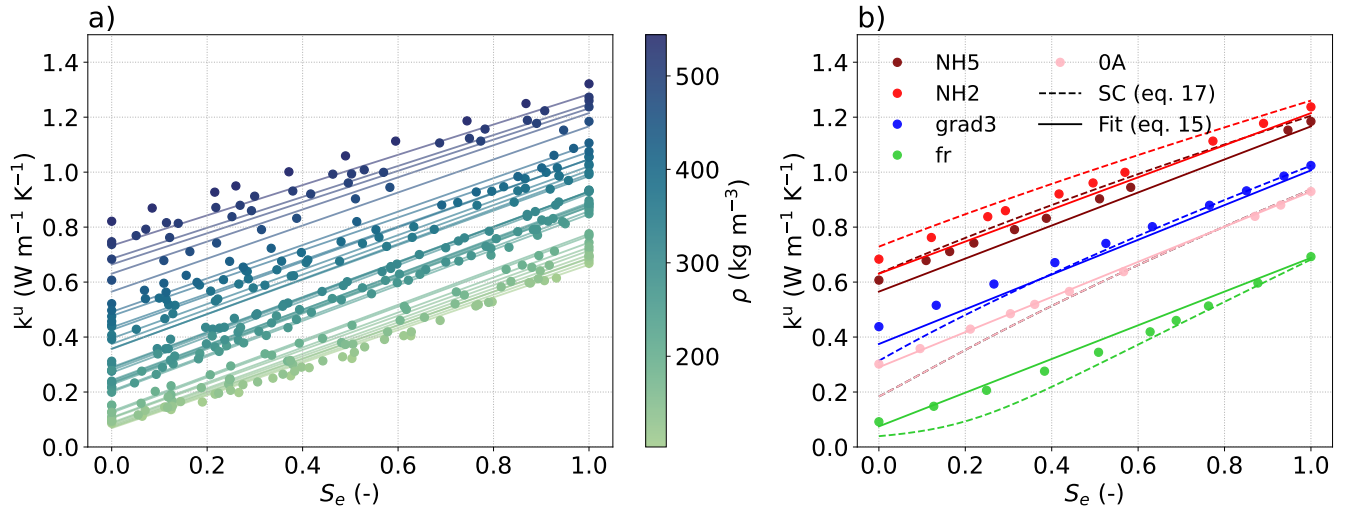
**Figure 9.** Numerical imbibition and drainage WRCs for a set of images whose VG shape parameters are the farthest from the regressions on the WRCs of Table 2 proposed in this work (E2B, for parameter  $\alpha_{VG}$  in imbibition, P11 for  $\theta_w^r$  in imbibition, grad3 for  $n_{VG}$  in 2 representative samples as a function of the effective saturation. The presented VG models are: imbibition and I23 for  $n_{VG}$  in drainage – see Fig. 7, Fig. 8 models from this work and related supplement tables), with the corresponding VG models from Daanen and Nieber (2009), Yamaguchi et al. (2010) and Yamaguchi et al. (2012). The new VG model suggested in this paper is represented with solid lines (see Table 2).



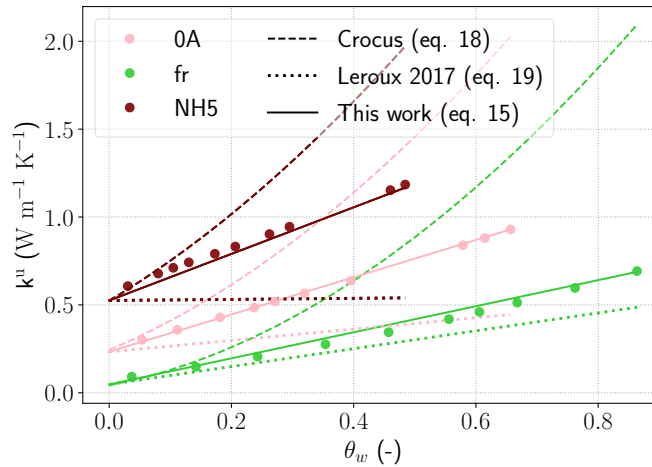
**Figure 10.** Effective relative water permeability as a function of the effective saturation for (a) all the whole set of snow samples, and (ab) and for the 5 reference samples (b). Computations were performed on the snow images from the drainage simulations only. The dry density of the snow samples is represented by the colorbar. The VGM models of relative permeability using the shape parameters fitted on the WRC of each image (provided in the supplementary materials) are shown by the solid lines.



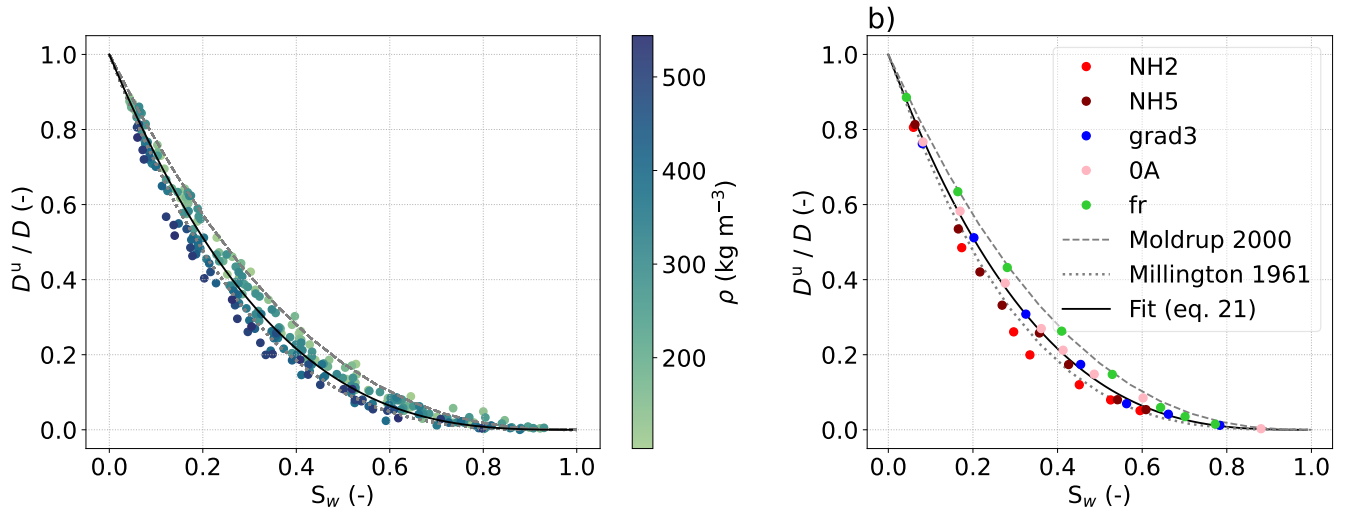
**Figure 11.** Unsaturated hydraulic conductivity  $K_w^u$  as a function of the effective saturation for (a) the whole set of snow samples, and (b) the 5 selected samples. Computations were performed on the snow images from the drainage simulations only. The dry density of the snow samples is represented by the colorbar. The VGM model, using the values of intrinsic permeability from Calonne et al. (2012) and the shape parameters from the regressions of this work given in Table 2, is shown for each sample by solid lines. The VGM model, using the values of intrinsic permeability from Calonne et al. (2012) and the regression from Yamaguchi et al. (2012) is shown with the dashed lines.



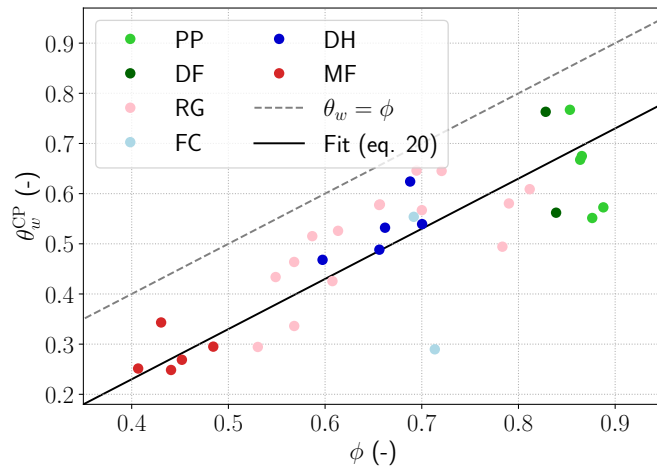
**Figure 12.** Unsaturation thermal conductivity  $k^u$  as a function of the effective saturation for (a) the whole set of snow samples, and (b) and for the 5 selected ones (b) samples. Computations were performed on the snow images from the drainage simulations only. The dry density of the snow samples is represented by the colorbar. The suggested regression is shown by solid lines and the self-consistent estimate for 3 phases is shown by dashed lines.



**Figure 13.** Comparison of the proposed regression of thermal conductivity of dry and wet snow with the regression from Yen (1981) extended for wet snow and the one proposed by Leroux and Pomeroy (2017), for three different snow samples.



**Figure 14.** Relative water vapor diffusivity  $D^u/D$  as a function of  $\theta_w/\phi$  the liquid water saturation  $S_w$  for drainage simulations of (a) the whole set of snow samples and (ab) and for the 5 selected ones (b) samples. The dry density of the snow samples is given by the colorbar. The proposed regression of unsaturated diffusivity is shown by a black solid line, and the models of Millington and Quirk (1961) and Moldrup et al. (2000) are shown with gray dotted and dashed lines respectively.



**Figure 15.** Values of the closed pore water content  $\theta_w^{CP}$  as a function of the snow porosity (circles). The colors represent the snow types. The saturation line  $\theta_w = \phi$  is represented with a dashed gray line and the proposed fit is shown with a solid black line.